The Timing of Choice-Enhancing Policies*

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Abstract

This paper analyzes the welfare consequences of choice-enhancing policies when a fraction of consumers are present-biased and a firm can change own pricing in response to the policies. In the model, a firm automatically enrolls consumers in a service and potentially exploits procrastination on their switching decisions. We show that a conventional choice-enhancing policy—which decreases consumers’ switching cost at the time they are enrolled—can be detrimental to consumer and social welfare. This is because time-consistent consumers are more likely to opt out of the service under the policy, and in response to that, the firm may increase its future prices for the service to exploit unsophisticated present-biased consumers. In contrast, an alternative policy—which decreases consumers’ switching cost when the firm charges a higher price for the service—does not have such a perverse effect of selecting unsophisticated consumers, and hence can increase consumer and social welfare. Our results highlight that the timing of facilitating an active choice matters when firms can respond to a policy.

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1 Introduction

Consumers often exhibit inertia about their product usage. One way for firms to exploit this inertia is to use an automatic enrollment or renewal, i.e., first putting consumers into a service with a free-trial period, and after that charging positive fees to consumers who do not opt out. Indeed, automatic enrollments or renewals to a service are prevalent in some industries. As examples, retail banks often promote a credit card with zero annual fee for the first year. Cell-phone companies offer a long-term contract with an automatic renewal. Also, many providers of an Internet connection in Germany have automatically enrolled consumers to own anti-virus options with some grace period when the consumers subscribe an Internet connection.¹

To protect unsophisticated consumers from such exploitation, policies facilitating consumers’ active choice have been discussed. Recent studies have shown that motivating consumers to make an active choice can increase consumer and social welfare (Carroll, Choi, Laibson, Madrian and Metrick 2009; Keller, Harlam, Loewenstein and Volpp 2011; Chetty, Friedman, Leth-Petersen, Nielsen and Olsen 2014). However, two issues on such policies have been underinvestigated. First, is there any adverse effect when firms can change their pricing in response to a policy? Second, if there are multiple instants of time at which consumers can opt out of a service, when should a policymaker motivate consumers to make an active choice?

This paper analyzes the welfare consequences of policies decreasing consumers’ switching costs when a firm can respond to the policies. In an illustrative version of our model, a firm automatically enrolls consumers to a service. A fraction of consumers are naive present-biased as in O’Donoghue and Rabin (1999a), whereas the rest of the consumers are time-consistent. Each consumer incurs a positive switching cost whenever opting out of the service. When a policymaker employs a policy, the switching cost in that period is reduced but may still be positive.²

We first show that, akin to the existing literature on naive present-bias, the firm may exploit naive consumers by charging a high price for the service. We then show that a policy which has been

¹ As a specific example, Kabel Deutschland, one of the largest providers in Germany, had the following automatic enrollment in 2010. The option included an anti-virus software, a firewall, and a parental control software. It had to be activated in a customer web portal. It was free for the first three months and cost €3.98 per month after that. The option could be canceled with a notice period of four weeks. See http://www.kabel-internet-telefon.de/news/7214-kabel-deutschland-mit-neuem-sicherheitscenter-kabelsicherheit-de (accessed March 1, 2015).

² The literature on active-choice policies has investigated the cases in which a policymaker either forces consumers to make an explicit choice or decreases their switching cost to zero. In contrast, we study the case in which a policy decreases consumers’ switching cost, but the consumers still decide whether or not to switch by themselves and the switching cost is still positive. We discuss interpretations and real-world applications of such a policy in Section 3.2.
used in some industries—decreasing the switching cost at the time consumers are enrolled into the
service—can decrease consumer and social welfare. The main logic is as follows. Consider the case in
which both time-consistent and naive consumers use the firm’s service without any policy. Suppose
that a policymaker decreases the switching cost when consumers are enrolled into the service.
Because naive consumers may procrastinate their switching decision, time-consistent consumers
are more likely to opt out of the firm’s service than naive present-biased consumers. When the
decrease of the switching cost by the policy is not sufficient, only time-consistent consumers may
opt out of the service under the policy. As a result, the firm may increase its prices for the service
in response to the policy to exploit naive consumers who still stay enrolled in the service.

In contrast, we show that an alternative policy—decreasing the switching cost whenever the
firm finishes its free-trial period or increases the price for the service—does not have such a perverse
effect of selecting naive consumers into the service, and hence can increase consumer and social
welfare even when the conventional policy does not work well. Our results indicate that the timing
of such a choice-enhancing policy matters for consumer and social welfare.

We then investigate the model which endogenizes the consumers’ decision of signing up to the
enrollment and in which the firm can possibly charge the prices in multiple periods. In the model, a
monopoly firm sells a base product which is necessary for the usage of the add-on service. Consumers
who decide to purchase the base product are automatically enrolled to the firm’s service. We show
that as the number of payment periods increases, the firm is more likely to exploit naive consumers.
In addition, the total payments of naive consumers are increasing and unbounded. Similarly to the
illustrative model, a conventional policy which decreases the switching cost at the time consumers
are enrolled can decrease both consumer and social welfare, and our alternative policy can increase
consumer and social welfare even in such a case. We also show that the alternative policy is as
effective as a policy which decreases the switching costs in all periods. We also discuss the effects
of imposing a deadline of consumers’ switching decision.

In extensions, we also discuss that how our results are robust to incorporating (i) competition
on the base product, (ii) sophistication and partial naivete in the sense of O’Donoghue and Rabin
(2001), and (iii) heterogeneous valuations for the base product or the additional service.

This paper belongs to the literature on behavioral public policy.\footnote{O’Donoghue and Rabin (2003, 2006) analyze the welfare effects of tax-subsidy policies under present bias and naivete. For the policy analysis on active choice, see Carroll, Choi, Laibson, Madrian and Metrick (2009), Keller,}
related to two theoretical literatures: pricing under naive present-bias and the equilibrium effects of policies. First, the literature in behavioral industrial organization has studied how firms can exploit consumers’ time inconsistency and naivete.4 Building upon the literature, we focus on the policy implications of enhancing an active choice and analyze how the timing of policies can affect consumer and social welfare. Second, recent theoretical and empirical literatures have analyzed the adverse effects of policies when consumers are inattentive.5 To the best of our knowledge, however, the timing of policies and their welfare effects have not been investigated in the literatures. Complementing the literatures, we highlight the adverse welfare effect of a conventional policy, analyze how the timing of policies affects welfare, and suggest an alternative policy which mitigates the adverse welfare effect and hence can improve welfare.

The rest of this paper is organized as follows. Section 2 provides an illustrative model and its analysis. Section 3 sets up the full model, in which consumers’ enrollment is endogenous and they possibly incur multiple payments for a service, and discusses its key assumptions. Section 4 analyzes the model and presents our results. Section 5 discusses extensions of the model. Section 6 concludes. Proofs are provided in the Appendix.

2 Illustrative Model: Setup and Analysis

To highlight our results and underlying intuitions in the simplest manner, this section sets up and analyzes an illustrative model.

2.1 Illustrative Model: Setup

There are one risk-neutral firm and a continuum of measure one of risk-neutral consumers. The firm provides a service which the consumers value at \( a > 0 \) in each \( t = 2, 3 \). A competitive fringe also

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4 See, for examples, DellaVigna and Malmendier (2004), Köszegi (2005), Gottlieb (2008), Heidhues and Köszegi (2010), and Heidhues and Köszegi (2014).

5 For the theoretical literature, see Armstrong, Vickers and Zhou (2009), Armstrong and Chen (2009), Piccione and Spiegler (2012), Grubb (2015), de Clippel, Eliaz and Rozen (2013), Ericson (2014), and Spiegler (2014). For the empirical literature, see Handel (2013), Grubb and Osborne (2015), and Duarte and Hastings (2012). Relatedly, based on Gabaix and Laibson’s (2006) hidden-attribute model, Kosfeld and Schüwer (2011) analyze the effect of increasing the fraction of sophisticated consumers in the market. They show that such an intervention can increase the fraction of consumers who take a costly step to avoid an extra payment and hence can lower welfare. In contrast, we focus on how firms’ equilibrium response to a policy can cause an adverse consequence and suggest an alternative policy when some consumers are naive present-biased.
provides a same-valued service. We normalize the production cost of the service—and hence the price of the competitive fringe—to be zero. Suppose for now that the firm has enrolled consumers into the service; in Section 3, we introduce a full model which endogenizes consumers’ enrollment. The price of the service is \( p^a \geq 0 \) and is charged in \( t = 3 \). In each \( t = 1, 2 \), consumers can switch from the firm to the competitive fringe for the service by incurring a switching cost \( k_t \). The firm offers a free trial (or a grace period) in \( t = 1 \); \( p^a \) is not charged in \( t = 3 \) if consumers opt out of the firm’s service either in \( t = 1 \) or in \( t = 2 \). At the beginning of the game, the policymaker decides whether or not to enact a choice-enhancing policy for each period. Without any policy, \( k_t = \bar{k} > 0 \) for all \( t \). If a policymaker enacts the policy in period \( t \), then the switching period of that period is reduced to \( k_t = \underline{k} \in (0, \bar{k}) \). Denote by \( \Delta_k := \underline{k}/\bar{k} \in (0, 1) \).

Following O’Donoghue and Rabin (1999a, 1999b), we assume that a fraction \( \alpha \) of consumers are naive present-biased whereas the remaining fraction of consumers are time-consistent.\(^6\) Let \( u_t \) denote a consumer’s period-\( t \) utility. In each period \( s = 1, 2 \), time-consistent consumers decide whether or not to opt out of the firm’s service based on the sum of their per-period utility \( \sum_{t=s}^{3} u_t \), and they correctly expect their future behavior. In contrast, naive present-biased consumers decide whether or not to opt out based on \( u_s + \beta \sum_{t=s+1}^{3} u_t \), where \( \beta \in (0, 1) \) represents the degree of their present bias. Also, these present-biased consumers believe that they will behave as if \( \beta = 1 \) in any period \( t > s \): they are naive about their future self-control.

We investigate perception-perfect equilibria in which each player maximizes her perceived utility in each subgame (O’Donoghue and Rabin 1999a). We evaluate consumer welfare based on each consumer’s long-run utility \( \sum_{t=1}^{3} u_t \). The timeline of the firm’s pricing and consumers’ decisions is described in Figure 1.

\(^6\) See Section 3.2 for evidence and discussions on the present bias and procrastination.
2.2 Illustrative Model: Analysis

We first characterize consumer behavior given prices and switching costs. Note that consumers do not take any action in $t = 3$. Note also that consumers never have an incentive to switch back from the competitive fringe to the firm. For notational simplicity, let $\beta^i$ be each consumer’s present biasness where $i \in \{TC, N\}$, $\beta^{TC} = 1$, and $\beta^N = \beta < 1$. In $t = 2$, consumers switch to the competitive fringe if and only if $-k_2 + \beta^i a > \beta^i(a - p^a)$ or equivalently $p^a > \frac{k_2}{\beta}$.

We next characterize consumer behavior in $t = 1$. Because naive consumers (wrongly) think that they will be time-consistent in $t = 2$, all consumers think that they will switch in period 2 if and only if $p^a > k_2$ when they do not opt out of the firm’s service in period 1. Given this belief, consumers’ switching behavior in period 1 can be divided into the following two cases. First, if $p^a \leq k_2$, consumers think they would keep using the firm’s service in period 2. Given this, they switch in period 1 if and only if $p^a > \frac{k_1}{\beta}$. Second, if $p^a > k_2$, consumers think they would switch in period 2. Given this, they switch in period 1 if and only if $\frac{k_2}{\beta} \geq k_1$.

We now analyze the optimal pricing of the firm and the effects of choice-enhancing policies. We first investigate the situation in which the policymaker does not employ any policy, i.e., $k_1 = k_2 = \overline{k}$.

The firm faces a trade-off between exploiting naive consumers at a high price ($p^a = \frac{1}{\beta} \overline{k}$) or selling its service to all consumers at a moderate price ($p^a = \overline{k}$). The result is summarized as follows:

**Lemma 1.** Suppose $k_1 = k_2 = \overline{k}$.

*If $\alpha > \beta$, the firm sets $p^a = \frac{1}{\beta} \overline{k}$. Time-consistent consumers switch either in period 1 or 2 and do not pay $p^a$, whereas naive consumers pay $p^a$. The profits of the firm are $\pi = \frac{2}{\beta} \overline{k}$.*

*If $\alpha \leq \beta$, the firm sets $p^a = \overline{k}$. All consumers pay $p^a$. The profits of the firm are $\pi = \overline{k}$.*

The intuition is simple. The firm is more likely to exploit naive consumers if there are more naive consumers (larger $\alpha$) or if naive consumers suffer from a severer present bias (smaller $\beta$). Consumer welfare is lower under such exploitation because naive consumers pay the high price which they have not anticipated to pay. Social welfare is also lower because time-consistent consumers incur $\overline{k}$ to switch the service.

We next investigate the situation in which the switching cost is decreased in the first period, i.e., $k_1 = \overline{k}$, $k_2 = \overline{k}$. This is the case if the policymaker employs the policy which reduces the switching cost at the time consumers are enrolled. The firm still faces the same type of trade-off as above. The equilibrium cut-off condition is different, however. On the one hand, time-consistent
consumers switch in period 1 if \( p^a > k \). On the other hand, naive consumers in period 1 prefer to switch in period 2 rather than to switch immediately if \(-k < -\beta k\) or equivalently \( \Delta_k > \beta \). In this case, the firm can set \( p^a = \frac{1}{\beta} k \) and naive consumers end up paying the price. The result is summarized as follows:

**Lemma 2.** Suppose \( k_1 = k, k_2 = k \).

(i) Suppose \( \Delta_k > \beta \). If \( \alpha > \beta \Delta_k \), the firm sets \( p^a = \frac{1}{\beta} k \). Time-consistent consumers switch in period 1 and do not pay \( p^a \), whereas naive consumers pay \( p^a \). The profits of the firm are \( \pi = \frac{\alpha}{\beta} k \). If \( \alpha \leq \beta \Delta_k \), the firm sets \( p^a = k \). All consumers pay \( p^a \). The profits of the firm are \( \pi = k \).

(ii) Suppose \( \Delta_k \leq \beta \). If \( \alpha > \beta \), the firm sets \( p^a_2 = \frac{1}{\beta} k \). Time-consistent consumers switch in period 1 and do not pay \( p^a \), whereas naive consumers pay \( p^a \). The profits of the firm are \( \pi = \frac{\alpha}{\beta} k \). If \( \alpha \leq \beta \), the firm sets \( p^a_2 = k \). All consumers pay \( p^a \). The profits of the firm are \( \pi = k \).

Lemma 2 (i) means that the firm may still be able to charge a high price and exploit naive consumers even when the switching cost in \( t = 1 \) is decreased. Intuitively, naive consumers procrastinate switching if the decrease of the switching cost in period 1 is not large (\( \Delta_k > \beta \)): they think that they do not switch in period 1 but will switch in period 2. In period 2, however, naive consumers again do not switch if \( p^a \leq \frac{1}{\beta} k \). In contrast, Lemma 2 (ii) shows that when \( \Delta_k \leq \beta \), the policy in \( t = 1 \) can decrease the price, and hence can increase consumer welfare. Intuitively, if naive consumers do not procrastinate switching, the firm needs to decrease its price in response to the policy.

Comparing Lemma 1 and Lemma 2 leads to our first main result:

**Proposition 1.** Suppose the policy is employed in \( t = 1 \). If \( 1 \geq \frac{\alpha}{\beta} > \Delta_k > \beta \), the policy increases the equilibrium price and decreases social welfare. If in addition \( \frac{\alpha}{\beta} + (1 - \alpha) \cdot \Delta_k > 1 \), it also decreases consumer welfare.

Proposition 1 highlights that the policy which decreases the switching cost at the time consumers are enrolled can lower social welfare. This occurs when the firm sells to both types of consumers without the policy (\( \beta \geq \alpha \)), the firm sells only to naive consumers with the policy (\( \alpha > \beta \Delta_k \)), and naive consumers procrastinate switching (\( \Delta_k > \beta \)). Intuitively, under these parameters the firm increases its price and starts exploiting naive consumers in response to the policy. The policy can also lower consumer welfare.
Figure 2 shows how the policy decreasing the switching cost in $t = 1$ changes the firm’s equilibrium pricing when the reduction of the switching cost is not sufficient (i.e., when $\Delta k > \beta$). There are three cases depending on the fraction of naive consumers. When most consumers are naive (i.e., $\alpha > \beta$), the firm sets a high price both before and after the policy. When most consumers are time-consistent (i.e., $\alpha \leq \beta \Delta k$), the firm always chooses a price which all consumers pay, and hence the policy decreases the equilibrium price. When the composition of consumers is in-between, however, the firm increases its price in response to the policy. This generates the adverse effect on consumer and social welfare as stated in Proposition 1.

We now investigate the situation in which the switching cost is decreased in the second period, i.e., $k_1 = \bar{k}, k_2 = \bar{k}$. This is the case if the policymaker decreases the switching cost whenever the firm starts charging a higher price. Similar to the above analysis, the firm faces a trade-off between exploiting naive consumers at a high price ($p^a = \frac{1}{\beta} \bar{k}$) or selling its service to all consumers at a moderate price ($p^a = k$). The result is summarized as follows:

**Lemma 3.** Suppose $k_1 = \bar{k}, k_2 = \bar{k}$. 
If $\alpha > \beta$, the firm sets $p^a = \frac{1}{\beta}k$. Time-consistent consumers switch in period 2 and do not pay $p^a$, whereas naive consumers pay $p^a$. The profits of the firm are $\pi = \frac{2}{\beta}k$.

If $\alpha \leq \beta$, the firm sets $p^a = k$. All consumers pay $p^a$. The profits of the firm are $\pi = k$.

Note that the circumstances under which the firm chooses to exploit the naive consumers ($\alpha > \beta$) are the same as the ones under no policy. By comparing Lemma 1 and Lemma 3, we have the following result:

**Proposition 2.** Suppose the policy is employed in $t = 2$. The policy always strictly increases consumer welfare and weakly increases social welfare. It strictly increases social welfare if $\alpha > \beta$.

Proposition 2 implies that the policy which decreases the switching cost when a firm increases the price (in this case, when a firm starts charging a positive fee) does not have the perverse effect of selecting naive consumers as described in Proposition 1. As depicted in Figure 2, such a policy always decreases the equilibrium price. Thus, it always strictly increases consumer welfare relative to the no-policy case, and also increases social welfare when a fraction of consumers switch in equilibrium.

Furthermore, the comparison between Lemma 2 and Lemma 3 leads to the following result:

**Proposition 3.** Under any parameters, both consumer and social welfare are weakly higher when enacting the policy in $t = 2$ than when enacting the policy in $t = 1$. Consumer welfare is strictly higher if $\frac{\alpha}{\beta} > \Delta_k > \beta$, and social welfare is strictly higher if in addition $1 \geq \frac{\alpha}{\beta}$.

Proposition 3 highlights that the timing of enacting the policy matters for both consumer and social welfare. If a policymaker enacts a choice-enhancing policy at the time consumers are enrolled, then a firm may change its pricing strategy in response to the policy, and hence the perverse welfare effect can occur. In contrast, as depicted in Figure 2, the policy when a firm increases the price (including the case in which the firm starts charging a positive fee) does not have such an adverse effect, and hence is welfare enhancing.

It is worth emphasizing that if $\frac{\alpha}{\beta} > \Delta_k > \beta$, the ex-post utility of time-consistent consumers is the same under the different policies whereas that of naive consumers strictly increases under the policy in $t = 2$. To illustrate the welfare effects clearly, we discuss and compare two extreme cases: $\alpha = 1$ and $\alpha = 0$. First, consider the case $\alpha = 1$. Note that if there are sufficiently many naive consumers, the inequality $\frac{\alpha}{\beta} > \Delta_k$ always holds. Hence, the timing of enacting the policy affects
consumer welfare whenever most consumers are naive and the reduction of the switching cost is not large (i.e., $\Delta_k > \beta$). Second, consider the case $\alpha = 0$. In this case, both policies increase consumer welfare. Contrasting the case $\alpha = 0$ with the case $\alpha = 1$ leads to an interesting result: the policy which decreases the switching cost when a firm increases the price is robust to the proportion of rational consumers, whereas the conventional policy which decreases the switching cost at the time consumers are enrolled is not. In this sense, our proposed policy is in line with “asymmetric paternalism” which benefits consumers who make errors while imposes no (or relatively little) harm on consumers who are fully rational (Camerer, Issacharoff, Loewenstein, O’Donoghue and Rabin 2003).

3 Model

This section introduces our full model. Section 3.1 sets up the model. Section 3.2 discusses the key assumptions throughout this paper.

3.1 Setup

There are $T + 1$ periods: $t = 0, 1, 2, \cdots, T$ where $T \geq 3$. One risk-neutral firm sells its products to a continuum of measure one of risk-neutral consumers. The firm produces two types of products: a base product and an add-on. Consumers value the base product at $v > 0$ and can consume it only once in $t = 1$. Consumers value the add-on at $a > 0$ in each $t = 2, \cdots, T$, where they can use the add-on only combined with the base product. If a consumer does not buy any product from the firm, she receives an outside option with utility $\bar{u} \in [0, v)$ in period $T$. While only the firm can produce the base product, a competitive fringe also provides an add-on with the same value.\(^7\) The production cost of the base product is $c^v \in (0, v - \bar{u})$. We normalize the production cost of the additional good—and hence the add-on price of the competitive fringe—to be zero.

The firm automatically enrolls consumers who buy the base product into the add-on service with a free trial or a grace period (i.e., the price for first-period add-on usage $p^a_2$ is zero). At the beginning of period 0, the firm sets and commits to its prices: a price of the base product $p^v \geq 0$ which is charged in period 1 and prices of the add-on $p^a_t \geq 0$ which are charged in $t = 3, \cdots, T$.\(^8\)

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\(^7\) In Section 5, we analyze a model incorporating competition on the base product.

\(^8\) Given the timing of payments described below, $p^a_2$ is indistinguishable from $p^v$ in the model whenever the firm automatically enrolls consumers. As discussed in Section 3.2, we set $p^a_2 = 0$ along with our real-world interpretation.
In each \( t = 1, 2, \cdots, T - 1 \), consumers can switch from the firm to the competitive fringe for the add-on by incurring a switching cost \( k_t > 0 \). Without any policy, \( k_t = \overline{k} > 0 \) for all \( t \). If the firm or a policymaker employs a policy in period \( t \), then the policy decreases the switching cost to \( k_t = \underline{k} \) where \( \underline{k} \in (0, \overline{k}) \). As an interpretation, \( \overline{k} - \underline{k} \) is a cost for cancellation which the firm or the policymaker can remove, whereas \( \underline{k} \) is the part of the switching cost the policymaker cannot remove (which includes, for example, a sign-up cost to the competitive fringe). Denote by \( \Delta_k := \frac{k}{\overline{k}} \in (0, 1) \). We assume that the add-on is sufficiently valuable for all consumers: \( a \geq \max\{ \frac{\overline{k}}{T - 1}, \underline{k} \} \).\(^9\)

Following O’Donoghue and Rabin (1999a, 1999b), we assume that a fraction \( \alpha \) of consumers are naive present-biased whereas the remaining fraction of consumers are time-consistent.\(^10\) Let \( u_t \) denote a consumer’s period-\( t \) utility. In each period \( s \), time-consistent consumers maximize \( u_s + \sum_{t=s+1}^{T} \delta^{t-s} u_t \) and correctly expect their future behavior. Naive present-biased consumers maximize \( u_s + \beta \sum_{t=s+1}^{T} \delta^{t-s} u_t \) in each period \( s \), where \( \beta \in (0, 1) \) represents the degree of their present bias. Also, these present-biased consumers believe that they will behave as if \( \beta = 1 \) in any period \( t > s \): they are naive about their future self-control. Following O’Donoghue and Rabin (1999a) and others, we investigate perception-perfect equilibria in which each player maximizes her perceived utility in each subgame. We evaluate consumer welfare based on each consumer’s long-run utility \( \sum_{t=0}^{T} \delta^t u_t \). In what follows, we set \( \delta = 1 \).

The timing of the game is as follows. In period 0, both the firm and the policymaker decide and commit whether or not to enact the policy for each period. If either or both of them enact the policy for period \( t \), the switching cost in period \( t \) is \( k_t = \underline{k} \); otherwise, it is \( k_t = \overline{k} \). Then, the firm sells the base product and commits to all prices \( (p^v, p^a_1, \cdots, p^a_T) \). After observing the prices and switching costs, consumers decide whether or not to buy the base product at the end of period 0.\(^11\) In period 1, consumers who bought the base product receive \( v \) and pay \( p^v \). They also decide whether or not to opt out of the firm’s add-on at switching cost \( k_1 \) incurred immediately. Then, in each period \( t = 2, \cdots, T \), consumers who use the add-on receive \( a \). Also in period \( t = 2, \cdots, T - 1 \),

\(^9\) The condition \( (T - 1)a \geq \overline{k} \) ensures that the add-on is valuable to consumers even when they plan to switch. The condition \( a \geq \frac{\overline{k}}{T - 1} \) means that when consumers opt out of the firm’s add-on, they prefer to take up the competitive fringe’s add-on rather than not to use the add-on. Section 5 discusses the case in which add-on demand is heterogeneous among consumers and \( a < \max\{ \frac{\overline{k}}{T - 1}, \underline{k} \} \) for some consumers.

\(^10\) As discussed in Section 5, our results are robust to incorporating partial sophistication on own future self-control.

\(^11\) Notice that the firm cannot increase its profits by offering a menu contract, because in \( t = 0 \) naive consumers believe that they will behave as if they were time-consistent.
if the consumers have not opted out of the firm’s add-on, then they decide either to opt out at switching cost $k_t$ incurred in period $t$ or to pay $p_{t+1}^a$ in period $t+1$. The game ends at the end of period $T$.

3.2 Discussion of Key Assumptions

This subsection discusses two key assumptions throughout this paper. First, a fraction of consumers may procrastinate their switching decisions.\textsuperscript{12} Second, policymakers cannot remove all switching costs.

**Procrastination** Recent empirical and experimental literature shows that people often procrastinate.\textsuperscript{13} In our model, consumers incur the switching cost now but the payment later. Due to the discrepancy of the timing, naive present-biased consumers may procrastinate their switching decisions. This assumption is plausible in our industrial examples because there is an immediate effort cost on a cancellation procedure whereas the change of the bill comes later (e.g., at the beginning of the next month). Moreover, the assumption on the timing can be relaxed when consumers possibly pay the add-on prices multiple times (i.e., when $T \geq 4$).\textsuperscript{14}

**Switching Costs** The literature on choice-enhancing and active-choice policies has focused on two cases: (i) the policy enables consumers to make a switching decision without incurring any switching cost or (ii) the policy forces all consumers to make a switching decision. In contrast to the literature, this paper analyzes the case in which a policy decreases consumers’ switching cost, but the switching cost is still positive and the consumers decide whether or not to switch by themselves. For example, suppose that a firm can automatically enroll its customers to an additional service and that the customers need to take an extra action (e.g., register their personal information) if they want to use the additional service of some other firm. In this case, a policymaker can decrease

\textsuperscript{12} Following O’Donoghue and Rabin (2001) and DellaVigna (2009), we classify that a consumer procrastinates if ex-ante the consumer anticipates switching the option in some period but she does not actually switch in that period.

\textsuperscript{13} See, for examples, Ariely and Wertenbroch (2002), DellaVigna and Malmendier (2006), and Skiba and Tobacman (2008).

\textsuperscript{14} In such a case, consumers may procrastinate opting out of the service and pay the add-on price today, but they may still think that they will opt out in the next period in order to avoid future payments. Hence, when $T \geq 4$, our results qualitatively hold even when consumers face the switching cost and the per-period add-on payment at the same time. In contrast, when $T = 3$ (i.e., when consumers pay the add-on price at most once), our timing assumption is crucial.
the switching cost (e.g., by mandating a simpler cancellation format), but cannot decrease it to zero due to the cost of new registration.

In practice, such automatic enrollments are not illegal because firms can offer free trials of the additional services. Along with this interpretation, we assume that the firm charges no additional price for the first usage of the add-on (i.e., \( p_a^0 = 0 \)).

Beyond automatic enrollments, our main logic and results can be extended to situations in which a firm cannot automatically enroll consumers into a service but the firm has a lower registration cost than other firms. For example, for customers of a retail bank, signing up for a credit card associated with the bank by using existing customer information is often easier than doing so at other firms. In such a case, \( \overline{k} \) is the difference of the registration costs between firms, and a policy decreases the registration cost of other firms by \( \overline{k} - \underline{k} \).

4 Analysis

This section analyzes the full model: consumers buy a base product and also possibly incur the add-on prices multiple times.

4.1 Consumer Behavior

Here we characterize consumer behavior given prices and switching costs. Note that consumers do not take any action in \( t = T \).

We first analyze the switching decision in \( t = T - 1 \). Suppose that consumers bought the base product and kept using the firm’s add-on. Then, consumers do not switch to the competitive fringe if and only if \(-k_{T-1} + \beta a \leq \beta^i(a - p_a^T)\) or equivalently \( p_a^T \leq \frac{k_{T-1}}{\beta^i} \).

We next analyze consumer behavior in period \( \tau < T - 1 \). Because naive consumers (wrongly) think that they are time-consistent, all consumers think that they will not switch in any future period if and only if \( \sum_{i=t+1}^{T} p_a^i \leq k_t \) for all \( t > \tau \). Given this belief, consumers’ switching behavior in period \( \tau \) can be divided into the following two cases. First, if \( \sum_{i=t+1}^{T} p_a^i \leq k_t \) for all \( t > \tau \), consumers do not switch in period \( \tau \) if and only if \( \sum_{i=t+1}^{T} p_a^i \leq \frac{k_{\tau}}{\beta^i} \) because they think that they will never switch in any future period \( t > \tau \). Second, if there exists \( t > \tau \) such that \( \sum_{i=t+1}^{T} p_a^i > k_t \), consumers think they will switch in period \( \hat{t} = \arg \min_{t \geq \tau + 1} (k_t + \sum_{i=\tau+1}^{t} p_a^i) \). Given \( \hat{t} \), they do not switch in period \( \tau \) if and only if \( k_{\tau} > \beta^i \left( k_{\hat{t}} + \sum_{i=\tau+1}^{\hat{t}} p_a^i \right) \). Given these, each consumer buys the
base product in $t = 0$ if and only if her perceived utility is equal to or greater than the outside option.

### 4.2 Optimal Pricing and Effects of Policies

We now analyze the optimal pricing of the firm and the effects of policies. Note that if no consumer had an option to opt out of the add-on, the firm would set its total price equal to its overall consumption value minus consumers’ outside option, i.e., $p^v + \sum_{t=3}^T p^a_t = v + (T - 1)a - \bar{u}$. For the clear comparison and the ease of notation, we denote the total surplus by $V_T := v + (T - 1)a - \bar{u}$.

We first investigate the situation in which switching costs are high in all periods, i.e., $k_t = \bar{k}$ for all $t \in \{1, \ldots, T - 1\}$. This is the case when the policymaker does not employ any policy. The firm faces a trade-off between exploiting naive consumers with a high add-on price or selling its add-on to all consumers with a moderate add-on price. Note that the add-on prices can be different for different periods. The result is summarized as follows:

**Lemma 4.** Suppose $k_t = \bar{k}$ for all $t \in \{1, \ldots, T - 1\}$.

If $\alpha + (T - 3)(1 - \beta)\alpha > \beta$, the firm sets $p^v = V_T - \bar{k}$, $p^a_t = \frac{1 - \beta}{\beta} \bar{k}$ in $t \in \{3, \ldots, T - 1\}$, and $p^a_T = \frac{1}{\beta} \bar{k}$. Time-consistent consumers switch in period 1, whereas naive consumers never switch. The profits of the firm are $\pi = V_T - c^v + \left(\frac{\alpha}{\beta} - 1\right) \bar{k} + (T - 3)\frac{\alpha}{\beta}(1 - \beta) \bar{k}$ and the long-run utility of each type of consumers is $u^N = \bar{u} - (T - 2)\frac{1 - \beta}{\beta} \bar{k}$ and $u^{TC} = \bar{u}$.

If $\alpha + (T - 3)(1 - \beta)\alpha \leq \beta$, the firm sets $p^v + \sum_{t=3}^T p^a_t = V_T$ with $\sum_{t=3}^T p^a_t \leq \bar{k}$. No consumer switches. The profits of the firm are $\pi = V_T - c^v$ and the long-run utility of each type of consumers is $u^N = u^{TC} = \bar{u}$.

The intuition is similar to the one in our illustrative model. The discrepancy of add-on prices between $p^a_T$ and $p^a_t$ where $t = 3, \ldots, T - 1$ is due to the different decision problems consumers face in different periods. In period $T - 1$, naive consumers cannot procrastinate their switching decision to the next period. In periods 2 to $T - 2$, naive consumers (wrongly) expect to cancel the subscription in the next period such that they do not have to make any further payments. Moreover, the cut-off when the firm decides to exploit the naive consumers is increasing in $T$.

**Corollary 1.** For given $\alpha$ and $\beta$, the firm is more likely to sell the add-on only to naive consumers as $T$ increases. Also, if $\alpha + (T - 3)(1 - \beta)\alpha > \beta$, the ex-post utility of naive consumers is strictly decreasing in $T$.
Corollary 1 shows that firms are more likely to exploit naive consumers when $T$ is larger. Furthermore, naive consumers’ utility in this case is strictly decreasing in $T$. In contrast to the classical argument that a monopolist can extract surplus from consumers only once, but akin to O’Donoghue and Rabin (1999b, 2001) that people may procrastinate for long periods, in our model the firm exploits naive consumers more as $T$ increases.

We next analyze the situation in which the switching cost is lower only in the first period, i.e., $k_1 = k$, $k_t = \overline{k}$ for all $t \in \{2, \cdots, T - 1\}$. This is the case if the policymaker employs the policy at the time consumers are enrolled in the add-on service. The result is summarized as follows:

**Lemma 5.** Suppose $k_1 = k$, $k_t = \overline{k}$ for all $t \in \{2, \cdots, T - 1\}$.

(i) Suppose $\Delta_k > \beta$. If $\alpha + (T - 3)(1 - \beta) > \beta \Delta_k$, the firm sets $p^v = V_T - k$, $p^a_t = \frac{1 - \beta}{\beta} k$ in $t \in \{3, \cdots, T - 1\}$, and $p^a_T = \frac{1}{\beta} \overline{k}$. Time-consistent consumers switch in period 1, whereas naive consumers never switch. The profits of the firm are $\pi = V_T - c^v + -k + \frac{\alpha}{\beta} [1 + (T - 3)(1 - \beta)] \overline{k}$ and the long-run utility of each type of consumers is $u^N = \bar{u} + k - \frac{1}{\beta} [1 + (T - 3)(1 - \beta)] \overline{k}$ and $u^{TC} = \bar{u}$.

If $\alpha + (T - 3)(1 - \beta) \alpha \leq \beta \Delta_k$, the firm sets $p^v + \sum_{t=3}^{T} p^a_t = V_T$ with $\sum_{t=3}^{T} p^a_t \leq k$. No consumer switches. The profits of the firm are $\pi = V_T - c^v$ and the long-run utility of each type of consumers is $u^N = u^{TC} = \bar{u}$.

(ii) Suppose $\Delta_k \leq \beta$. If $\alpha > \beta$, the firm sets $p^v = V_T - k$, $p^a_t = \frac{1 - \beta}{\beta} k$ in $t \in \{3, \cdots, T - 1\}$, and $p^a_T = \frac{1}{\beta} \overline{k}$. Time-consistent consumers switch in period 1, whereas naive consumers never switch. The profits of the firm are $\pi = V_T - c^v + (\frac{\alpha}{\beta} - 1) \overline{k}$ and the long-run utility of each type of consumers is $u^N = \bar{u} - \frac{1 - \beta}{\beta} \overline{k}$ and $u^{TC} = \bar{u}$.

If $\alpha \leq \beta$, the firm sets $p^v + \sum_{t=3}^{T} p^a_t = V_T$ with $\sum_{t=3}^{T} p^a_t \leq k$. No consumer switches. The profits of the firm are $\pi = V_T - c^v$ and the long-run utility of each type of consumers is $u^N = u^{TC} = \bar{u}$.

Similarly to Lemma 2, the policy which decreases the switching cost at the time consumers are enrolled may change the cut-off and lead the firm to exploit naive consumers. Analogously to the illustrative model, comparing Lemmas 4 and 5 leads to the following result:

**Proposition 4.** Suppose that a policymaker enacts a policy in $t = 1$. If $1 \geq \frac{\alpha + (T - 3)(1 - \beta) \alpha}{\beta} > \Delta_k > \beta$, the policy increases the equilibrium add-on prices and decreases both consumer and social welfare.

Along with Proposition 1, the policy which decreases the switching cost at the time consumers are enrolled into the add-on service can lower social welfare. Different from Proposition 1, however,
the policy also decreases consumer welfare whenever it decreases social welfare. This is because under the full model, the firm needs to appropriately discount its base-product price in order to attract consumers.

We now investigate an alternative policy. Interestingly, merely imposing a low switching cost in period 2 is not sufficient to unambiguously improve welfare. This is because the firm would simply set a low $p^2_3$ and start exploiting consumers afterwards.\textsuperscript{15} Hence, we propose a policy in which the policymaker forces the firm to lower the switching cost whenever it increases the add-on price. This is the case if a firm needs to get an additional consent from consumers for charging a higher price. As an example, suppose that the firm sets $p^2_3 > 0$. Since the add-on price increases from $p_2^2 = 0$ to $p^2_3 > 0$, the policy requires to lower the consumers’ switching cost in $t = 2$ by letting consumers know the price increase and forcing the firm to distribute a simple cancellation format.

The equilibrium outcomes when enacting this alternative policy are summarized as follows:

**Lemma 6.** Suppose $k_t = k$ for any $t$ which satisfies $p^0_t > p^0_{t-1}$ with $p^0_2 = 0$.

If $\alpha + (T - 3)(1 - \beta)\alpha > \beta$, the firm sets $p^v = V_T - k$, $p^u_t = \frac{1 - \beta}{\beta}k$ in $t \in \{3, \ldots, T - 1\}$, and $p^u_T = \frac{1}{\beta}k$. Time-consistent consumers switch in period 1, whereas naive consumers never switch.

The profits of the firm are $\pi = V_T - c^v + \left(\frac{\alpha}{\beta} - 1\right)k + (T - 3)\frac{\alpha}{\beta}(1 - \beta)k$ and the long-run utility of each type of consumers is $u^N = \bar{u} - (T - 2)\frac{1 - \beta}{\beta}k$ and $u^{TC} = \bar{u}$.

If $\alpha + (T - 3)(1 - \beta)\alpha \leq \beta$, the firm sets $\sum_{t=3}^{T}p^u_t = V_T$ with $\sum_{t=3}^{T}p^u_t \leq k$. No consumer switches. The profits of the firm are $\pi = V_T - c^v$ and the long-run utility of each type of consumers is $u^N = u^{TC} = \bar{u}$.

Interestingly, under the policy in Lemma 6, the firm may have an incentive to voluntarily decrease its switching cost to $k$ in the periods after it is forced to do so by the policy; this makes naive consumers more likely to believe that they will switch in future, and hence makes them more likely to procrastinate their switching decision.\textsuperscript{16} This has the policy implication that a policymaker

\textsuperscript{15} Formally, consider the case in which naive consumers face a switching decision in period 2, $k_2 = k$, and $k_t = \bar{k}$ for all $t \geq 3$. In this case, naive consumers (wrongly) think that they will switch in period 3 if $p^0_3 = \frac{1 - \beta}{\beta}k$. Given that, they do not switch in period 2 if $-k \leq -\beta(p^0_2 + \bar{k})$. Hence, if $\Delta_k > \beta$, the firm can make naive consumers procrastinate their switching decisions by lowering its add-on price in the period after the low switching cost.

\textsuperscript{16} To see this, suppose that naive consumers face a switching decision in period $t$ with $k_t = \frac{k}{2}$ due to the increase of the add-on price and the policy. In period $t$, the condition for naive consumers to procrastinate switching to the next period is $-k \leq -\beta(p^0_{t+1} + k_{t+1})$. Notice that naive consumers always switch in period $t$ if $\Delta_k \leq \beta$ and $k_{t+1} \leq \bar{k}$. Hence, when $\Delta_k \leq \beta$, the firm decreases $k_{t+1}$ from $k$ to $\bar{k}$ voluntarily in order to lead naive consumers to procrastinate their switching decisions. Moreover, even when $\Delta_k > \beta$ the firm has an incentive to decrease $k_{t+1}$ under the policy; see the proof of Lemma 6 for details.
does not need to enforce a lower switching cost in every period, as we show in Proposition 6 later.

Analogously to the illustrative model, comparing Lemmas 4 and 6 leads to the following result:

**Proposition 5.** Suppose the policymaker enacts a policy which requires the firm to lower its switching cost whenever it increases the add-on price. The policy always weakly increases consumer and social welfare. It strictly increases consumer and social welfare if \( \alpha + (T - 3)(1 - \beta)\alpha > \beta \).

Note that the policy in Proposition 5, which requires the firm to lower the switching cost whenever the firm raises add-on prices, is more likely to increase social welfare as \( T \) becomes larger.

Furthermore, the following result shows that it is not necessary to force the firm to reduce the switching cost in every period in order to improve welfare; a milder intervention as in Proposition 5 has the same consequence.

**Proposition 6.** Equilibrium outcomes under a policy that forces the firm to reduce the switching cost whenever the firm increases the add-on price are the same as under a policy that forces the firm to reduce the switching cost in every period.

Hence, our suggested policy may be preferable when there is any cost for forcing firms to reduce the switching cost in practice.

### 4.3 Deadlines

So far, we have shown the consequences of policies which decrease consumers’ switching costs in certain periods. Now we examine an alternative policy intervention. Specifically, in this subsection we analyze an extended model in which the policymaker can sufficiently increase switching costs in certain periods so that consumers cannot cancel their contract in those periods. By doing so, the policymaker can impose a deadline of switching decisions to consumers. In our basic model, it is optimal for the policymaker to prevent consumers from switching after the first two periods:

**Proposition 7.** Assume that the policymaker can prohibit consumers to switch in certain periods: the policymaker chooses \( \mathcal{T} \subseteq \{2, \cdots, T - 1\} \) such that \( k_t = \infty \) for all \( t \in \mathcal{T} \). Then, choosing \( \mathcal{T}^* = \{3, \cdots, T - 1\} \) maximizes consumer and social welfare. If \( \alpha + (T - 3)(1 - \beta)\alpha > \beta \), consumer welfare is strictly larger than choosing \( \mathcal{T} = \emptyset \). If in addition \( \beta > \alpha \), social welfare is also strictly larger than choosing \( \mathcal{T} = \emptyset \).
The intuition of Proposition 7 is as follows: since consumers are not able to cancel after the second period under the policy, naive consumers cannot falsely believe that they will switch in a future period.\(^{17}\) Given this, they will not procrastinate their switching decision if the total future payment for the add-on is high. This finding is in line with the theoretical literature which analyzes the effects of imposing deadlines (O’Donoghue and Rabin 1999b, Herweg and Müller 2011).

Proposition 7 implies that \((T + 1)\)-period models can be reduced to the four-period model when the policymaker can impose an optimal deadline. In such a case, \(p^a_T\) is interpreted as the sum of all payments that have to be made after the second period. If the policymaker cannot regulate the prices directly but can change the switching costs, decreasing the switching cost in the second period and imposing the deadline in the second period maximize social welfare in our basic model. Namely, when the policymaker cannot regulate the prices directly, imposing \(k_2 = \bar{k}\) and \(\mathcal{T}^* = \{3, \cdots, T - 1\}\) becomes the optimal policy.\(^{18}\)

Unlike a choice-enhancing or an active-choice policy which decreases the switching costs, however, one should be very cautious about imposing such a strict deadline in practice. For example, imposing a deadline may decrease welfare if add-on values or switching costs are changing over time. Also, as we will discuss in Section 6, imposing a deadline can be welfare harmful when consumers also have other psychological biases.

Furthermore, imposing a deadline might not be feasible as the firm might be able to circumvent the deadline by (pretendedly) changing the product features of the add-on such that consumers receive extraordinary termination rights. Corollary 2 states that the firm indeed has an incentive to do so:

**Corollary 2.** Assume that the policymaker can prohibit consumers to switch in certain periods: she can choose \(\mathcal{T} \subseteq \{2, \cdots, T - 1\}\) such that \(k_t = \infty\) for all \(t \in \mathcal{T}\). If \(\alpha + (T - 3)(1 - \beta)\alpha > \beta\), then profits of the firm under \(\mathcal{T} = \emptyset\) are strictly higher than under \(\mathcal{T} = \{3, \cdots, T - 1\}\).

Consequently, if the firm can credibly commit to pretendedly change its terms and conditions of the add-on, the deadline policy in Proposition 7 may not be effective and the policymaker can only force the firm to lower the switching cost in certain periods—as we analyzed in the previous

\(^{17}\) Note that imposing a deadline in \(t = 1\) is also optimal, although the deadline in \(t = 1\) does not seem to be legal in practice because consumers who get a free trial do not have an option to cancel the service later.

\(^{18}\) Note that when the policymaker can regulate the prices directly, simply imposing \(p^a_t = 0\) for all \(t\) maximizes the social welfare.
subsection.

5 Extensions

This section discusses extensions and modifications of the model. For brevity, we discuss the case in which consumers pay the add-on price at most once (i.e., \( T = 3 \)) throughout this section. We discuss in turn a model incorporating (i) competition on the base product, (ii) the possibility that naive consumers are partially aware of their self-control problems, or (iii) heterogeneous base-product values or add-on values among consumers.

5.1 Competition on Base Product

In the model, we have assumed that one firm can provide a base product. In this subsection, we analyze the case in which \( N \geq 2 \) firms sell the homogeneous base product. We focus on a symmetric pure-strategy equilibrium in which all firms offer the same contract on the equilibrium path and equally split each type of consumers in the case of tie-breaking.

Under competition on the base product, market outcomes depend on whether setting negative prices is feasible or not. We first discuss the case in which firms can set negative prices without incurring any additional cost:

**Proposition 8.** Suppose that there are \( N \geq 2 \) firms selling the base product and that \( p^v \in \mathbb{R} \).

Then, all firms earn zero profits in any equilibrium. Under any parameters, both consumer and social welfare are weakly higher when enacting the policy in \( t = 2 \) than when enacting the policy in \( t = 1 \). Consumer welfare is strictly higher if \( \frac{\alpha}{\beta} > \Delta_k > \beta \), and social welfare is strictly higher if in addition \( 1 \geq \frac{\alpha}{\beta} \).

Intuitively, if firms can completely compete down their base-product prices (i.e., if firms can set \( p^v < 0 \)), they will do so as in the standard Bertrand-type price competition. Although all profits from exploitation will be passed on to consumers and all firms earn zero profits, the timing of policies still matters as in Section 2. In addition, under competition a cross-subsidization from naive consumers to time-consistent consumers may occur, because the presence of naive consumers decreases the equilibrium base-product price (Gabaix and Laibson 2006).
In practice, however, firms may not be able to profitably set overly low prices (i.e., negative base-product prices). Heidhues, Kőszeigi and Murooka (2012, 2014) investigate how the possibility of arbitrage can endogenously generate a price floor of the base product.\(^{19}\) In such a case, firms may earn positive profits even under competition:

**Proposition 9.** Suppose that there are \(N \geq 2\) firms selling the base product and that \(p^v \geq 0\).

When the policy is enacted in \(t = 2\) or when the policy is enacted in \(t = 1\) and \(\Delta_k \leq \beta\), a positive-profit equilibrium in which \((p^v = 0, p^a_3 = \frac{1}{\beta}k)\) exists if \(\frac{1}{N}(\frac{\alpha}{\beta}k - c^v) > \max\{k - c^v, 0\}\). When the policy is enacted in \(t = 1\) and \(\Delta_k > \beta\), a positive-profit equilibrium in which \((p^v = 0, p^a_3 = \frac{1}{\beta}k)\) exists if \(\frac{1}{N}(\frac{\alpha}{\beta}k - c^v) > \max\{k - c^v, 0\}\).

Under any parameters, both consumer and social welfare are weakly higher when enacting the policy in \(t = 2\) than when enacting the policy in \(t = 1\). Consumer and social welfare is strictly higher if \(\Delta_k > \beta\) and \(\frac{1}{N}(\frac{\alpha}{\beta}k - c^v) > \max\{k - c^v, 0\} \geq \frac{1}{N}(\frac{\alpha}{\beta}k - c^v)\).

To see the intuition, suppose that \(k_1 = k_2 = k > \frac{\beta}{\alpha}c^v\). Then, each firm earns profits \(\frac{1}{N}(\frac{\alpha}{\beta}k - c^v) > 0\) by setting \((p^v = 0, p^a_3 = \frac{k}{\beta})\) and charging the add-on price only to naive consumers. If a firm deviates and charges the add-on price to both naive and time-consistent consumers, it would earn profits at most \(k - c^v\). Hence, such deviations may not be profitable for the firms.\(^{20}\) The effects on policies are qualitatively the same as those in Section 2.

### 5.2 Partial Naivete and Sophistication

So far, we have assumed that naive consumers are fully unaware of their naivete. In this subsection, we discuss the models in which naive consumers are partially or fully aware of their self-control problems.

First, following O’Donoghue and Rabin (2001), consider the case in which a fraction \(\alpha\) of consumers are partially naive: in \(t = 1\), they think that their present bias in \(t = 2\) will be equal to \(\hat{\beta} \in (\beta, 1)\). The remaining fraction of consumers are time-consistent. Proposition 10 shows that none of our results will change no matter how small the degree of naivete is:

\(^{19}\) See also Armstrong and Vickers (2012) and Grubb (2015) for the analysis under price floors.

\(^{20}\) Here, the logic of the existence of the positive-profit equilibrium is close to that of Heidhues, Kőszeigi and Murooka (2012), although our model is dynamic and firms do not have an option to educate naive consumers.
Proposition 10. Suppose that a fraction $\alpha$ of consumers are partially naive whereas the remaining fraction of consumers are time-consistent. Then, for any $\hat{\beta} \in (\beta, 1]$, all equilibrium outcomes in Section 2 remain the same.

To see the intuition, suppose that the firm sets $p^3 = k_2 / \beta$. Note first that the consumer behavior in $t = 2$ does not depend on $\hat{\beta}$. In $t = 1$, partially naive consumers think that they will switch in $t = 2$ if and only if $p^3 > k_2 / \hat{\beta}$. Since $k_2 / \beta > k_2 / \hat{\beta}$ for any $\hat{\beta} > \beta$, consumers think they will switch in $t = 2$. Given this, they do not switch in $t = 1$ if and only if $k_1 \geq \beta^i k_2$—consumer behavior both in $t = 1$ and $t = 2$ do not depend on $\hat{\beta}$.

Note that the firm sets $p^3 = k_2 / \beta$ whenever naive consumers procrastinate. Hence, akin to Heidhues and Köszegi (2010), the firm can make partially naive consumers procrastinate and can exploit them by setting the same add-on price. Consequently, all equilibrium outcomes are the same for any $\hat{\beta} \in (\beta, 1]$.

Next, consider the case in which consumers who are fully aware of their self-control problems ($\beta = \hat{\beta} < 1$) are also in the market. Although the presence of these sophisticated consumers change the condition in which the firm chooses to exploit, our policy implications still hold in the sense that the policy in $t = 1$ can be worse than no policy welfare whereas the policy in $t = 2$ is better than no policy. The detailed analysis is provided in the Supplementary Material.

5.3 Heterogeneous Demand

We now discuss the cases in which the consumers’ valuation for the base product or for the add-on is heterogeneous. The detailed analysis of each of the following cases is provided in the Supplementary Material.

If the valuation for the base product or for the add-on is heterogeneous, the equilibrium base-product price may be different. Similarly to Grubb (2015), under downward-sloping demand, a choice-enhancing or an active-choice policy may increase the equilibrium base-product price. The intuition is as follows. As in a simple monopoly problem, a firm faces the trade-off between charging

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21 Specifically, Heidhues and Köszegi (2010) show that in a general contracting setting, for any $\hat{\beta} > \beta$ the ex-ante incentive compatibility constraint (in our model, the condition that partially naive consumers procrastinate switching in $t = 1$) slacks so long as the ex-post incentive compatibility constraint (in our model, the condition that partially naive consumers do not switch in $t = 2$) binds.

22 Nocke and Peitz (2003) analyze the durable-good market under the presence of sophisticated present-biased consumers.
a high price for the base product (but only serving few consumers) and serving many consumers by setting a low price for the base product (but only making a small profit per consumer). In addition to the profits from the base product, the firm makes extra profits from the add-on. If a policy reduces the profits from the add-on, serving many consumers becomes less profitable for the firm. Hence, the policy may increase the base-product prices. This effect would not arise under competition on the base product, however.

Also, additional inefficiencies can arise. When the valuation for the base product is less than $c_v - 2a + \bar{u}$ for some naive consumers, the firm may sell the base product to these consumers at the price below the production cost in order to enroll them in the add-on. Similarly, when the valuation for the add-on is less than $\max\{\frac{k}{2}, k\}$ for some consumers, some consumers would not buy the base product at the equilibrium prices of Section 4. In this case, the firm may set lower equilibrium prices. In contrast, when the valuation for the add-on is larger than or equal to $\max\{\frac{k}{2}, k\}$, the equilibrium prices of Section 4 do not change.

6 Conclusion

We investigate the welfare consequences of policies which reduce consumers’ switching cost when a firm can change its strategy in response to a policy. We show that a conventional policy—reducing the switching cost at the time consumers are enrolled into a service—can decrease consumer and social welfare. We also suggest an alternative policy—reducing the switching cost when a firm charges a higher price for the service—which (weakly) increases consumer and social welfare compared to no policy or compared to the conventional policy. Our welfare and policy implications shed light on the design of choice-enhancing and active-choice policies. Beyond the model, the logic of our model and its policy implications seem applicable when rational consumers are more responsive to the change of an economic environment than naive consumers.

We conclude by discussing a couple of questions related to but beyond the scope of this paper. First, how to detect consumer naivete and an adverse policy effect from market data is both theoretically and practically important. One difficulty is that automatic enrollment itself may not be harmful to consumer and social welfare. For example, naive consumers may procrastinate taking up a valuable additional service if there is a registration cost and no automatic enrollment. In such a case, automatic enrollment itself is valuable, though it also creates the possibility for a firm to
exploit consumers as analyzed in this paper. In general, the way of detecting consumer naivete can be complicated and depend on the nature of the industry. As a potential future direction, investigating the usage or activation data as well as the purchase data could be helpful to identify consumer naivete and exploitation.

Second, this paper focuses on the present bias as a source of procrastination. Although the present bias is one of the most prevalent behavioral biases and our policy implications seem applicable when rational consumers are more responsive than naive consumers (except for deadlines as we clarified in Section 4.3), how to identify the type of consumer bias from data is an important issue. Also, optimal policies would depend on the type of consumer biases. Identifying the type of consumer biases from market data and investigating an optimal policy in a model with different consumer biases are left for future research.
Appendix: Proofs

Proof of Lemma 1.

Before the proof, we explicitly describe the consumer switching behavior. Notice that consumers who are still enrolled in period 2 do not switch in period 2 if and only if \( p^a \leq \frac{k_2}{\beta} \). In period 1, both types of consumers think that they would switch in period 2 if and only if \( p^a \leq k_2 \). Given these plans, the analysis can be divided into two cases. If \( p^a \leq k_2 \), consumers do not plan to switch in period 2 and hence do not switch in period 1 if and only if \( p^a \leq k_1 \). If \( p^a > k_2 \), consumers plan to switch in period 2 and hence do not switch in period 1 if and only if \( k_1 > \beta^i k_2 \). Being consistent with the full model, let \( \bar{u} \) denote the baseline utility of consumers.

Now we prove Lemma 1. In what follows, we analyze a slightly more general case in which \( k_1 = k_2 = k \). We divide the analysis into two cases.

First, suppose that the firm sells the service only to naive consumers. In this case, the firm sets \( p^a = \frac{1}{\beta} k \). Naive consumers do not switch, whereas time-consistent consumers switch either in period 1 or in period 2. The profits of the firm are \( \pi = \frac{\alpha}{\beta} k \) and the long-run utility of each type of consumers is \( u^N = \bar{u} - \frac{1}{\beta} k + 2a \) and \( u^{TC} = \bar{u} - k + 2a \).

Second, suppose that the firm sells the service to both naive and time-consistent consumers. In this case, the firm sets \( p^a = k \). The profits of the firm are \( \pi = k \) and the long-run utility of each type of consumers is \( u^N = u^{TC} = \bar{u} - k + 2a \).

By comparing the above two cases, we obtain the result.

Proof of Lemma 2.

(i) Note that time-consistent consumers do not switch in period 1 if and only if \( p^a \leq k \). Naive consumers do not switch in period 1 because \( -k < -\beta k \).

First, suppose that the firm sells the service only to naive consumers. In this case, the firm sets \( p^a = \frac{1}{\beta} k \). Naive consumers do not switch, whereas time-consistent consumers switch in period 1. The profits of the firm are \( \pi = \frac{\alpha}{\beta} k \) and the long-run utility of each type of consumers is \( u^N = \bar{u} - \frac{1}{\beta} k + 2a \) and \( u^{TC} = \bar{u} - k + 2a \).

Second, suppose that the firm sells the service to both naive and time-consistent consumers. In this case, the firm sets \( p^a = k \). The profits of the firm are \( \pi = k \) and the long-run utility of each type of consumers is \( u^N = u^{TC} = \bar{u} - k + 2a \).
By comparing the above two cases, we obtain the result.

(ii) Note that time-consistent consumers do not switch in period 1 if and only if \( p^a \leq \frac{1}{\beta} k \). Naive consumers do not switch in period 1 if and only if \( p_2^a \leq \frac{1}{\beta} k \).

First, suppose that the firm sells the service only to naive consumers. In this case, the firm sets \( p^a = \frac{1}{\beta} k \). Naive consumers do not switch, whereas time-consistent consumers switch in period 1. The profits of the firm are \( \pi = \frac{\alpha}{\beta} k \) and the long-run utility of each type of consumers is \( u^N = \bar{u} - \frac{1}{\beta} k + 2a \) and \( u^{TC} = \bar{u} - k + 2a \).

Second, suppose that the firm sells the service to both naive and time-consistent consumers. In this case, the firm sets \( p^a = k \). The profits of the firm are \( \pi = k \) and the long-run utility of each type of consumers is \( u^N = u^{TC} = \bar{u} - k + 2a \).

By comparing the above two cases, we obtain the result.

\[ \square \]

**Proof of Proposition 1.**

The conditions in which the policy in \( t = 1 \) increases the equilibrium price and decreases social welfare, \( 1 \geq \frac{\alpha}{\beta} > \Delta_k > \beta \), are immediate from Lemma 1 and Lemma 2.

Given \( 1 \geq \frac{\alpha}{\beta} > \Delta_k > \beta \), the total consumer surplus under no policy is \( \bar{u} + 2a - k \), whereas under the policy in \( t = 1 \) it is \( \bar{u} + 2a - \alpha \frac{1}{\beta} k - (1 - \alpha) k \). Comparing these two cases, we get the condition in which the policy in \( t = 1 \) decreases consumer welfare if and only if \( \frac{\alpha}{\beta} + (1 - \alpha) \cdot \Delta_k > 1 \).

\[ \square \]

**Proof of Lemma 3.** Note that consumer behavior in each case is described in the proof of Lemma 1. Again, we divide the analysis into two cases.

First, suppose that the firm sells the service only to naive consumers. In this case, the firm sets \( p^a = \frac{1}{\beta} k \). Naive consumers do not switch, whereas time-consistent consumers switch in period 2. The profits of the firm are \( \pi = \frac{\alpha}{\beta} k \) and the long-run utility of each type of consumer is \( u^N = \bar{u} - \frac{1}{\beta} k + 2a \) and \( u^{TC} = \bar{u} - k + 2a \).

Second, suppose that the firm sells the service to both naive and time-consistent consumers. In this case, the firm sets \( p^a = k \). The profits of the firm are \( \pi = k \) and the long-run utility of each type of consumers is \( u^N = u^{TC} = \bar{u} - k + 2a \).

By comparing the above two cases, we obtain the result.

\[ \square \]

**Proof of Proposition 2.**

Immediate from Lemma 1 and Lemma 3. \[ \square \]
Proof of Proposition 3.

Immediate from Lemma 2 and Lemma 3.

Proof of Lemma 4.

Before the proof, we explicitly describe the consumer behavior on the purchase of the base product in $t = 0$. Since naive consumers (wrongly) think that they will behave as if they were time-consistent, the participation constraint in $t = 0$ is identical to all consumers. Given the switching decisions regarding the add-on in the main text, each consumer takes up the base product in $t = 0$ if and only if one (or both) of the following two conditions is satisfied; (i) the total perceived utility of buying the base product and the add-on from the monopoly firm exceeds the outside option: $\beta^t[v + (T - 1)a - p^\nu - \sum_{t=3}^T p^a_t] \geq \beta^t\bar{u}$, (ii) the total perceived utility of buying the base product and switching in period $\hat{t}$ exceeds the outside option for some $\hat{t} \in \{2, \ldots, T - 1\}$: $\beta^t[v + (T - 1)a - p^\nu - k_{\hat{t}} - \sum_{t=3}^\hat{t} p^a_t] \geq \beta^t\hat{u}$. Note that (i) is equivalent to $p^v + \sum_{t=3}^T p^a_t \leq V_T$.

It is easy to show that the firm sells its add-on (i.e., charges $p^a_t$) to some consumers in every period: $p^a_t \leq k_t$. It is also easy to show that if time-consistent consumers pay $p^a_t$, then naive consumers also pay $p^a_t$. From the above two participation constraints in $t = 0$, we can divide the firm’s maximization problem into two cases: $p^v + \sum_{t=3}^T p^a_t \leq V_T$ and $V_T - \sum_{t=3}^T p^a_t < p^v \leq V_T - \min_{\hat{t}}[k_{\hat{t}} + \sum_{t=3}^\hat{t} p^a_t]$. In the former case, it is optimal for the firm to sells the add-on to both naive and time-consistent consumers. In the latter case, the firm sells the add-on only to naive consumers from period $\hat{t}$ on.

Now we prove Lemma 4. In what follows, we analyze a slightly more general case in which $k_t = k$ for all $t$. We divide the analysis into two cases.

First, suppose that the firm sells the add-on only to naive consumers. In this case, the maximal add-on price the firm can charge to naive consumers is $p^a_T = \frac{1-\beta}{\beta^3}k$ as we showed in Lemma 1. Given this, in period $t \leq T - 2$ naive consumers prefer to switch in the next period $t + 1$ to switch in the current period $t$ if and only if $p^a_t \leq \frac{1-\beta}{\beta^3}k$ as described in the main text. As a result, the firm sets $p^v = V_T - k$, $p^a_T = \frac{1-\beta}{\beta^3}k$, and $p^a_t = \frac{1-\beta}{\beta^3}k$ for all $t \in \{3, \ldots, T - 1\}$. Naive consumers do not switch, whereas time-consistent consumers switch in period 1. The profits of the firm are $\pi = V_3 - e^v + (\frac{\alpha}{\beta} - 1)k + (T - 3)\frac{\alpha}{\beta}(1 - \beta)k$ and the long-run utility of each type of consumers is $u^N = \bar{u} - (T - 2)\frac{1-\beta}{\beta^3}k$ and $u^{TC} = \bar{u}$.

Second, suppose that the firm sells the add-on to both naive and time-consistent consumers.
In this case, the firm sets \( p^v + \sum_{t=3}^{T} p_t^a = V_T \) with \( \sum_{t=3}^{T} p_t^a \leq k \). The profits of the firm are \( \pi = V_T - c^v \) and the long-run utility of each type of consumers is \( u^N = u^{TC} = \bar{u} \).

By comparing the above two cases, we obtain the result. \(\Box\)

**Proof of Lemma 5.**

Note that consumer behavior in each case is described in the proof of Lemma 4.

(i) Notice that time-consistent consumers do not switch in period 1 if and only if \( \sum_{t=3}^{T} p_t^a \leq k \).

Because \(-k < -\beta k\), naive consumers do not switch in period 1.

First, suppose that the firm sells the add-on only to naive consumers. In this case, the firm sets \( p^v = V_T - k \), \( p^a_T = \frac{1}{\beta} k \), and \( p^a_t = \frac{1-\beta}{\beta} k \) for all \( t \in \{3, \ldots, T-1\} \) as in Lemma 4. Naive consumers do not switch, whereas time-consistent consumers switch in period 1. The profits of the firm are \( \pi = V_T - c^v - k + \frac{\alpha}{\beta}[1 + (T-3)(1-\beta)]k \) and the long-run utility of each type of consumers is \( u^N = \bar{u} + k - \frac{1}{\beta}[1 + (T-3)(1-\beta)]k \) and \( u^{TC} = \bar{u} \).

Second, suppose that the firm sells the add-on to both naive and time-consistent consumers. In this case, the firm sets \( p^v + \sum_{t=3}^{T} p_t^a = V_T \) with \( \sum_{t=3}^{T} p_t^a \leq k \). The profits of the firm are \( \pi = V_T \) and the long-run utility of each type of consumers is \( u^N = u^{TC} = (T-1)a - k \).

By comparing the above two cases, we obtain the result.

(ii) Notice that time-consistent consumers do not switch in period 1 if and only if \( \sum_{t=3}^{T} p_t^a \leq k \).

Naive consumers do not switch in period 1 if and only if \( \beta \sum_{t=3}^{T} p_t^a \leq k \) because given \(-k \geq -\beta k\) naive consumers always prefer to switch in period 1 rather than to switch in any subsequent period.

First, suppose that the firm sells the add-on only to naive consumers. In this case, the firm sets \( p^v = V_T - k \) and \( \sum_{t=3}^{T} p_t^a = \frac{1}{\beta} k \). Naive consumers do not switch, whereas time-consistent consumers switch in period 1. The profits of the firm are \( \pi = V_T - c^v + \frac{\alpha}{\beta} k \) and the long-run utility of each type of consumers is \( u^N = \bar{u} - \frac{1-\beta}{\beta} k \) and \( u^{TC} = \bar{u} \).

Second, suppose that the firm sells the add-on to both naive and time-consistent consumers. In this case, the firm sets \( p^v + \sum_{t=3}^{T} p_t^a = V_T \) with \( \sum_{t=3}^{T} p_t^a \leq k \). The profits of the firm are \( \pi = V_T - c^v \) and the long-run utility of each type of consumers is \( u^N = u^{TC} = \bar{u} \).

By comparing the above two cases, we obtain the result. \(\Box\)

\(23\) In addition to \((p^v = V_T - k, \sum_{t=3}^{T} p_t^a = k)\), there are multiple equilibria for charging a higher \( p^v \) and a lower \( \sum_{t=3}^{T} p_t^a \). We can pin down the equilibrium base-product price by assuming that a tiny fraction of consumers exit the market at the end of \( t = 1 \) and cannot use the add-on. The same argument can be applied to the subsequent lemmas.
Proof of Proposition 4.

Immediate from Lemma 4 and Lemma 5.

Proof of Lemma 6.

Note that consumer behavior in each case is described in the proof of Lemma 4. We divide the analysis into two cases.

We first analyze the case in which the firm sells the add-on to both naive and time-consistent consumers. In this case, the firm sets \( p^v + \sum_{t=3}^{T} p_t^o = V_T \) with \( \sum_{t=3}^{T} p_t^o \leq k \). The profits of the firm are \( \pi = V_T - c^v \) and the long-run utility of each type of consumers is \( u^N = u^{TC} = \bar{u} \).

Second, suppose that the firm sells the add-on only to naive consumers. In this case, the firm sets \( p^v = V_T - k \) and \( p_T^o = \frac{1}{\beta}k \), and \( p_t^o = \frac{1-\beta}{\beta}k \) for all \( t \in \{3, \cdots, T-1\} \). In this case, the firm voluntarily reduces the switching cost to \( k \) in any period after the firm is forced to do so by the policy. Suppose that \( k_t = k \) and that the firm decreases \( k_{t+1} \). On the one hand, this makes naive consumers more likely to believe that they will switch in future, and hence makes them more likely to procrastinate their switching decision by relaxing the constraint of not switching in period \( t \): \( k \geq \beta(p_t^o + k_{t+1}) \). On the other hand, it tightens the constraint of not switching in period \( t + 1 \): \( k_{t+1} \geq \beta(p_{t+1}^o + k_{t+2}) \). However, the latter constraint is not binding because the firm has to decrease its switching cost whenever charging a higher price by the policy. To show this, suppose that the firm didn’t decrease \( k_{t+1} \) voluntarily. In such a case, the firm can charge at most \( p_{t+1}^o \leq \frac{1}{\beta}k - k \), which is less than \( \frac{1-\beta}{\beta}k \). To make use of the relaxed constraint \( k_{t+1} \geq \beta(p_{t+2}^o + k_{t+2}) \), the firm would have to increase the price \( p_{t+1}^o \). By doing so, however, the firm has to reduce \( k_{t+1} \) and can charge at most \( p_{t+2}^o \leq \frac{1}{\beta}k - k_{t+2} \leq \frac{1-\beta}{\beta}k \) because of the policy. Hence, compared to the situation in which the firm sets the switching cost to \( k \) in any period after the policy is implemented, the firm cannot increase its profits by setting a higher switching cost under the policy. Given that, the firm charges a positive add-on price in \( t = 2 \), and then keeps the add-on prices constant with setting a low \( k \). Naive consumers do not switch, whereas time-consistent consumers switch in period 1. The profits of the firm are \( \pi = V_T - c^v + (\frac{a}{\beta} - 1)k + (T-3)\frac{a}{\beta}(1-\beta)k \) and the long-run utility of each type of consumers is \( u^N = \bar{u} - (T-2)\frac{1-\beta}{\beta}k \) and \( u^{TC} = \bar{u} \).

By comparing the above two cases, we obtain the result.

Proof of Proposition 5.

Immediate from Lemma 4 and Lemma 6.
Proof of Proposition 6.

Immediate from Lemma 4 with \( k_t = \overline{k} \) for all \( t \) and Lemma 6.

Proof of Proposition 7.

Note first that time-consistent consumers’ utility is not affected by the policy and is always \( u^{TC} = \bar{u} \).

Let \( t \) and \( \overline{t} \) be the first and the last period of a sequence of periods such that \( k_t = \infty \) for all \( t \in \{t, \ldots , \overline{t}\} \). Then, naive consumers do not switch in period \( t - 1 \) if and only if \( k_{t - 1} \geq \beta \left( \sum_{l=t-1}^{t} p_t^a + k_T \right) \) for some \( \tau \in \{t, \ldots , T - 1\} \) or \( k_{t - 1} \geq \beta \left( \sum_{l=t}^{T} p_t^a \right) \). So the maximum total payment the firm can charge (weakly) decreases as increasing the number of periods in which consumers cannot switch. Charging lower prices potentially benefits naive consumers and potentially increases social welfare when time-consistent consumers do not switch anymore.

Analogous to Lemma 4 with \( T = 3 \), if \( t = 2 \) is the last period in which a consumer can cancel the contract and if \( \alpha > \beta \), then the firm sets \( p^v = V_T - \overline{k} \) and \( \sum_{t=3}^{T} p_t^a = \frac{1}{2} \overline{k} \). Naive consumers do not switch, whereas time-consistent consumers switch either in period 1 or period 2. The profits of the firm are \( \pi = V_T - c^v + (\frac{\alpha}{\beta} - 1)\overline{k} \) and the long-run utility of each type of consumers is \( u^N = \bar{u} - \frac{1 - \beta}{\beta} \overline{k} \) and \( u^{TC} = \bar{u} \). If \( t = 2 \) is the last period in which a consumer can cancel the contract and if \( \alpha \leq \beta \), then the firm sets \( p^v + \sum_{t=3}^{T} p_t^a = V_T \). The profits of the firm are \( \pi = V_T - c^v \) and the long-run utility of each type of consumers is \( u^N = u^{TC} = \bar{u} \).

Comparing this to Lemma 4 delivers the result.

Proof of Proposition 8.

Suppose that a symmetric equilibrium exists in which firms earn positive profits. Then, each firm can profitably deviate by offering the same add-on price and a slightly lower base-product price, because the deviating firm can attract all consumers and each consumer’s behavior about the add-on purchase does not change—a contradiction.

As firms make zero profits in equilibrium, the base-product price equals the production cost minus the total profits from the add-on. Similar to the analysis in Section 2, the outcomes are summarized as follows:

First, suppose that \( k_1 \geq k_2 \). If \( \alpha > \beta \), there exists an equilibrium in which \( (p^v = c^v - \frac{\alpha}{\beta} k_2, p_3^a = \frac{1}{\beta} k_2) \). If \( \alpha \leq \beta \), there exists an equilibrium in which \( (p^v = c^v - k_2, p_3^a = k_2) \).
Second, suppose that \( k_1 = \hat{k} < k = k_2 \) and \( \Delta_k > \beta \). If \( \alpha > \beta \Delta_k \), there exists an equilibrium in which \( (p^v = c^v - \frac{\alpha}{\beta}k, p_3^0 = \frac{1}{\beta}k) \). If \( \alpha \leq \beta \Delta_k \), there exists an equilibrium in which \( (p^v = c^v - \hat{k}, p_3^0 = \hat{k}) \).

Third, suppose that \( k_1 = \hat{k} < k = k_2 \) and \( \Delta_k \leq \beta \). If \( \alpha > \beta \), there exists an equilibrium in which \( (p^v = c^v - \frac{\alpha}{\beta}k, p_3^0 = \frac{1}{\beta}k) \). If \( \alpha \leq \beta \), there exists an equilibrium in which \( (p^v = c^v - \hat{k}, p_3^0 = \hat{k}) \).

By comparing the above three cases, we obtain the result.

\[ \square \]

**Proof of Proposition 9.**

The argument in the proof of Proposition 8 implies that in any positive-profit equilibrium firms set \( p^v = 0 \). Also, if all consumers pay the add-on price, then the standard Bertrand-type price competition argument leads to \( p^v + p_3^0 = c^v \). On the other hand, If only naive consumers pay the add-on price, then the firms may be able to earn positive profits because of the constraint \( p^v \geq 0 \). To see it, consider a candidate equilibrium \( (p^v = 0, p_3^0 = \frac{1}{\beta}k_2) \). If a firm deviates from the candidate equilibrium and charges the add-on price to both naive and time-consistent consumers, the deviating firm can charge the total payment at most \( p^v + p_3^0 = \min\{k_1, k_2\} \) in order to attract these consumers. The analysis of the case in which \( (p^v = 0, p_3^0 = \frac{1}{\beta}k_1) \) is a candidate equilibrium is the same. Similar to the previous analysis, the outcomes are summarized as follows:

First, suppose that the policy is enacted in \( t = 2 \) or when the policy is enacted in \( t = 1 \) and \( \Delta_k \leq \beta \). If \( \frac{1}{N}(\frac{\alpha}{\beta}k - c^v) > \max\{k - c^v, 0\} \), there exists a positive-profit equilibrium in which \( (p^v = 0, p_3^0 = \frac{1}{\beta}k) \). If \( \frac{1}{N}(\frac{\alpha}{\beta}k - c^v) \leq \max\{k - c^v, 0\} \), there exists a zero-profit equilibrium in which \( p^v + p_3^0 = c^v \).

Second, suppose that the policy is enacted in \( t = 1 \) and \( \Delta_k > \beta \). If \( \frac{1}{N}(\frac{\alpha}{\beta}k - c^v) > \max\{k - c^v, 0\} \), there exists a positive-profit equilibrium in which \( (p^v = 0, p_3^0 = \frac{1}{\beta}k) \). If \( \frac{1}{N}(\frac{\alpha}{\beta}k - c^v) \leq \max\{k - c^v, 0\} \), there exists a zero-profit equilibrium in which \( p^v + p_3^0 = c^v \).

By comparing the above two cases, we obtain the result.

\[ \square \]

**Proof of Proposition 10.**

Note that actual consumer behavior in \( t = 2 \) does not change because \( \hat{\beta} \) is not relevant to her actual decision in \( t = 2 \). In \( t = 1 \), partially naive consumers think that they will not switch in \( t = 2 \) if and only if \( p_3^0 \leq k_2/\hat{\beta} \). Conditional on this belief, consumers’ switching behavior in \( t = 1 \) can be divided into the following two cases. First, if \( p_3^0 \leq k_2/\hat{\beta} \), consumers think they will not switch in \( t = 2 \). Given this, they do not switch in \( t = 1 \) if and only if \( p_3^0 \leq k_1/\beta \). Second, if \( p_3^0 > k_2/\hat{\beta} \),
consumers think they will switch in $t = 2$. Given this, they do not switch in $t = 1$ if and only if $k_1 \geq \beta k_2$. Note that $k_2/\hat{\beta} < k_2/\beta$ for any $\hat{\beta} \in (\beta, 1]$ and that the firm always sets $p^a_3 = k_2/\beta$ whenever naive consumers procrastinate as in Section 2. Hence, akin to Heidhues and Kőszegi (2010), the firm can make partially naive consumers procrastinate and can exploit them by setting the same add-on price as in Section 2 which is irrespective of $\hat{\beta} \in (\beta, 1]$. □
References


