Learning about one’s own type in two-sided search

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Abstract

This paper is an analysis of a two-sided search model in which agents are vertically heterogeneous and some agents do not know their own types. Agents who do not know their own types update their beliefs about their own types through the offers or rejections that they receive from others. In the belief-updating process, an agent who is unsure of his or her own type frequently behaves as if he or she were an over- or underconfident agent. In this paper, we show that this apparent over- or underconfidence influences both the individual’s and other agents’ matching behaviors. We show, especially, that the apparent overconfidence of some agents prevents the lowest-type agents from matching. However, the apparent underconfidence of some agents does not affect the matching of the lowest-type agents.

JEL Classification Numbers: D82, D83, J12

Key Words: two-sided search; imperfect self-knowledge; overconfidence; looking-glass self
1 Introduction

The “looking-glass self” has been the dominant concept in sociology and social psychology for the development of the self. The idea, attributed to Cooley (1902), is that people form their self-views by observing how others treat them. That is, others are significant as the “mirrors” that reflect images of the self. Although there is much literature on the “looking-glass self” in the field of sociology and social psychology, the topic has received little attention in economics.\(^1\)

In this paper, we introduce the “looking-glass self” to a two-sided search model and study the implications of the “looking-glass self” in the search behavior. We construct a model in which searchers do not know their own types, although they know the types of others. They then update their beliefs about their own types when they receive offers or rejections from others. For example, workers in search of an employer are evaluated by employers on their abilities when they meet. When a worker is young in terms of experience, his or her self-assessment is based on limited experience. On the other hand, employers may have considerable experience in evaluating workers. At this time, when a young worker observes an offer or rejection from an employer, he or she learns something about his or her own type. Of course, when an experienced worker searches for a new job that is very similar to his or her previous job, he or she may have a more accurate self-view of his or her ability than employers. However, such situations are not considered in this paper. The key feature of this study is that others have better information about agents’ types than the agents themselves. Similarly, in the search for a marriage partner, a single agent is evaluated with regard to his or her marital charms by a member of the opposite sex when they meet. When an agent is young, his or her self-assessment is based on limited experience, such as academic achievement and family background. However, since marital charm is composed of various elements, an individual of the opposite sex may have better assessments of the agents’ charm than the agents themselves.\(^2\) Hence, when an agent observes an offer or rejection from a member of the opposite sex, he or she infers something about his or her own type. In this paper, we show that this looking-glass self influences both their own and other agents’ search behaviors.

We consider the basic framework of Burdett and Coles (1997), which is a two-sided search model with complete information. Their model can treat the marriage market, the labor market, the housing market, and other markets in which heterogeneous buyers and sellers search for the right trading partner. Using the marriage market interpretation, the model is described as follows. Single agents are vertically heterogeneous, i.e., there exists a ranking of marital charm (types). Single men/women enter the market in order to look for a marital partner. When a man and a woman meet, an opponent’s type can be recognized. The agent’s optimal search strategy has the reservation level property, i.e., he or she continues

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\(^1\)As we discuss below, in economics, Bénabou and Tirole (2003), Ishida (2006), and Swank and Visser (2006) consider “the looking-glass self” in principal-agent models.

\(^2\)Marital charm is defined by various elements, including quality, attraction, intelligence, height, age, education, income, position at work, social status and family background, in much of the literature regarding marriage.
searching until he or she meets a member of the opposite sex that is at least as good as the predetermined threshold, called the “reservation level,” which depends on the agent’s search cost and the type distribution of agents. If a man and a woman meet and both agents propose, they marry and leave the market. If at least one of the two decides not to propose, they separate and continue to search for another partner. Given these settings, the marriage pattern (i.e., who marries whom) in the market is determined. This marriage pattern becomes a kind of positive assortative matching.\(^3\)

When an agent (she) is unsure of her own type, she often behaves as if she were an over- or underconfident agent in her learning process. The woman with imperfect self-knowledge may reject a man whom she accepts when she knows her own type. Therefore, this woman will apparently overestimate her actual type. However, agents in this study are unbiased and rational, since this apparent overconfidence is generated due to the correct belief-updating process in the sense that there is no error in an agent’s processing information.\(^4\) Then, agents in this paper have no false information and follow Bayes’s rule in updating their posterior beliefs about their own types. Likewise, we use the term apparent underconfidence if a woman with imperfect self-knowledge accepts a man whom she rejects when she knows her own type.

Our results are as follows. The apparent overconfidence in an agent’s learning processes generates two externalities: the first is direct externality: the rejection from an apparently overconfident woman delays the timing of marriage of the man who is directly rejected by her when they meet. If there are many apparently overconfident women in the market, the second externality is generated: the men who are now rejected by the apparently overconfident women accept another lower type of women who are rejected by these men when all agents know their own types. We call this change in an agent’s behavior due to the apparent overconfidence of other agents indirect externality. Moreover, in a two-sided search framework, the women who are now accepted by these men also reject the men whom they accept when all agents know their own types. Then, the indirect externality spreads across the market. As a result, the lowest-type agents cannot marry.

On the other hand, the apparent underconfidence does not generate the indirect externality. This is because if the indirect externality of apparent underconfidence occurs; in other words, if the men who are now accepted by apparently underconfident women reject the women whom these men accept when all agents know their own types, the offers from these men to the apparently underconfident women inform these apparently underconfident women that they are higher-type women than they think. As a result, the apparent underconfident women have the incentives to reject these men. Hence, even if there are many apparently underconfident women, the men who are now accepted by these women always accept the women whom they accept when all agents know their own types. Therefore, the apparent

\(^3\)Positive assortative matching is said to hold if the characteristics (types and marital charm) of those who match are positively correlated. Becker (1973) found strong empirical evidence of a positive correlation between the characteristics of partners.

\(^4\)In contrast, ‘overconfidence’ is generally generated due to some errors in an agent’s processing information. For example, overconfidence would occur when an agent overestimates the actual distributions of types in the market.
underconfidence has only the *direct externality*: the acceptance by an apparently underconfident woman makes the future partner better off, as she increases the value of the match to the partner.

Our results also show that, when there are agents with an imperfect self-knowledge under the cloning assumption and the assumption of non-transferable utility, multiple equilibria can arise in some parameter ranges. By contrast, when all agents know their own types under the cloning assumption and the assumption of non-transferable utility, a unique equilibrium always arises (see Burdett and Coles (1997)). In our multiple equilibrium case, marriage patterns are determined by all agents’ expectations about the behaviors of agents with imperfect self-knowledge.

The result in which the lowest-type agents cannot marry in an apparent overconfidence case is consistent with the recent data of educational assortative marriage patterns in the United States and Japan. In the U.S. and Japan, the percentage of never married men/women is increasing and, in particular, that of never married men/women with a low level of education is notably high.\(^5\) According to the U.S. Census Bureau data, in 2006, the percentage of never-married individuals at age 35-44 was 24% for men with high school education or less and 14% for women with high school education or less. On the other hand, the percentage of never-married individuals at age 35-44 was 14% for men with some college education or more and 12% for women with some college education or more. Moreover, in Japan, the decline in marriage has been most pronounced among less-educated men at age 35-39 (Raymo and Iwasawa 2005). Our results suggest that the apparent overconfidence accelerates the increase in the proportion of never-married men/women with a low level of education, since education is one of the elements of charm.

**Related literature**

Early psychologists and sociologists thought that the self was built on reflected assessments—people form their self-views by observing how others treat them. James (1890), who set the stage for the idea of “looking-glass self”, argued that the self was a product and reflection of social life. The idea of the “looking-glass self” was introduced by Cooley (1902). He expanded the idea that the self develops by referencing other people in the social environment. Cooley maintained that the person observes how others view him- or herself and then incorporates those views into the self-view. Mead (1934) further developed the idea of Cooley (1902).\(^6\) Following this long tradition, most researchers in psychology and sociology accepted that others are significant as the “mirrors” which construct and modify the self-view (for example, Goffman (1959), Baumeister (1982, 1986), Wicklund and Gollwitzer (1982), Gollwitzer (1986), Rhodewalt (1986), Schlenker (1986), Swann (1987, 1990, 1996), Cole (1991), Kenny

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\(^5\) Schwartz and Mare (2005) explain that the trend of educational assortative marriage in the U.S. is led by the declining economic standing of men with a low level of education from the late 1970s through the mid-1990s. Nosaka (2009) shows that an increase in the wages of highly educated women leads to an increase in the number of unmarried individuals among persons with low education in a marriage model.

\(^6\) In his view, people are affected not only by how they think significant others respond to them but also by how they think their entire social group does.
and Depaulo (1993), Swann, De La Ronde, and Hixon (1994), Bartusch and Matsueda (1996), Kelly (2000), and Tice and Wallace (2003)).

In economics, recent work has introduced the idea of “looking-glass self” (for example, Bénabou and Tirole (2003), Ishida (2006), and Swank and Visser (2006)). Bénabou and Tirole (2003), who presented the principal-agent model, assume that, whereas the principal knows the agent’s type, an agent has imperfect knowledge about his own type. As the principal prefers to offer the challenging task when facing an agent with high ability, this offer becomes the signal that the principal trusts the agent. Thus, giving a challenging task increases the agent’s self-confidence. Given that effort and ability are usually complements, the more confidence the agent has with regard to his ability, the more effort he exerts. Swank and Visser (2006) focus on the quality of the principal’s information about the agent’s type and show that the quality of the principal’s information determines whether or not a delegation can be used as a means of communicating this information to an agent. Ishida (2006) applies the framework of Bénabou and Tirole (2003) to promotion policies. He endogenizes the degree of information asymmetry and derives dynamic implications. In contrast with these studies, we apply the idea of the “looking-glass self” to the two-sided search model and not to the principal-agent model.

In our model, some women with imperfect self-knowledge behave as if they were over- or underconfident agents. Our paper is then related to the studies of over- or underconfidence. A study by Dubra (2004) is the most closely related one to ours; Dubra examines the implications of overconfidence in the search behaviors of workers. The difference from our model is that his is a one-sided search and an overconfident worker has false prior belief (then, overconfident workers in his model overestimate their chances of finding a better offer). In this paper, although apparently overconfident agents do not know their own types, they do not have false prior beliefs; i.e., they do not over- or underestimate the type distributions in the market. Moreover, the two-sided problem generates the indirect externality of apparent overconfidence. There are several recent studies about overconfidence of workers in addition to the study by Bénabou and Tirole (2003) and Dubra (2004) reported earlier. Camerer and Lovallo (1999) show that overconfidence about relative ability leads to excessive business entry by creating experimental entry games. Santos-Pinto and Sobel (2005) show that a subjective positive/negative self-image arises when different people have different opinions about how skills determine ability. Furthermore, Benoît and Dubra (2008) show that, even if everyone perfectly understands the level of skills in the population, they are apparently overconfident in a signaling model. Our agents are similar to agents in their model in the sense that they are unbiased and rational, have correct information, and are Bayesian. However, we examine the influence of this rational learning process on agents’ search behaviors.

A body of literature supports the looking-glass self theory with respect to the evaluative quality of the self’s attributes. Pinhey, Rubinstein, and Colfax (1997) found that overweight people were not significantly happy in cultures in which thinness was valued (see also Ross (1994) and Cioffi (2000)). Ichiyama (1993) conducted an experiment and showed that others’ actual assessments of interpersonal behavior were linked to self-assessments and reflected assessments (what participants think others think of them). Furthermore, Jussim, Soffin, Brown, Ley, and Kohlhoff (1992) conducted several experiments and found that, when people were fully aware of how others viewed them, their self-perceptions were indeed affected by reflected assessments.
This paper is organized as follows. Section 2 is a description of the basic framework for our analysis. In Section 3, first, we derive a perfect sorting equilibrium in which only persons of the same type marry if all agents know their own types as a benchmark case. Next, we examine the case of a perfect sorting equilibrium with imperfect self-knowledge. Thirdly, we investigate the apparent overconfidence case. Finally, we examine an apparent underconfidence case. In Section 4, we analyze the number of marriages and social welfare generated by marriages. In Section 5, we discuss the extensions of the model. Section 6 is the conclusion.

2 The basic framework

In this section, we present a basic framework for our analysis in this study. Throughout, we restrict our attention to the steady state.

Let us assume that there are a large and equal number of men and women in a marriage market. Let \( N \) denote the participating men/women in this market. An agent in the market wishes to marry a member of the opposite sex.

Finding a marriage partner always involves a time cost. It is difficult for agents to meet someone of the opposite sex in the market. Let \( \alpha \) denote the rate at which a single individual contacts a member of the opposite sex, where \( \alpha \) is the parameter of the Poisson process.

It is assumed that agents are ex-ante heterogeneous and all agents have the same ranking about a potential partner in the marriage market. Let \( x_k \) denote the type (charm) of a single man/woman \( k \) in the market; it is assumed to be a real number.

When both sexes meet, each agent can instantly recognize the opponent’s type \( x_k \) and then decide whether or not to propose. We assume that both agents submit their offers or rejections simultaneously in order to simplify our analysis. If at least one of the two decides not to propose, they return to the marriage market and search for another partner. If both agents propose, they marry and leave the marriage market permanently. We assume that, if a couple marries, he or she obtains a utility flow equal to the spouse’s type per unit of time and vice versa. That is, utilities are non-transferable: there is no bargaining for the division of the total marital utility. Let us assume that people live forever and there is no divorce.

Let us assume that \( x_k \) is drawn from \( F_i(x), i = m, w \), which denotes the distribution of actual types among men (m)/women (w) in the market. Here, \( F_m(x) \) and \( F_w(x) \) need not be symmetric among men and women. All agents know \( F_m(x) \) and \( F_w(x) \).

An equilibrium is a steady state in which all agents maximize their expected discounted utilities given that they have correct expectations about the strategies of all others in the market. A steady state requires that the exit rate of each type equals the entry rate of new agents of that type. To simplify the analysis, we assume that, if a pair marries and leaves the market, two identical agents enter the market at once (see, for example, MacNamara and Collins (1990), Morgan (1994), Burdett and Coles (2001), Bloch and Ryder (2000), and Cornelius (2003)).

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8Some other assumptions of “inflow” have been considered by Burdett and Coles (1999). With other
### 3 Analysis

In this section, first, to show the externality of learning process, we derive the conditions under which the economy is at a perfect sorting equilibrium, in which only persons of the same type marry, if all agents know their own types as a benchmark. In later subsections, we study three cases with *imperfect self-knowledge* (i.e., agents do not know their own types perfectly) and compare these three cases with the benchmark case.

#### 3.1 Perfect self-knowledge—Benchmark result

In this subsection, we consider the *perfect self-knowledge* case in which all agents know their own types. To simplify the analysis, let us assume that there are three types of men/women according to charm: high (*H*), middle (*M*), and low (*L*). A participant in a marriage market belongs to one of these types. Let $x_H/r$ denote the (discounted) utility of marrying a high-type agent; similarly, $x_M/r$ and $x_L/r$ represent the utilities of marrying a middle-type agent and a low-type agent, respectively, where $r > 0$ is the discount rate. We assume that $x_H > x_M > x_L > 0$. That is, in any equilibrium, all agents would like to marry a high-type agent. Both sexes are assumed to obtain zero utility flow while they are single.

Let $\lambda_H^i (i = m, w)$ denote the share of high-type men/women in the marriage market. Similarly, $\lambda_M^i$ and $\lambda_L^i$ are the share of men/women who belong to the middle- and low-type, respectively, where $\lambda_H^i + \lambda_M^i + \lambda_L^i = 1$. Here, $\lambda_k^i (k = H, M, L)$ of each sex ($i = m, w$) need not be symmetric among men and women.

First, we consider the decision of a high-type man. He decides whether to accept or reject a woman of the middle- or low-type. The expected discounted lifetime utility of a single high-type man $V_H$ becomes

$$
rv_H = a\lambda_H^w \left( \frac{x_H}{r} - V_H \right) + a\lambda_M^w \left[ \max \left( V_H, \frac{x_M}{r} \right) - V_H \right] + a\lambda_L^w \left[ \max \left( V_H, \frac{x_L}{r} \right) - V_H \right].
$$

(1)

A high-type man ($k = H$) meets a high-type woman with probability $a\lambda_H^w$, and they marry. However, if a high-type man meets a middle- (low-) type woman with probability $a\lambda_M^w$ ($a\lambda_L^w$), he compares $x_M/r$ ($x_L/r$) and $V_H$ and then decides whether or not to propose. By this comparison, we can obtain the optimal strategy of the high-type man given $\alpha$ and $F_i (x)$. This optimal strategy has the feature of the reservation level for rejecting other types; that is, a $k$-type man accepts any offer $x_{k'} \geq R_k$ from a $k'$-type woman, where $R_k = rv_{k'}$. Likewise, reasonable assumptions of inflow, such as exogenous inflow, the analysis is very complicated in our framework with learners. Moreover, now, our attention is focused not on the change of the type distribution, which is derived under an assumption of inflow, but on the marriage pattern (i.e., who marries whom) in a steady state when there are agents with imperfect self-knowledge. We then apply the “cloning assumption” to our model for technical simplicity.

As we will discuss in detail in Appendix C, we assume not two but three types of agents in order to show the indirect effect (indirect externality) of the learning process: even if the agents with perfect self-knowledge do not directly meet agents with imperfect self-knowledge, these agents with perfect self-knowledge may change their marriage behavior due to the existence of agents with imperfect self-knowledge in the market.
we can obtain the optimal reservation strategies of each type of men/women.

We restrict our attention to the next equilibrium in this article in order to show the influences of the learning process on a marriage market.

**Definition 1** In the perfect sorting equilibrium (PSE), high-type agents marry within their group, as do middle-type agents and low-type agents.

In the PSE, men and women of the same type marry. Therefore, we can consider that high-type agents who marry within their group form the first cluster of marriages, middle-type agents who marry within their group form the second cluster of marriages, and low-type agents who marry within their group form the third cluster of marriages in this equilibrium.

We now define the following situation as a benchmark case: if all agents know their own types, the PSE occurs. The following proposition shows the sufficient conditions for the PSE.

**Proposition 1** (PSE) Let us assume that all agents recognize their own types. The economy is at the PSE if

\[ x_M < R_H^* \equiv \frac{\alpha \lambda_H^i x_H}{\alpha \lambda_H^i + r}, \quad i = m, w, \]  

and

\[ x_L < R_M^* \equiv \frac{\alpha \lambda_M^i x_M}{\alpha \lambda_M^i + r}, \quad i = m, w. \]  

**Proof.** See, Appendix A. ■

The inequality (2) can be rewritten as the condition of parameter \( \lambda_H^i \). Therefore, Proposition 1 means that, with constant \( \alpha \), if \( \lambda_H^i \) is large enough or if the difference between \( x_H \) and \( x_M \) is large enough (\( \alpha \lambda_H^i > \frac{x_M}{\alpha \lambda_H^i + x_M} \)), a high-type agent turns down a middle- and a low-type opposite sex agent \( i \) in the market (\( x_M < R_H^* \)). Conversely, if there are sufficiently few high-type opposite sex agents \( i \) or if (\( x_H - x_M \)) is small enough (\( \alpha \lambda_H^i \leq \frac{x_M}{\alpha \lambda_H^i + x_M} \)), a high-type agent accepts a middle-type opposite sex agent \( i \) (\( x_M \geq R_H^* \)). A similar discussion can be done for parameter \( \lambda_M^i \) by inequality (3). If \( \lambda_H^i \) and \( \lambda_M^i \) are small enough to satisfy \( R_H^* \leq x_L \) and \( R_M^* \leq x_L \), all agents obtain the same expected discounted lifetime utility: \( V_L = V_M = V_H < \frac{x_L}{r} \). In this case, all types accept each other, and then all agents marry the first agent of the opposite sex that they meet.\(^10\)

It is noteworthy that, if \( r = 0 \), then \( x_M < R_H^* \) and \( x_L < R_M^* \) hold. Therefore, the equilibrium is the PSE when \( r = 0 \).

In the following subsections, we introduce the imperfect knowledge about agents’ own types into the benchmark case. To investigate the externality of the learning process, in the

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\(^{10}\)Four possible steady-state equilibrium outcomes can be considered when all agents have perfect self-knowledge: Equilibrium (i) agents of the same type marry (PSE); Equilibrium (ii) agents of the high-type and the middle-type form the first cluster of marriages, and agents of the low-type form the second cluster of marriages; Equilibrium (iii) agents of the high-type form the first cluster of marriages, and agents of the middle- and low-type form the second cluster of marriages; and Equilibrium (iv) all agents marry the first person of the opposite sex they meet (they form one cluster of marriage). From the proof of Proposition 1, equilibrium (ii) occurs when \( R_H \leq x_M, R_M > x_L \). Equilibrium (iii) occurs when \( R_H > x_M, R_M \leq x_L \). Equilibrium (iv) occurs when \( R_H \leq x_M, R_M \leq x_L \).
following subsections, we consider the case in which the condition in Proposition 1 is satisfied: $x_M < R^*_H$ and $x_L < R^*_M$ hold.\footnote{For other parameter ranges, it is difficult to show the indirect effect (indirect externality) of the learning process. We will discuss this in detail in Appendix C.}

### 3.2 Imperfect self-knowledge

In this subsection, we introduce the imperfect self-knowledge into the benchmark case. In an agent’s learning process, she/he will use a different strategy from that in the benchmark case. We then consider the following three cases in this subsection. First, we consider the case of PSE with imperfect self-knowledge, in which, even if there are some agents who do not know their own types, only persons of the same type marry. Next, we consider the case of apparent overconfidence, in which some women with imperfect self-knowledge accept the men whom they reject when they know their own types. Finally, we consider the case of apparent underconfidence, in which some women with imperfect self-knowledge reject the men whom they accept when they know their own types.

Let us assume that all agents understand the type distribution $F_m(x)$ and $F_w(x)$ and that all men know their own types. However, no women initially know their own types when they have just entered the marriage market.\footnote{This one-sided imperfect knowledge assumption can make the influence of imperfect self-knowledge clearer than the two-sided imperfect knowledge assumption. We discuss this in detail in Section 5. This one-sided imperfect knowledge assumption describes the following situations: in the context of the labor market, a firm has more information than a worker about its own type, since the firm will generally have more experience than the worker. In the context of the marriage market, this assumption describes a situation in which it would be easier for men than for women to obtain objective data on their own charm, such as income, position at work, and social status, in cultures in which more men than women work outside the home.} We refer to them as ‘$k_0$-type’ women, where $k (k = H, M, L)$ represents their actual type. Therefore, a woman who does not know her own type has a belief about her own type.

A woman with imperfect self-knowledge may learn something about her actual type after a meeting with a man. For example, let us consider that high-type men accept only high-type women and this is common knowledge among all agents. If a woman is rejected by a high-type man, she then learns that she is either a middle- or a low-type following a meeting with him. Therefore, a woman’s belief about her own type depends on men whom she met in the past. As a result, there are different kinds of women with different beliefs even if they belong to the same actual type. Let $G_m(x)$ and $G_w(x)$ denote the distribution of men according to differences in their beliefs and that of women according to differences in their beliefs, respectively. Let us assume that all agents also know $G_m(x)$ and $G_w(x)$ (we later show that $G_m(x)$ and $G_w(x)$ depend on $\alpha$ and $F_i(x)$, which is common knowledge among all agents). Therefore, all agents choose optimal strategies using $G_m(x)$ and $G_w(x)$. Moreover, let us assume that any woman’s prior belief about her own type is $G_w(x)$. Since all men know their own types, $G_m(x) = F_m(x)$. Let us assume that a woman updates the belief about her own type by Bayes’s rule, when the offer or rejection from a man carries information about her actual type. Throughout, we assume that individuals with the same beliefs use the same matching strategies.
3.2.1 PSE with imperfect self-knowledge

In this subsubsection, we discuss the PSE with imperfect self-knowledge (we call this equilibrium ‘PSEI’). First, we investigate the optimal strategies of men when there are women with imperfect self-knowledge. Men decide their optimal strategies given women’s behaviors (who accepts (or rejects) whom) in the market. We will use the term “behavior” to distinguish it from “strategy” in the following sections. In our model with discrete types, even if an agent lowers his reservation utility strategy, this does not guarantee that he accepts a woman whom he has rejected previously. Therefore, the statement that an agent changes his behavior means that he changes the type of women whom he is willing to accept. The strategy (and behavior) of each type is common knowledge among all agents, as all agents know $G_i(x), i = m, w$.

In the PSEI, $k_0$-type ($k = H, M, L$) women always reject middle-type men. Moreover, all high-type women (including high-type women with imperfect self-knowledge) always reject middle-type men, all middle-type women (including middle-type women with imperfect self-knowledge) always reject low-type men, and some low-type women always accept low-type men from the definition of the PSEI.

Given the above women’s behaviors, men decide their optimal strategies. A high-type man has the same reservation level as a high-type man in the PSE, since all women want to marry high-type men. Given this, the decision of a middle-type man is as follows. Let us assume that a fraction $\eta \in (0, 1)$ of middle-type women accept middle-type men and a fraction $\zeta \in (0, 1)$ of low-type women accept middle-type men. The following lemma applies to a middle-type man.

**Lemma 1** Let us assume that $R^*_H > x_M, R^*_M > x_L,$ and that $\eta \in (0, 1)$ of middle-type women accept middle-type men and $\zeta \in (0, 1)$ of low-type women accept middle-type men. If

$$x_L < (\geq) R^*_M = \frac{\alpha \eta \lambda_M x_M}{(r+\alpha \eta \lambda_M)},$$

a middle-type man rejects (accepts) a low-type woman. In this case, the reservation level of a middle-type man for a low-type woman decreases, in contrast with the benchmark result, i.e., $R^*_M > R^p_{Mm}$.

**Proof.** See Appendix A. ■

This lemma means that the rejections of middle-type men by some middle-type women lower the reservation utility level of a middle-type man for a low-type woman.

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13 Otherwise, the PSEI does not occur as men and women of different types marry.
14 When $k_0$-type women reject middle-type men and when high-type men reject middle-type women, there are always some middle- and low-type women who accept middle-type men. For example, if $k_0$-type women $(k = M, L)$ are rejected by high-type men, they always accept middle-type men (otherwise, their expected discounted lifetime utility becomes zero). Of course, other middle- or low-type women may also accept middle-type men, as we show later.
15 Here, the first subscript of $R^p_{Mm}$ denotes an agent’s type, and the second subscript of $R^p_{Mm}$ means a “man.”

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We can rewrite \( R^w_{Mm} > (\leq) x_L \) as \( \alpha \eta \lambda^w_M > (\leq) \frac{R^w_{Lm}}{\eta \lambda^w_M}. \) Therefore, with constant \( \alpha \), if \( R^w_{Mm} > x_L \) is small enough \( (\alpha \eta \lambda^w_M > \frac{R^w_{Lm}}{\eta \lambda^w_M}) \), a middle-type man accepts a low-type woman. Conversely, if there are enough middle-type women who accept middle-type men \( (\alpha \eta \lambda^w_M > \frac{R^w_{Lm}}{(x_M-x_L)}) \), a middle-type man turns down a low-type woman. Let us assume that \( R^w_{Mm} > x_L \) in this subsubsection in order to focus on the PSEI.

When \( R^w_{Mm} > x_L \), the middle-type man’s rate of contact with a woman whom he wishes to marry is \( \alpha \eta \lambda^w_M \). Then, a middle-type man’s time (duration) until meeting such a woman is \( \frac{1}{\alpha \eta \lambda^w_M} \). Therefore, his time until marriage is delayed due to the rejections from middle-type women with imperfect self-knowledge, since that in the benchmark case is \( \frac{1}{\alpha \lambda^w_M} \). This delay of marriage is the direct negative externality of learning.

Next, we investigate the optimal strategies of women. Women decide their optimal strategies given men’s behaviors. In the PSEI, high-type men reject middle-type women \( (R^H_H > x_M) \), middle-type men reject low-type women \( (R^w_{Mm} > x_L) \), and low-type men accept low-type women. Let us confirm the information carried by rejections or offers by men about types of women in order to obtain the strategies of women. A rejection by a high-type man means that the rejected woman is a middle- or low-type. Conversely, a proposal by a high-type man suggests that the woman who receives it is a high-type. Likewise, a proposal by a middle-type man means that the woman who receives it is a high- or middle-type. On the other hand, a rejection by a middle-type man means that the rejected woman is a low-type. A proposal by a low-type man carries no information about the types of women.

Therefore, we can consider the learning process of women. Figure 1 contains a description of it. The outline box for each type in Figure 1 represents the share of each type of women \( \lambda^w_k \), \( k = H, M, L \). First, we consider the learning process of high-type women. If an \( H_0 \)-type woman meets a high-type man, then she learns that she is a high-type, leaving the market with him. When another \( H_0 \)-type woman meets a middle-type man, she always rejects him in the PSEI. However, she learns that she is a high- or middle-type because she received the offer by that middle-type man. We refer to the woman who believes that she is either a high- or middle-type as an ‘\( HM \)-type’ woman (the subscript represents her remaining possible types). Now, we assume that an \( HM \)-type woman rejects a middle-type man (otherwise, the PSEI does not occur). If an \( HM \)-type woman meets a high-type man, she leaves the market with him and learns that she is a high-type. As a result, there are two kinds of high-type women according to different beliefs: \( H_0 \)-type and \( HM \)-type. Here, let \( \theta \in (0, 1) \) denote the share of \( HM \)-type women in high-type women.

Likewise, we can consider the learning process of middle-type and low-type women (see Figure 1).\(^\text{16}\) Here, for the middle-type women, let \( \mu \in (0, 1) \), \( \gamma (1 - \phi) \in (0, 1) \), and \( \gamma \phi \)}

\(^{16}\) The learning process of middle-type women is as follows. An \( M_0 \)-type woman learns that she is a high- or middle-type after she meets a middle-type man. Then, she becomes an ‘\( M_{HM} \)-type’ woman. An \( M_{HM} \)-type woman rejects a middle-type man in the PSEI, similarly to an \( HM \)-type woman. Then, an \( M_{HM} \)-type woman becomes an ‘\( M_{MM} \)-type’ woman after meeting with a high-type man who rejects her. An \( M_{MM} \)-type woman leaves the market with a middle-type man when they meet. On the other hand, another \( M_0 \)-type woman becomes an ‘\( M_{ML} \)-type’ woman by meeting with a high-type man. An \( M_{ML} \)-type woman rejects a low-type man in the PSEI. Then, if an \( M_{ML} \)-type woman meets a middle-type man, she marries him and learns that she is a middle-type.
\( \in (0, 1) \) denote the share of \( M_{ML} \)-type women, \( M_{HM} \)-type women, and \( M_{-} \)-type women in middle-type women, respectively. For the low-type women, let \( \psi (1 - \nu) \in (0, 1) \), \( \psi \nu \in (0, 1) \), and \( \kappa \in (0, 1) \) denote the share of \( L_{ML} \)-type women, \( L_{L1} \)-type women, and \( L_{L2} \)-type women in low-type women, respectively.\(^{17}\) From these shares \((\theta, \mu, \gamma, \phi, \psi, \nu, k)\) and \( \lambda_k \), \( G_w (x) \) is given.\(^{18}\)

Under these settings, the optimal strategies of \( l_{ML} \)-type \((l = M, L)\), \( j_{HM} \)-type \((j = H, M)\), and \( k_0 \)-type \((k = H, M, L)\) women are obtained in the next lemma.

**Lemma 2** Let us assume that a high-type man rejects a middle-type woman \((R_H^* > x_M)\), a middle-type man rejects a low-type woman \((R_{Mm}^* > x_L)\), and a low-type man accepts a low-type woman. If

\[
x_{L} < (\geq) \frac{\lambda_{ML}^n \alpha x_M (r + \alpha \lambda_n^m)}{r (r + \alpha \lambda_n^m + \alpha \lambda_n^m) + \alpha^2 \lambda_n^m \lambda_M^m p_{ML}} \equiv R_{l_{ML}}^p, \tag{5}
\]

an \( l_{ML} \)-type \((l = M, L)\) woman rejects (accepts) a low-type woman, where \( R_{l_{ML}}^p < R_{Mm}^* \). If

\[
x_{M} < (\geq) \frac{\lambda_{MH}^n \alpha x_H (r + \alpha \lambda_n^m)}{r (r + \alpha \lambda_n^m + \alpha \lambda_n^m) + \alpha^2 \lambda_n^m \lambda_M^m p_{HM}} \equiv R_{j_{HM}}^p = R_{k_0}^p, \tag{6}
\]
a \( j_{HM} \)-type \((j = H, M)\) or a \( k_0 \)-type \((k = H, M, L)\) woman rejects (accepts) a middle-type man, where \( R_{j_{HM}}^p = R_{k_0}^p < R_H^* \).

**Proof.** See Appendix A. \( \blacksquare \)

Here, \( p_{ML} = \frac{\mu \lambda_n^m}{\mu \lambda_n^m + \nu (1 - \nu) \lambda_M^m} \) is the probability that the actual type of an \( l_{ML} \)-type woman is the middle-type and \( p_{HM} = \frac{\theta \lambda_n^m}{\theta \lambda_n^m + \gamma (1 - \phi) \lambda_M^m} \) is the probability that the actual type of a \( j_{HL} \)-type woman is the high-type.

This lemma implies as follows: women with imperfect self-knowledge assign probabilities to their own types. Therefore, the reservation utility levels of \( M_{ML} \)-type women, \( H_{HM} \)-type women, and \( H_0 \)-type women are lowered, in contrast with the benchmark results. On the other hand, the reservation utility levels of \( L_{ML} \)-type women, \( M_{HM} \)-type women, and \( k_0 \)-type women \((k = M, L)\) are raised, in contrast with the PSE.

When \( r = 0 \), \( R_{l_{ML}}^p = R_M^* = x_M \) holds. Therefore, an \( l_{ML} \)-type woman always prefers to meet a middle-type man over accepting a low-type man in order to have the chance to confirm her actual type.\(^{19}\) This is because, if the actual type of an \( l_{ML} \)-type woman is a low-type, she would marry a low-type man sooner or later regardless of her behavior. At this time, she obtains the same value when she is single regardless of her behavior due to

---

\(^{17}\)The learning process of low-type women is as follows. An \( L_0 \)-type woman becomes the ‘\( L_{ML} \)-type’ after she is rejected by a high-type man. An \( L_{ML} \)-type woman rejects a low-type man in the PSE, similarly to an \( M_{ML} \)-type woman. Thus, an \( L_{ML} \)-type woman becomes the ‘\( L_{L1} \)-type’ by meeting with a middle-type man. Another \( L_0 \)-type woman becomes the ‘\( L_{L2} \)-type,’ after she meets a middle-type man.

\(^{18}\)As we show in the following, these shares depend on \( F_i(x) \) \((i = m, w)\) in a steady state. Therefore, we can rewrite these shares using \( \lambda_i^m \) \((k = H, M, L)\).

\(^{19}\)An \( L_{ML} \)-type woman will be apparently overconfident as she raises her reservation utility to reject a low-type man. This is because she accepts him when she knows her own type. We will analyze the case of apparent overconfidence in Subsubsection 3.2.2.
a lack of time-consuming cost \( (r = 0) \).\textsuperscript{20} Hence, the possibility that the actual type of an \( l_{ML} \)-type woman is a low-type does not affect her own decision. Consequently, the decision of an \( l_{ML} \)-type woman is the same as that of a middle-type woman with perfect self-knowledge.

If \( r > 0 \), the possibility that the actual type of an \( l_{ML} \)-type woman is a low-type affects her own decision. The agents with imperfect self-knowledge need to take into account the time-consuming cost due to the learning process.\textsuperscript{21} When an \( l_{ML} \)-type woman is a low-type, she is refused by a middle-type man. It is then desirable for an \( M \)-type woman to thoroughly understand her own type.\textsuperscript{22} Therefore, the reservation level of an \( l_{ML} \)-type \( (l = M, L) \) woman \( R^{p}_{l_{ML}} \) is lower than that in the case of \( r = 0 \).\textsuperscript{23}

A similar discussion could be presented for \( R^{p}_{jHM} = R^{p}_{k_{0}} = R^{*}_{H} \) when \( r = 0 \).

Lemma 2 means that an \( l_{ML} \)-type woman rejects (accepts) a low-type man if there are enough (few enough) middle-type men or if \( p_{ML} \) is sufficiently large (sufficiently small). It is noteworthy that the reservation utilities of women depend on not only enough (few enough) middle-type men or if \( p_{HM} \) is sufficiently large (sufficiently small).

Lemma 2 also shows that the reservation level of a \( k_{0} \)-type woman for a middle-type man is the same as that of a \( j_{HM} \)-type woman. If the actual type of a \( k_{0} \)-type woman is the low-type, she is rejected by high- and middle-type men regardless of her own behavior. Therefore, the decision of a \( k_{0} \)-type woman regarding whether or not to accept a middle-type man does not depend on the possibility that her actual type is the low-type. Hence, even if a \( k_{0} \)-type woman becomes a \( j_{HM} \)-type woman by meeting a middle-type man, her decision does not change.

Finally, we derive sufficient conditions for the PSEI. In the PSEI, \( x_{L} < R^{p}_{Mm} ( < R^{*}_{M} ) \), \( x_{L} < R^{p}_{l_{ML}} ( < R^{*}_{L} ) \), and \( x_{M} < R^{p}_{jHM} = R^{p}_{k_{0}} ( < R^{*}_{H} ) \) hold. Moreover, the steady state requires that the inflow into each state be equal to the outflow in Figure 1:

\[
\alpha \lambda^{m}_{M} (1 - \theta) \lambda^{W}_{H} = \alpha \lambda^{l}_{H} \theta \lambda^{W}_{H}, \quad (7)
\]

\[
\alpha \lambda^{m}_{H} (1 - \mu - \gamma) \lambda^{W}_{M} = \alpha \lambda^{l}_{M} \mu \lambda^{W}_{M}, \quad (8)
\]

\[
\alpha \lambda^{m}_{M} (1 - \mu - \gamma) \lambda^{W}_{L} = \alpha \lambda^{l}_{H} \gamma (1 - \phi) \lambda^{W}_{M} = \alpha \lambda^{m}_{M} \gamma \phi \lambda^{W}_{M}, \quad (9)
\]

\[
\alpha \lambda^{m}_{H} (1 - \psi - \kappa) \lambda^{W}_{L} = \alpha \lambda^{l}_{M} \psi (1 - \nu) \lambda^{W}_{M} = \alpha \lambda^{m}_{L} \psi \nu \lambda^{W}_{M}, \quad (10)
\]

\[
\alpha \lambda^{m}_{M} (1 - \psi - \kappa) \lambda^{W}_{L} = \alpha \lambda^{l}_{M} \kappa \lambda^{W}_{L}. \quad (11)
\]

Equation (7) means that the rate at which \( H_{0} \)-type women learn that they are the high- or the middle-type (that is, \( H_{0} \)-type women become \( H_{HM} \)-type women) equals the rate at which \( H_{HM} \)-type women marry high-type men. Equation (8) means that the rate at which \( M_{0} \)-type

\textsuperscript{20}When \( r = 0 \), \( V^{p}_{ML} \) in (31) equals \( V^{p}_{LML} \) in (33).

\textsuperscript{21}Therefore, in our model, it is possible that a woman with imperfect self-knowledge could marry before thoroughly understanding her own type.

\textsuperscript{22}When \( r > 0 \), \( V^{p}_{ML} \) in (31) is always larger than \( V^{p}_{LML} \) in (33).

\textsuperscript{23}If an \( l_{ML} \)-type woman lowers her \( R^{p}_{l_{ML}} \) to accept a low-type man whom she rejects when she knows her own type, an \( M_{ML} \)-type woman will apparently underestimate her own type. We will analyze this case of apparent underconfidence in Subsubsection 3.2.3.
women become $M_{ML}$-type women equals the rate at which $M_{ML}$-type women marry middle-type men. The first equality of (9) suggests that the rate at which $M_0$-type women become $M_{HM}$-type women equals the rate at which $M_{HM}$-type women become $M_M$-type women. The second equality of (9) means that the rate at which $M_{HM}$-type women become $M_M$-type women equals the rate at which $M_M$-type women marry middle-type men. Similarly, for the low-type women, their inflow into each state is equal to the outflow.\footnote{The first equality of (10) means that the rate at which $L_0$-type women become $L_{ML}$-type women equals the rate at which $L_{ML}$-type women become $L_{L1}$-type women. The second equality of (10) means that the rate at which $L_{ML}$-type women become $L_{L1}$-type women equals the rate at which $L_{L1}$-type women marry low-type men. Equation (11) means that the rate at which $L_0$-type women become $L_{L2}$-type women equals the rate at which $L_{L2}$-type women marry low-type men and leave the market.}

From (7) - (11), we then obtain the next proposition for the PSEI.

**Proposition 2** (PSEI) Let us assume that $x_M < R^*_M$ and $x_L < R^*_M$ hold. If

\[
R^p_{Mm} = \frac{\lambda^m_{Mm} \alpha \lambda^m_{Mm} x_M}{r(\lambda^m_{MM} + \lambda^m_{MM}) + \alpha \lambda^m_{MM}} > x_M,
\]

\[
R^p_{ML} = \frac{\alpha \lambda^m_{ML} (\lambda^m_{ML} + \lambda^m_{ML}) \lambda^m_{ML} \lambda^m_{ML} x_M (r + \alpha \lambda^m_{ML})}{\lambda^m_{ML} (\lambda^m_{ML} + \lambda^m_{ML}) (r + \alpha \lambda^m_{ML} + \lambda^m_{ML} (r + \alpha \lambda^m_{ML} + \alpha \lambda^m_{ML}))} > x_M,
\]

\[
R^p_{JHM} = R^p_{k_0} = \frac{\alpha \lambda^m_{Mm} (\lambda^m_{Mm} + \lambda^m_{Mm}) x_M (r + \alpha \lambda^m_{Mm})}{\lambda^m_{Hm} (\lambda^m_{Mm} + \lambda^m_{Hm}) (r + \alpha \lambda^m_{Mm} + \lambda^m_{Hm} (r + \alpha \lambda^m_{Mm} + \alpha \lambda^m_{Hm})))} > x_M,
\]

there exists the PSE with imperfect self-knowledge (PSEI) in which high-type agents form the first cluster of marriages, middle-type men and $M_{ML}$- and $M_M$-type women, the second cluster, and low-type men and $L_{L1}$- and $L_{L2}$-type women, the third cluster.

**Proof.** See Appendix A. ■

The implications of Proposition 2 are as follows: if there are enough high-type men or high-type women ($R^p_{JHM} = R^p_{k_0} > x_M$), a middle-type man is rejected by a $k_0$- or a $j_{HM}$-type ($j = H, M$) woman. A middle-type man rejects a low-type woman when there are enough $M_M$- and $M_{ML}$-type women ($R^p_{Mm} > x_L$) who accept middle-type men.\footnote{The share of $M_M$- and that of $M_{ML}$-type women in the market depend on $\lambda^m_{Hm}$. Then, if there are many high-type men, there will be many $M_M$- and $M_{ML}$-type women.} When there are enough middle-type men ($R^p_{l_{ML}} > x_L$), an $l_{ML}$-type ($l = M, L$) woman rejects a low-type man. However, when an $L_{ML}$-type woman rejects a low-type man, she becomes an $L_{L1}$-type woman sooner or later due to being rejected by a middle-type man. Then, a low-type woman who believes that she is a low-type (namely, she is $L_{L1}$- or $L_{L2}$-type) accepts a low-type man. As a result, the PSEI occurs. It is noteworthy that the first cluster of marriages is not influenced by women who are unaware of their own types.

### 3.2.2 Case of apparent overconfidence

In this subsubsection, we consider the case of apparent overconfidence: A woman with imperfect self-knowledge apparently overestimates her actual type if she rejects a man whom she accepts when she knows her own type. However, apparently overconfident agents in this study are unbiased and rational, as this apparent overconfidence is generated due to the
correct belief-updating process in the sense that agents have no false information and are Bayesian. Thus, our apparent overconfidence is different from true overconfidence, which is generally generated due to some errors in an agent’s processing information. Similarly, we will use the term apparent underconfidence in the next subsubsection.

We show that the apparent overconfidence of some women generates two externalities. The rejection from an apparently overconfident woman delays the timing of marriage of the man who is directly rejected by her relative to that in the benchmark case. Then, the apparent overconfidence has a direct (negative) externality. Furthermore, the apparent overconfidence of some women may have an indirect externality: when there are many women who are apparently overconfident in the market, the men who are now rejected by apparently overconfident women change their behaviors; i.e., they accept another lower type of women whom, when all agents know their own types, they reject. Given this, in a two-sided search, the women who are now accepted by these men may also change their behaviors. In this subsubsection, we show that the indirect externality of the apparent overconfidence prevents the lowest-type agents from marrying in an equilibrium. For this, we will find a Type I equilibrium (T1E) in which high-type men reject middle-type women, middle- and low-type men accept low-type women, $k_0$-type ($k = H, M, L$) women reject middle-type men, and $l_{ML}$-type ($l = M, L$) women reject low-type men. Then, $M_0$- and $L_0$-type women are apparently overconfident. Since apparently overconfident $M_0$-type women reject middle-type men, middle-type men accept low-type women.

First, we investigate the optimal strategies of men. Now, a high-type man has the same reservation level as a high-type man in the PSE because all women want to marry high-type men. The option of a middle-type man is to marry or to turn down a low-type woman, as a high- or $M_0$-type woman rejects him in the T1E. In the same manner as in Lemma 1, the reservation level of a middle-type man for a low-type woman $R_{Mm}^{T1} = \frac{\alpha M_H x_M}{(r + \alpha M_H)} (< R_M)$ is immediately obtained. Let us assume that $R_{Mm}^{T1} = x_L$ in the following analysis in order to consider the T1E. A low-type man also accepts a low-type woman because a middle-type man accepts a low-type woman.

Next, we investigate the strategies of women given men’s behaviors. Since a middle- or a low-type man accepts any woman, the woman who receives his proposal learns nothing about her own type. Then, only when a woman meets a high-type man does she learn something about her own type. Figure 2 describes the women’s learning processes. Then, there are five kinds of women according to different beliefs: $k_0$-type women ($k = H, M, L$) and $l_{ML}$-type women.

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26 The following analysis shows that there are no $j_{HM}$-type women in the T1E.

27 Of course, we can consider another equilibrium in order to describe the apparently overconfident behavior. However, at this time, we cannot show that the indirect externality of apparent overconfidence prevents the lowest-type agents from marrying in our model with three types of agents (for details, see Appendix C). Hence, we focus on the T1E.

28 The learning process of women is as follows. If a high-type woman is accepted by a high-type man, she leaves the market with him and knows that she is a high-type. After a middle- or a low-type woman is rejected by a high-type man, she becomes an $l_{ML}$-type woman ($l = M, L$). However, she does not learn any more about her own type, since the offer from a middle- or a low-type man carries no information about the types of women.
women \((l = M, L)\). Here, \(\tau \in (0, 1)\) and \(\varpi \in (0, 1)\) denote the share of \(M_{ML}\)-type women in middle-type women and that of \(L_{ML}\)-type women in low-type women, respectively. The optimal strategies of women are obtained in the next lemma.

**Lemma 3** Let us assume that a high-type man rejects a middle-type woman \((R_H^* > x_M)\) and a middle- or a low-type man accepts a low-type woman \((R_{ML}^{T1} \leq x_L < R_H^*)\). At this time, an \(l_{ML}\)-type woman \((l = M, L)\) rejects a low-type man \((R_{ML}^{T1} = R_L^*)\). On the other hand, a \(k_0\)-type woman rejects (accepts) a middle-type man if

\[
x_M < \left( \frac{\lambda_H^m \alpha x_H(r + \alpha \lambda_H^m) p_{T1}^{T1} \lambda_H^m \lambda_H^m}{r (r + \alpha x_H(r + \alpha \lambda_H^m) + \alpha^2 \lambda_H^m \lambda_H^m) p_{T1}^{T1}} \right) \leq R_{k_0}^{T1},
\]

where \(R_{k_0} < R_H^*\).

**Proof.** See Appendix A.

Here, \(r_H^{T1} \equiv \frac{\lambda_H^m + x_H(r + \alpha x_H) \lambda_H^m \lambda_H^m}{r (r + \alpha x_H(r + \alpha \lambda_H^m) + \alpha^2 \lambda_H^m \lambda_H^m) p_{T1}^{T1}}\) is the probability that the actual type of a \(k_0\)-type woman is the high-type. Then, Lemma 3 suggests that a \(k_0\)-type woman rejects a middle-type man if there are enough high-type men or high-type women. On the other hand, since a middle-type man accepts a low-type woman, an \(M_{ML}\)-type woman and an \(L_{ML}\)-type woman face the same problem. As a result, an \(l_{ML}\)-type \((l = M, L)\) woman turns down a low-type man, as there are enough middle-type men \((x_L < R_L^*)\).

Given matching strategies of men and women, let us derive sufficient conditions for the \(T1E\) in which \(R_{ML}^{T1} \leq x_L\) and \(R_{k_0}^{T1} > x_M\) hold. The steady state requires that

\[
\begin{align*}
n \lambda_M^v \lambda_M^v (1 - \tau) \lambda_M^v &= \alpha \lambda_M^v \rho \lambda_M^v, \quad (13) \\
n \lambda_M^v \lambda_M^v (1 - \varpi) \lambda_M^v &= \alpha \lambda_M^v \rho \lambda_M^v, \quad (14)
\end{align*}
\]

hold from Figure 2.\(^{29}\)

We then obtain the following proposition. In this equilibrium, the indirect externality of apparent overconfidence of \(M_0\)-type women prevents the lowest-type men from marrying.

**Proposition 3** (\(T1E\)) Let us assume that \(x_M < R_H^*\) and \(x_L < R_L^*\) hold. If

\[
(R_H^* > R_{k_0}^{T1}) \Rightarrow R_{ML}^{T1} = \frac{\lambda_H^m \lambda_H^m \alpha x_H(r + \alpha \lambda_H^m)(\lambda_H^m + \lambda_H^m)}{r (r + \alpha x_H(r + \alpha \lambda_H^m) + \alpha \lambda_H^m \lambda_H^m) p_{T1}^{T1}} > x_M
\]

and

\[
R_{ML}^{T1} = \frac{\lambda_H^m \lambda_H^m a x_M(\lambda_H^m + \lambda_H^m)}{r (\lambda_H^m + \lambda_H^m) + a \lambda_H^m \lambda_H^m} \leq x_L,
\]

there exists the \(T1E\) in which high-type agents form the first cluster of marriages, middle-type men and \(M_{ML}\)-type women, the second cluster, and middle-type men and \(L_{ML}\)-type women, the third cluster. In this equilibrium, low-type men can never marry.

\(^{29}\)Equation (13) means that the rate at which \(M_0\)-type women become \(M_{ML}\)-type women equals the rate at which \(M_{ML}\)-type women marry middle-type men. Likewise, Equation (14) means that the rate at which \(L_0\)-type women become \(L_{ML}\)-type women equals the rate at which \(L_{ML}\)-type women marry middle-type men.
Proof. See Appendix A. ■

The implications of Proposition 3 are as follows: when there are enough high-type men or high-type women \((R_{k_0}^T > x_M)\), a \(k_0\)-type woman rejects a middle-type man. \(M_0\)- and \(L_0\)-type women become \(M_{ML}\)- and \(L_{ML}\)-type women, respectively, after they meet high-type men. If there are few enough \(M_{ML}\)-type women \((R_{Mm}^T \leq x_L)\) who accept middle-type men, a middle-type man accepts a low-type woman. This leads an \(M_{ML}\)- or an \(L_{ML}\)-type woman to refuse the offer by a low-type man \((R_{ML} = R_{M}^* > x_L)\). As a result, a low-type man can never marry.\(^{30}\)

In the next subsubsection, we investigate a case in which some middle-type women do not know their own types and then accept low-type men, i.e., a case of apparent underconfidence.

### 3.2.3 Case of apparent underconfidence

In this subsubsection, let us consider the case of apparent underconfidence: a woman apparently underestimates her own type when she accepts a man whom, when she knows her own type, she rejects. From Lemma 2, if \(r > 0\) is large, a woman with imperfect knowledge would tend to underestimate her own type.

In this subsubsection, we investigate the externalities of the apparent underconfidence of some women. The apparent underconfidence of a woman makes the future partner better off, as she increases the value of the match to the partner. Then, the apparent underconfidence will have direct (positive) externality. However, the apparent underconfidence has no indirect externality. To see this by proof by contradiction, we have to investigate all cases in which indirect externality occurs: if there are enough apparently underconfident women, the men who are now accepted by these apparently underconfident women reject the women whom they accept when all agents know their own types. In this subsubsection, in order to reuse the framework of the PSEI in the latter half of this subsubsection, we treat a case in which since apparently underconfident \(M_{ML}\)-type women accept low-type men, low-type men reject low-type women. The proofs of the other cases are shown in Appendix B, as we can qualitatively obtain the same results as in Lemma 4 in all remaining cases.

Let us consider the following case: high-type men reject middle-type women and middle-type men reject low-type women. Moreover, low-type men reject low-type women since apparently underconfident \(M_{ML}\)-type women accept low-type men. At this time, the means of the proposal and rejection by a high-type man are the same as those in the PSEI. A rejection from a middle- or low-type man means that the woman who is rejected by him is a low-type. An offer from a middle- or a low-type man means that the woman who is accepted by him is either a high- or a middle-type. Then, there are at least \(k_0\)-type \((k = H, M, L)\), \(j_{HM}\)-type \((j = H, M)\), and \(l_{ML}\)-type \((l = M, L)\) women in the market.

The next Lemma shows that the indirect externality of the apparent underconfidence of \(M_{ML}\)-type women does not occur.

\(^{30}\)The share of \(l_{ML}\)-type women \((l = M, L)\) depends on the share of high-type men \(\lambda_M^H\). However, even if \(F_m(x)\) and \(F_w(x)\) are symmetric, \(R_{Mm}^T \leq x_L\) and \(R_{k_0}^T > x_M\) hold in some parameter ranges (see Example 1).
Lemma 4 Let us assume that $x_M < R^*_H$ and $x_L < R^*_M$. When there are apparently underconfident $M_{ML}$-type women who accept low-type men, a low-type man always accepts a low-type woman.

Proof. See Appendix A. ■

This lemma implies that the apparent underconfidence of $M_{ML}$-type women does not have indirect externality. If a low-type man rejects a low-type woman due to the expectation to marry an apparently underconfident $M_{ML}$-type woman, the proposal from him to an $M_{ML}$-type woman informs her that she is the middle-type. Furthermore, if the actual type of an $l_{ML}$-type woman is the low-type, she is rejected by a middle- or a low-type man regardless of her behavior. Then, the possibility that an $l_{ML}$-type woman is a low-type does not affect her decision to accept or reject a low-type man. As a result, an $l_{ML}$-type woman has the incentive to reject a low-type man, as she prefers to have the chance to learn her actual type than to accept a low-type man. This contradicts the assumption that an $l_{ML}$-type woman accepts a low-type man. Therefore, a low-type man does not change his behavior even if there are enough apparently underconfident $M_{ML}$-type women for low-type men to reject low-type women.

Next, we show that the apparent underconfidence has direct externality in a steady state. Let us consider a Type 2 equilibrium (T2E) in which high-type men reject middle-type women, middle-type men reject low-type women, low-type men accept low-type women, $k_0$- and $j_{HM}$-type ($k = H, M, L, j = H, M$) women reject middle-type men, and $l_{ML}$-type ($l = M, L$) women accept low-type men. The acceptance of a low-type woman by a low-type man gives him a chance to marry an $M_{ML}$-type woman.

Let us investigate the strategies of men given the above women’s behaviors. A high-type man has the same reservation level as a high-type man in PSE. In the same manner as in Lemma 1, the reservation level of a middle-type man for a low-type woman $R^T_{Mm} = \frac{\alpha M \lambda M x_M}{(r + \alpha M \lambda M)}$ ($< R^*_M$) is obtained. The option of a low-type man is to marry or to turn down a low-type woman, as apparently underconfident $M_{ML}$-type women accept low-type men. Let us assume that $\varphi \in (0, 1)$ of middle-type women accept low-type men. Thus, the reservation level of a low-type man for a low-type woman is obtained in the same manner as $R^T_{Mm}$. His reservation level is $R^T_{Lm} = \frac{\alpha M \lambda M x_M}{r + \alpha M \lambda M}$, where $R^T_{Lm} < R^*_M$. We assume that $R^T_{Mm} > x_L > R^T_{Lm}$ in the following analysis in order to consider the T2E.\(^{31}\)

Since the behaviors of men are the same as those in the PSEI, the information about women’s types from the proposals or rejections by men is also the same as that in the PSEI. Then, the learning processes of women are also the same as those in the PSEI (see Figure 3). Let $\theta_2 \in (0, 1)$, $\gamma_2 (1 - \phi_2) \in (0, 1)$, $\gamma_2 \phi_2 \in (0, 1)$, $\mu_2 \in (0, 1)$, $\psi_2 (1 - \nu_2) \in (0, 1)$, $\psi_2 \nu_2 \in (0, 1)$, and $\kappa_2 \in (0, 1)$ denote the share of $H_{HM}$-type women, that of $M_{HM}$-type women, that of $M_{M}$-type women, that of $M_{ML}$-type women, that of $L_{ML}$-type women, that of $L_{L1}$-type women, and that of $L_{L2}$-type women, respectively. Hence, the optimal strategies of an $l_{ML}$-type, a $j_{HM}$-type, and a $k_0$-type woman are obtained in the next lemma.

\(^{31}\)At this time, $\varphi < \eta$ is required. However, the following analysis shows that $\varphi < \eta$ holds.
Lemma 5 Let us assume that a high-type man rejects a middle-type woman ($R_H^* > x_M$), a middle-type man rejects a low-type woman ($R_M^* > R_{ML}^2 > x_L$), and a low-type man accepts a low-type woman ($R_{LM}^2 < x_L$). If

$$x_L < (>) R_{ML}^2 = \frac{\lambda_M^0 \alpha x_M (r + \alpha \lambda_M^w p_{ML}^2)}{r (r + \alpha \lambda_M^w + \alpha \lambda_{ML}^w) + \alpha^2 \lambda_M^w \lambda_{ML}^w R_{ML}^2},$$

an $l_{ML}$-type woman ($l = M, L$) rejects (accepts) a low-type man, where $R_{ML}^2 < R_M^*$. If

$$x_M < (>) R_{HM}^2 = \frac{\lambda_H^0 \alpha H (r + \alpha \lambda_H^w p_{HM}^2)}{r (r + \alpha \lambda_H^w + \alpha \lambda_{HM}^w) + \alpha^2 \lambda_H^w \lambda_{HM}^w R_{HM}^2} = R_{k0}^2,$$

a $j_{HM}$-type ($j = H, M$) and a $k_0$-type woman reject (accept) a middle-type man, where $R_{HM}^2 = R_{k0}^2 < R_M^*$.  

Proof. See Appendix A. 

Here, $p_{ML} = \frac{\mu \lambda_M^w}{\mu \lambda_M^w + \psi (1 - \psi) \lambda_L^w}$ is the probability that the actual type of an $l_{ML}$-type woman is the middle-type and $p_{HM} = \frac{\theta \lambda_H^w}{\theta \lambda_H^w + \gamma (1 - \gamma) \lambda_M^w}$ is the probability that the actual type of a $j_{HL}$-type woman is the high-type. Lemma 5 implies the following: Equations (15) and (16) have the same form as Equations (5) and (6), respectively. However, the difference between Figures 1 and 3 is whether or not $l_{ML}$-type women accept low-type men. Therefore, $G_w (x)$ in the T2E will be different from that in the PSEI.

In the T2E, $x_M < R_{HM}^2 = R_{k0}^2$, $x_L > R_{ML}^2$, and $R_{LM}^2 > x_L > R_{LM}^2$ hold. Moreover, the steady state requires that the inflow into each state be equal to the outflow in Figure 3:

$$\alpha \lambda_M^m (1 - \theta_2) \lambda_H^w = \alpha \lambda_M^w \theta_2 \lambda_H^w,$$  

(17)  

$$\alpha \lambda_H^m (1 - \mu_2 - \gamma_2) \lambda_M^w = \alpha (\lambda_M^m + \lambda_L^m) \mu_2 \lambda_M^w,$$  

(18)  

$$\alpha \lambda_M^m (1 - \mu_2 - \gamma_2) \lambda_L^w = \alpha \lambda_M^w \gamma_2 (1 - \phi_2) \lambda_M^w = \alpha \lambda_M^w \mu_2 \phi_2 \lambda_M^w,$$  

(19)  

$$\alpha \lambda_H^m (1 - \psi_2 - \kappa_2) \lambda_L^w = \alpha (\lambda_M^m + \lambda_L^m) \psi_2 (1 - \nu_2) \lambda_L^w,$$  

(20)  

$$\alpha \lambda_M^m (1 - \psi_2) \lambda_L^w = \alpha \lambda_M^w \psi_2 \nu_2 \lambda_L^w,$$  

(21)  

$$\alpha \lambda_M^m (1 - \psi_2) \lambda_L^w = \alpha \lambda_M^w \kappa_2 \lambda_L^w.$$  

(22)

All equations, except (18) and (20), have the same forms as (7) and (9)-(11) in the PSEI.  

Equation (18) means that the rate at which $M_0$-type women become $M_{ML}$-type women equals the rate at which $M_{ML}$-type women marry middle- or low-type men. Equation (20) means that the rate at which $L_0$-type women become $L_{ML}$-type women equals the rate at which $L_{ML}$-type women become $L_{LL}$-type women and the rate at which $L_{ML}$-type women marry low-type men.

32From (17) - (22), we have the next proposition for the T2E. In this equilibrium, apparent underconfidence has direct externality.
Proposition 4 (T2E) Let us assume that

\[
R^T_{Lm} = \frac{\alpha^2 M^2 x_M}{\lambda_H^2 (r+\alpha\lambda_L^M + \lambda_H^W)} \leq x_L < R^T_{Mm} = \frac{\alpha^2 M^2 x_M + \lambda_H^W}{\lambda_H^2 (r+\alpha\lambda_L^M + \lambda_H^W)} < R^*_M,
\]

\[
x_M < R^T_{JHM} = R^T_{k0} = \frac{\alpha^2 M^2 x_M}{\lambda_H^2 (r+\alpha\lambda_L^M + \lambda_H^W)} < R^*_H,
\]

\[
x_L \geq R^T_{MML} = \frac{\alpha^2 M^2 x_M}{\lambda_H^2 (r+\alpha\lambda_L^M + \lambda_H^W)} \leq x_L < R^T_{Mm} = \frac{\alpha^2 M^2 x_M + \lambda_H^W}{\lambda_H^2 (r+\alpha\lambda_L^M + \lambda_H^W)} < R^*_M.
\]

There exists the T2E in which high-type agents form the first cluster of marriages, middle-type men and \(M_{ML}\)-type women, the second cluster, low-type men and \(M_{ML}\)-type women, the third cluster, and low-type men and \(L_{ML}\)-type, \(L_{L1}\)-type, and \(L_{L2}\)-type women, the fourth cluster.

Proof. See Appendix A. ■

The implications of Proposition 4 are as follows: in the T2E, there is the direct externality of apparent underconfidence. Since there is no indirect externality of apparent underconfidence in this equilibrium, all agents can marry. If there are enough high-type men and high-type women \((x_M < R^T_{JHM})\), a middle-type man is rejected by a \(k_0\)-type and a \(j_{HM}\)-type woman. Moreover, a large \(\lambda_H^M\) implies a large share of \(M_{-}\) and \(M_{ML}\)-type women in middle-type women. If there are enough \(M_{-}\) and \(M_{ML}\)-type women \((x_L < R^T_{Mm})\), a middle-type man rejects a low-type woman. Furthermore, if there are few enough \(M_{ML}\)-type women satisfying \(R^T_{MML} \leq x_L < R^T_{Mm}\), an \(L_{ML}\)-type woman accepts a low-type man. This is because she assigns low probability to being a middle-type woman. However, a low-type man accepts a low-type woman, as there are few enough \(M_{ML}\)-type women \((R^T_{Lm} \leq x_L)\). Therefore, all agents can marry sooner or later.

When all agents know their own types under the cloning assumption and the assumption of non-transferable utility, a unique equilibrium always occurs (see Burdett and Coles (1997)). However, if there are agents with imperfect self-knowledge under the cloning assumption and the non-transferable utility, it is possible that multiple equilibria can occur. From Propositions 2, 3, and 4, in some parameter ranges, multiple equilibria can occur: both the PSEI and the T2E can exist. To clarify this, we consider the next example.

Example 1 Let us assume that \(2\alpha > 3r\) and that \(F_m(x)\) and \(F_w(x)\) are discrete uniform distributions: \(\lambda_i^k = \frac{1}{\beta}, \ (i = m, w, \ k = H, M, L)\). At this time, the sufficient conditions for the PSEI become \(R^P_{JHM} = R^P_{k0} = \frac{(3r+\alpha)2\alpha x_H}{18r\alpha+2\alpha^2+27r^2} > x_M\) and \(R^P_{Mm} = \frac{\alpha x_M}{9r+\alpha} > R^P_{Lm} = \frac{12r\alpha+\alpha^2+18r}{12r\alpha+\alpha^2+18r} > x_L\) from Proposition 2. Similarly, the sufficient conditions for the T1E are \(R^T_{JHM} = R^T_{k0} = \frac{(3r+\alpha)2\alpha x_H}{12r\alpha+\alpha^2+18r^2} > x_M\) and \(R^T_{Mm} = \frac{\alpha x_M}{9r+\alpha} < x_L\) from Proposition 3. The sufficient conditions for the T2E are \(x_M < R^T_{JHM} = R^T_{k0} = \frac{(3r+\alpha)2\alpha x_H}{66r\alpha+\alpha^2+99r^2}\) and \(R^T_{Mm} = \frac{\alpha x_M}{27r+\alpha} < x_L < R^T_{MML} = \frac{\alpha x_M}{27r+\alpha} < x_M < R^T_{Lm} = \frac{\alpha x_M}{27r+\alpha} < x_L < R^T_{Mm} - R^T_{Lm} = -\frac{\alpha x_M}{27r+\alpha} < 0\), the T2E and the T1E do not hold. Moreover, as \(R^T_{Mm} = R^T_{Mm}\), the PSEI and the T1E
do not hold either.\footnote{It is noteworthy that $R_{k0}^{T2} = R_{pMm}^{p}$ always holds from Proposition 2 and 3.} However, since $x_M < R_{k0}^{T2} (< R_{k0}^{p})$ and $R_{k0}^{kM} < x_L < R_{lML}^{p} (< R_{Mm}^{T2})$, the PSEI and the T2E hold.

Figure 4 and 5 show how the outcomes in Example 1 depend on $x_H$, $x_M$ and $x_L$ while holding all other parameters constant. There is an overlap between the two equilibria for $x_M < R_{k0}^{T2}$ and $R_{k0}^{T2} < x_L < R_{lML}^{p}$.

The intuition of multiple equilibria is as follows: marriage patterns are determined by all agents’ expectations about the behaviors of agents with imperfect self-knowledge. If all agents expect that $l_{ML}$-type ($l = M, L$) women will accept low-type men, these expectations form their prior beliefs $G_w(x)$. Then, the marriage pattern of the T2E arises. On the other hand, if all agents expect that $l_{ML}$-type women will reject low-type men, the marriage pattern of the PSEI arises through their prior belief $G_w(x)$. The welfare implication of these two steady states is obtained in Example 2 in Section 4.

Propositions 3 and 4 and Lemma 4 suggest that, whereas apparent overconfidence has an indirect externality, apparent underconfidence does not have indirect externality. This difference depends on the agents whom the indirect externality firstly affects. In the case of apparent underconfidence, some middle-type women with imperfect self-knowledge accept low-type men. Given this, if a low-type man rejects a low-type woman, his offer to an apparently underconfident middle-type woman informs her that she belongs to a higher type than she believes. Then, the indirect externality of apparent underconfidence does not occur. On the other hand, in the case of apparent overconfidence, even if a middle-type man accepts a low-type woman due to the existence of many apparently overconfident middle-type women, the acceptance of a low-type woman by a middle-type man makes a low-type woman better off. Therefore, the indirect externality of apparent overconfidence remains.\footnote{In the T1E, since middle-type men accept the lowest type of women (that is, low-type women), his offer carries no information about types of women. Then, a low- or a middle-type woman learns nothing from his offer. However, if there is a lower type than ‘low-type’ and if a middle-type man accepts a ‘low-type’ woman, a low- or a middle-type woman will learn something about her own type from the acceptance by a middle-type man. However, this acceptance still makes a low-type woman better off. Then, the indirect externality of apparent overconfidence will remain in this case.}

In the next section, we investigate the total number of marriages and the welfare of the economy.

4 Welfare and the number of marriages

In this section, we investigate whether the existence of women with imperfect self-knowledge improves the welfare of the economy relative to the benchmark case. To do so, let us examine the overall number of marriages and the overall welfare from new marriages that take place in the marriage market at any point in time.

First, we investigate the number of marriages at the PSE as a benchmark. In the PSE, a high-type man meets a high-type woman with probability $\alpha \lambda_H^w$, and there are $\lambda_H^m N$ number of high-type men in the market. Then, the number of marriages among high-type agents is
\( \alpha \lambda_H^w \lambda_H^m N \). In the same way, we obtain the number of marriages of middle-type \( \alpha \lambda_M^m \lambda_M^w N \) and low-type \( \alpha \lambda_L^w \lambda_L^m N \). Therefore, the overall number of marriages in the marriage market \( T^* \) is

\[
T^* = \alpha \lambda_H^m \lambda_H^w N + \alpha \lambda_M^m \lambda_M^w N + \alpha \lambda_L^w \lambda_L^m N. \tag{23}
\]

The number of marriages in the PSE (\( T^p \)), the T1E (\( T^{T1} \)), and the T2E (\( T^{T2} \)) can be derived similarly (see also Figure 1-3). Therefore, we obtain

\[
T^p = \alpha \lambda_H^m \lambda_H^w N + \alpha \left( \frac{\lambda_H^m}{\lambda_H^w + \lambda_M^w} \right) \lambda_M^w \lambda_M^m N + \alpha \left( \frac{\lambda_H^m}{\lambda_L^w + \lambda_M^w} \right) \lambda_L^w \lambda_L^m N, \tag{24}
\]

\[
T^{T1} = \alpha \lambda_H^m \lambda_H^w N + \alpha \left( \frac{\lambda_H^m}{\lambda_H^w + \lambda_M^w} \right) \lambda_M^w \lambda_M^m N + \alpha \left( \frac{\lambda_H^m}{\lambda_L^w + \lambda_M^w} \right) \lambda_L^m \lambda_L^w N, \tag{25}
\]

and

\[
T^{T2} = \alpha \lambda_H^m \lambda_H^w N + \alpha \left( \frac{\lambda_H^m}{\lambda_H^w + \lambda_M^w} \right) \lambda_M^w \lambda_M^w N + \alpha \left( \frac{(\lambda_H^m)^2}{(\lambda_H^w + \lambda_M^w)(\lambda_L^w + \lambda_M^w)} \right) \lambda_M^w \lambda_L^m N + \alpha \left( \lambda_H^m + \lambda_M^m \right) \lambda_L^m \lambda_L^w N. \tag{26}
\]

Next, we explore overall welfare. If a high-type man marries a high-type woman, each of them obtains the utility of marriage \( x_H \). Hence, the aggregation of high-type agents’ utilities from marriage is \( 2 \alpha \lambda_H^m \lambda_H^w x_H N \) in the PSE. Similarly, we obtain \( 2 \alpha \lambda_M^m \lambda_M^w x_M N \) for the middle-type and \( 2 \alpha \lambda_L^w \lambda_L^m x_L N \) for the low-type. As a result, the welfare in the PSE (\( W^* \)) is

\[
W^* = \alpha \lambda_H^m \lambda_H^w (2x_H) N + \alpha \lambda_M^m \lambda_M^w (2x_M) N + \alpha \lambda_L^w \lambda_L^m (2x_L) N. \tag{27}
\]

The welfare in the PSE (\( W^p \)), the T1E (\( W^{T1} \)), and the T2E (\( W^{T2} \)) can be derived similarly. Hence,

\[
W^p = \alpha \lambda_H^m \lambda_H^w (2x_H) N + \alpha \left( \frac{\lambda_H^m}{\lambda_H^w + \lambda_M^w} \right) \lambda_M^w \lambda_M^m (2x_M) N + \alpha \left( \frac{\lambda_H^m}{\lambda_L^w + \lambda_M^w} \right) \lambda_L^m \lambda_L^w (2x_L) N, \tag{28}
\]

\[
W^{T1} = \alpha \lambda_H^m \lambda_H^w (2x_H) N + \alpha \left( \frac{\lambda_H^m}{\lambda_H^w + \lambda_M^w} \right) \lambda_M^m \lambda_M^w (2x_M) N + \alpha \left( \frac{\lambda_M^m}{\lambda_H^w + \lambda_M^w} \right) \lambda_L^m \lambda_L^w (x_M + x_L) N, \tag{29}
\]

and

\[
W^{T2} = \alpha \lambda_H^m \lambda_H^w (2x_H) N + \alpha \left( \frac{\lambda_H^m}{\lambda_H^w + \lambda_M^w} \right) \lambda_M^w \lambda_M^m (2x_M) N + \alpha \left( \frac{(\lambda_H^m)^2}{(\lambda_H^w + \lambda_M^w)(\lambda_M^w + \lambda_L^w)} \right) \lambda_M^w \lambda_L^m (x_M + x_L) N + \alpha \left( \lambda_H^m + \lambda_M^m \right) \lambda_L^m \lambda_L^w (2x_L) N. \tag{30}
\]

hold. From these, the next lemma is immediately obtained.

**Proposition 5** The number of marriages and the welfare in the PSE are higher than those in the PSE, i.e., \( T^* > T^p \) and \( W^* > W^p \), respectively.

**Proof.** See Appendix A. □
From this proposition, the welfare and the number of marriages in the PSEI are lower than those in the PSE under any \( F_i(x), \ i = m, w \). This is because the marriages of all agents, except those of high-type men and some high-type women who meet high-type men in their first encounter, are delayed due to the refusal by the women who are learning their own types.\(^{35}\) As a result, their marriages decrease, and the overall welfare then decreases. From this proposition, in the PSEI, the policy which promotes women’s learning can improve the overall welfare.

On the other hand, the comparisons of the overall welfare and the overall number of marriages between the PSE and T1E are ambiguous. The number of marriages and the welfare of middle-type women in the T1E are always lower than those in the PSE, since their marriages are delayed due to their own learning. Moreover, the number of marriages and the welfare of low-type men in the T1E are also lower because these men cannot get married. However, since middle-type men accept both middle- and low-type women in the T1E, their number of marriages and welfare also increase or decrease as a whole, depending on the shapes of \( F_m(x) \) and \( F_w(x) \) and \( x_k(k = M, L) \). Moreover, since low-type women marry middle-type men, their number of marriages and welfare also increase or decrease as a whole, depending on \( F_m(x) \), \( F_w(x) \), and \( x_k(k = M, L) \). If the overall welfare in the T1E is larger than that in the PSE, the policy that promotes the learning by apparently overconfident women in the T1E will lower the overall welfare.

The comparisons of the overall welfare and the overall number of marriages between the PSE and T2E are also ambiguous. The number of marriages and the welfare of low-type women in the T2E are always lowered due to their own learning. The number of marriages and the welfare of middle-type men in the T2E also decrease compared to those in the PSE, as these men are rejected by \( M_0 \) and \( M_{HM} \)-type women. \( M_{ML} \)-type women who marry low-type men obtain lower utilities than middle-type women with perfect self-knowledge. However, \( M_{ML} \)-type women in the T2E can marry earlier than middle-type women in the PSE, since they accept middle- and low-type men. Therefore, the number of marriages and welfare of middle-type women in the T2E then increase or decrease as a whole, depending on \( F_m(x) \), \( F_w(x) \), and \( x_k(k = M, L) \). Since low-type men marry middle- and low-type women in the T2E, their number of marriages and welfare also increase or decrease depending on \( F_m(x) \), \( F_w(x) \), and \( x_k(k = M, L) \). Hence, if the overall welfare in the T2E is larger than that in the PSE, the policy that informs apparently underconfident women of their own types in the T2E will lower the overall welfare.

It is noteworthy that, in both the T1E and the T2E, the number of marriages and the welfare of high-type men/women are not influenced by the imperfect self-knowledge of women.

\(^{35}\) The duration until marriage of each agent can be obtained easily. In the PSE, the duration until marriage of \( k \)-type man (woman) is \( \frac{1}{\alpha x_k^{M_{ML}}}(i = m, w, k = H, M, L) \). However, in the PSEI, the duration until marriage of a middle-type man is \( \frac{1}{\alpha x_k^{M_{ML}}} \), that of a low-type man is \( \frac{1}{\alpha x_k^{M_{ML}}} \), that of an \( H_{HM} \)-type woman is \( \frac{1}{\alpha x_k^{M_{ML}}} \), that of an \( M_{ML} \)-type woman is \( \frac{1}{\alpha x_k^{M_{ML}}} \), that of an \( M_{M} \)-type woman is \( \frac{1}{\alpha x_k^{M_{ML}}} \), that of an \( L_{LM} \)-type woman is \( \frac{1}{\alpha x_k^{M_{ML}}} \), and that of an \( L_{LM} \)-type woman is \( \frac{1}{\alpha x_k^{M_{ML}}} \). Therefore, their marriages are delayed by the learning process of women.
Finally, we compare the number of marriages and the welfare in the PSEI with those in the T2E. As we show in Example 1, the PSEI and the T2E hold in some parameter ranges.

**Example 2** Let us assume that $\lambda_i^k = \frac{1}{3}$, $(i = m, w, k = H, M, L)$. At this time, the number of marriages in the PSE, the PSEI, and the T2E are $T^* = \frac{2}{3}N\alpha$, $T^p = \frac{2}{3}N\alpha$, and $T^{T2} = \frac{27}{159}N\alpha$, respectively, from (23), (24), and (26). Then, $T^* > T^{T2} > T^p$. The welfare of marriages in the PSE, the PSEI, and the T2E are $W^* = \frac{2}{3}N\alpha(x_H + x_L + x_M)$, $W^p = \frac{1}{9}N\alpha(2x_H + x_L + x_M)$, and $W^{T2} = \frac{1}{159}N\alpha(42x_H + 31x_L + 21x_M)$ respectively, from (27), (28), and (30). Hence, $W^* > W^{T2} > W^p$.

When $3\alpha > 2r$ and $F_m(x)$ and $F_w(x)$ are discrete uniform distributions, multiple equilibria arise. At this time, the PSEI and the T2E are not Pareto-rankable: apparently underconfident women prefer the PSEI, and low-type men prefer the T2E. However, since apparently underconfident women accept middle- and low-type men in the T2E, the overall number of marriages in the T2E increases relative to that in the PSEI. As a result, the overall welfare in the T2E also increases relative to that in the PSEI. However, since $\lambda_i^k = \frac{1}{3}$, the welfare and the number of marriages in the PSE are larger than those in the T2E. Then, in the T2E, the policy that informs apparently underconfident women of their own types can improve the overall welfare when $\lambda_i^k = \frac{1}{3}$ ($i = m, w, k = H, M, L$).

5 Discussion—Two-sided imperfect self-knowledge

In this paper, we assume one-sided imperfect self-knowledge: no women initially know their own types, whereas all men know their own types. This one-sided imperfect self-knowledge assumption is important in order to clarify the influence of imperfect self-knowledge. From Lemma 2, the uncertainty of an agent’s own type affects her own expected life utility. Moreover, the existence of others with imperfect self-knowledge also affects agents’ expected life utilities from Lemma 1. We can analyze these two influences on the expected life utility of an agent separately, under the assumption of the one-sided imperfect self-knowledge. The one-sided imperfect self-knowledge assumption describes the situations below. In the context of the labor market, a firm has more information about its own type than a worker, since the firm will generally have more experience than the worker. In the context of the marriage market, when more men work outside the home than women, it will be easier for men than for women to get the objective data on their own charm, such as income, position at work, and social status.

Although two-sided imperfect self-knowledge—all men and women initially lack knowledge of their own types—is a nontrivial extension, our results suggest that, if two-sided imperfect self-knowledge is assumed in the apparent overconfidence case, the reservation level of any
agent (‘he’) will be simultaneously affected by the following two factors: (i) the large share of apparently overconfident women who now reject his type and (ii) the uncertainty of his own type. The first element always lowers his reservation level from Lemma 1. For the second element, as we show in Lemma 2, his reservation level decreases or increases relative to that in the case of perfect self-knowledge. On the other hand, in the case of apparent underconfidence, only the uncertainty of his own type will affect his reservation level. Hence, two-sided imperfect self-knowledge will make the analysis more complex. Such work is left for future research.

6 Concluding remarks

We analyze a two-sided search model in which we presume that no women initially know their own types and then learn their own types from the offers or rejections by men. With this learning process, the two-sided aspect of a search problem generates a significant interest. We show, especially, that the apparent overconfidence of some agents prevents the lowest-type agents from matching in an equilibrium. However, the apparent underconfidence of some agents does not affect the matching of the lowest-type agents.

We conclude with a discussion of some possible further extensions of this model. First, this paper assumes that there is no divorce. However, when a woman marries a man before thoroughly understanding her own type, she may learn about her actual type after she gets married. In this case, the divorce rate will be influenced by this learning in marriage. Next, we assume three types of agents. If we consider a model in which there are \( n \) type agents and many clusters of marriages, the learning process about one’s own type will be more complex. However, if there are \( n \) type agents and three clusters of marriages are generated by a large enough \( \alpha \), our results also apply to this case.

References


Appendix A

Proof of Proposition 1: If a high-type agent turns down a middle-type agent of the opposite sex $i$ ($=m,w$), $V_H > \frac{x_M}{r}$. From (1), this high-type agent’s discounted lifetime utility when he or she is single becomes

$$rv_H = \alpha \lambda_H^i \left( \frac{x_H}{r} - rv_H \right).$$

On the other hand, when he or she accepts a middle-type agent $i$ and turns down a low-type agent $i$, i.e., $r \frac{z_M}{r} > V_H > \frac{x_L}{r}$, his/her value function is

$$rv_H = \alpha \lambda_H^i \left( \frac{x_H}{r} - rv_H \right) + \alpha \lambda_M^i \left( \frac{x_M}{r} - V_H \right).$$

If $V_H > V_H^a$ is satisfied, a high-type agent refuses a middle-type opposite sex agent $i$. This inequality $V_H > V_H^a$ means that

$$x_M < R_H^* \equiv \frac{\alpha \lambda_H^i x_H}{\alpha \lambda_H^i + r}.$$ If $x_M \geq R_H^*$, a high-type agent proposes to a middle-type agent $i$.

Under inequality (2), we can obtain the condition for a middle-type agent to reject a low-type opposite sex agent $i$ by the same process as that described above. Consequently, we have

$$x_L < R_M^* \equiv \frac{\alpha \lambda_M^i x_M}{\alpha \lambda_M^i + r}.$$ Proof of Lemma 1: The reservation level of a middle-type man for a low-type woman can be calculated as follows: now, a fraction $\eta \in (0,1)$ of middle-type women and a fraction $\zeta \in (0,1)$ of low-type women accept middle-type men. If a middle-type man turns down a low-type woman ($V_{Mm}^r > x_L/r$), his value function becomes

$$rv_{Mm}^r = \alpha \eta \lambda_M^w \left( \frac{x_M}{r} - V_{Mm}^r \right),$$

where the first subscript of $V$ denotes an agent’s type and the second subscript of $V$ means a “man.”

Conversely, when a middle-type man proposes to a low-type woman ($V_{Mm}^a \leq x_L/r$),

$$rv_{Mm}^a = \alpha \eta \lambda_M^w \left( \frac{x_M}{r} - V_{Mm}^a \right) + \alpha \zeta \lambda_L^w \left( \frac{x_L}{r} - V_{Mm}^a \right).$$

Hence, we have his reservation utility level for declining a low-type woman, $\frac{\alpha \lambda_M^i x_M}{r + \alpha \lambda_M^i}$.
\( R_{Mm}^p \): Compared to the PSE, we have

\[
R_{Mm}^p - R_{M}^* = -\frac{r\alpha\lambda_M^p x_M (1-\eta)}{(r+\alpha\lambda_M^p)(r+\alpha\gamma\lambda_M^p)} < 0.
\]

\[\text{Proof of Lemma 2:} \] An \( l_{ML} \)-type woman thinks that her actual type is a middle-type with probability \( p_{MML} = \frac{\mu\lambda_{MML}^{\psi(1-\nu)}}{\mu\lambda_{MML} + \psi(1-\nu)\lambda_{M}^{\psi}} \) and low-type with probability \( p_{LML} = \frac{\psi(1-\nu)\lambda_{L}^{\psi}}{\mu\lambda_{MML} + \psi(1-\nu)\lambda_{M}^{\psi}} \) (see Figure 1). The reservation level of an \( l_{ML} \)-type woman for a low-type man can be calculated as follows: if an \( l_{ML} \)-type woman turns down a low-type man \((V_{ML}^r > x_L/r)\), her value function becomes

\[
\begin{align*}
\hat{V}_{ML}^r &= p_{MML} \left[ \alpha\lambda_M^m \left( \frac{x_M}{r} - V_{MML}^r \right) \right] + p_{LML} \left[ \alpha\lambda_M^m \left( V_{LL} - V_{LML}^r \right) \right] \\
V_{LL} &= \alpha\lambda_L^m \left( \frac{x_L}{r} - V_{LL} \right). 
\end{align*}
\]

(31)

where \( \hat{V}_{ML}^r \equiv p_{MML} V_{MML}^r + p_{LML} V_{LML}^r \). The second term in Equation (31) means that, if the actual type of an \( l_{ML} \)-type woman is a low-type, she learns that she is a low-type by meeting a middle-type man. She then changes her value function to (32), since she is accepted by a low-type man.

When an \( l_{ML} \)-type woman accepts a low-type man, her value function is

\[
\begin{align*}
\hat{V}_{ML}^a &= p_{MML} \left[ \alpha\lambda_M^m \left( \frac{x_M}{r} - V_{MML}^a \right) \right] + p_{LML} \left[ \alpha\lambda_M^m \left( V_{LL} - V_{LML}^a \right) \right] \\
&\quad + p_{LML} \left[ \alpha\lambda_M^m \left( V_{LL} - V_{LML}^a \right) \right] + \alpha\lambda_L^m \left( \frac{x_L}{r} - V_{LL}^a \right),
\end{align*}
\]

(33)

where \( \hat{V}_{ML}^a \equiv p_{MML} V_{MML}^a + p_{LML} V_{LML}^a \). From (31)-(33), the reservation level of an \( l_{ML} \)-type woman for a low-type man is

\[
R_{ML}^p = \frac{\lambda_M^m \alpha x_M (r+\alpha\lambda_M^m)p_{MML}}{(r+\alpha\lambda_M^m)(r+\alpha\gamma\lambda_M^p)p_{MML}}.
\]

Compared to the benchmark case, we have

\[
R_{ML}^p - R_{M}^* = -\frac{r\lambda_M^m \alpha x_M (r+\alpha\lambda_M^m)(1-p_{MML})}{(r+\alpha\lambda_M^m)(r+\alpha\gamma\lambda_M^p)+(\alpha^2\lambda_M^m \gamma p_{MML})} < 0.
\]

On the other hand, the reservation level of a \( j_{HM} \)-type woman for a middle-type man can be calculated as follows: if a \( j_{HM} \)-type woman turns down a middle-type man \((V_{HM}^r > x_M/r)\), her value function becomes

\[
\begin{align*}
\hat{V}_{HM}^r &= p_{HM} \left[ \alpha\lambda_M^m \left( \frac{x_M}{r} - V_{HM}^r \right) \right] + p_{MHM} \left[ \alpha\lambda_M^m \left( V_{MM} - V_{MHM}^r \right) \right] \\
V_{MM} &= \alpha\lambda_M^m \left( \frac{x_M}{r} - V_{MM} \right),
\end{align*}
\]

(34)

where \( \hat{V}_{HM}^r \equiv p_{HM} V_{HM}^r + p_{MHM} V_{HM}^r \), \( p_{HM} \equiv \frac{\delta\lambda_M^m}{\delta\lambda_M^m + \gamma(1-\phi)\lambda_M^p} \) and \( p_{MHM} \equiv \frac{\gamma(1-\phi)\lambda_M^m}{\delta\lambda_M^m + \gamma(1-\phi)\lambda_M^p} \).
The first term in the second square bracket in (34) implies that, if the actual type of a \( j_{HM} \)-type woman is the middle-type, she learns that she is a middle-type after a meeting with a high-type man. She then changes her value function to (35) as middle-type men accept middle-type women.

When she accepts a middle-type man, her value function is

\[
\hat{V}^a_{jHM} = p_{HM} \left[ \alpha \lambda^m_H \left( \frac{x_H}{r} - V^a_{HM} \right) + \alpha \lambda^m_M \left( \frac{x_M}{r} - V^a_{HM} \right) \right] + p_{MHM} \left[ \alpha \lambda^m_H \left( V^a_{MM} - V^a_{MHM} \right) + \alpha \lambda^m_M \left( \frac{x_M}{r} - V^a_{MHM} \right) \right],
\]

where \( \hat{V}^a_{jHM} \equiv p_{HM} V^a_{HM} + p_{MHM} V^a_{MHM} \). Therefore, the reservation level of a \( j_{HM} \)-type woman for a middle-type man is

\[
R^p_{j_{HM}} = \frac{\lambda^m_H^\alpha \lambda^m_H \left( r + \alpha \lambda^m_M \right) p_{HM}}{r \left( r + \alpha \lambda^m_H \right) + \alpha^2 \lambda^m_H \lambda^m_M, p_{HM}}.
\]

Compared to the benchmark case, we have

\[
R^p_{j_{HM}} - R^*_H = -\frac{\left( r \lambda^m_H^\alpha \lambda^m_H \left( r + \alpha \lambda^m_M \right) \right)(1-p_H)}{r \left( r + \alpha \lambda^m_H \right) + \alpha^2 \lambda^m_H \lambda^m_M, p_{HM}} < 0.
\]

Given the behaviors of all agents except \( k_0 \)-type women, we can obtain the lifetime utility of a \( k_0 \)-type woman. If she rejects a middle-type man, her value function is

\[
r\hat{V}^r = p_{H_0} \left[ \alpha \lambda^m_H \left( \frac{x_H}{r} - V^r_H \right) + \alpha \lambda^m_M \left( \hat{V}^r_{JHM} - V^r_H \right) \right] + p_{M_0} \left[ \alpha \lambda^m_M \left( \hat{V}^r_{MML} - V^r_M \right) + \alpha \lambda^m_M \left( \hat{V}^r_{JHM} - V^r_M \right) \right] + p_{L_0} \left[ \alpha \lambda^m_H \left( \hat{V}^r_{MLL} - V^r_L \right) + \alpha \lambda^m_M \left( V^r_M - V^r_L \right) \right],
\]

where \( \hat{V}^r \equiv p_{H_0} V^r_H + p_{M_0} V^r_M + p_{L_0} V^r_L \), \( p_{H_0} = \frac{1}{(1-\theta)\lambda^m_H^0 + (1-\mu-\gamma)\lambda^m_M^0 + (1-\psi-\kappa)\lambda^m_L^0} \), \( p_{M_0} = \frac{1}{(1-\theta)\lambda^m_H^0 + (1-\mu-\gamma)\lambda^m_M^0 + (1-\psi-\kappa)\lambda^m_L^0} \), \( \lambda^m_H = 1 \).

The second term in the third square bracket in (37) implies that, if the actual type of a \( k_0 \)-type woman is the low-type, she learns that she is the low-type after meeting a middle-type man. She then changes her value function to (32) as a low-type man accepts a low-type woman. The second term in the first (or the second) square bracket in Equation (37) means that, if the actual type of a \( k_0 \)-type woman is the high- (or middle-) type, she learns that she is the high- or the middle-type after meeting a middle-type man. She then changes her optimal strategy to \( R^p_{j_{HM}} \). Likewise, the first term in the second (or the third) square bracket in Equation (37) means that, if the actual type of a \( k_0 \)-type woman is the middle- (or low-) type, she learns that she is the middle- or the low-type after meeting a high-type man. She then changes her optimal strategy to \( R^p_{i_{ML}} \).
If a $k_0$-type woman accepts a middle-type man, her value function becomes

$$r V^a = p H_0 \left[ \alpha \lambda_H^m \left( \frac{x_H}{r} - V_H^a \right) + \alpha \lambda_M^m \left( \frac{x_M}{r} - V_M^a \right) \right] + p M_0 \left[ \alpha \lambda_H^m \left( V_{ML} - V_M^a \right) + \alpha \lambda_M^m \left( \frac{x_M}{r} - V_M^a \right) \right] + p L_0 \left[ \alpha \lambda_H^m \left( V_{ML} - V_L^a \right) + \alpha \lambda_M^m \left( V_L - V_L^a \right) \right],$$

where $V^a = p H_0 V_H^a + p M_0 V_M^a + p L_0 V_L^a$. Therefore, the reservation level of a $k_0$-type woman for a middle-type man is

$$R_{k_0}^p = R_{jHM}^p > (\leq) x_M.$$

If $R_{k_0}^p > (\leq) x_M$, a $k_0$-type woman rejects (accepts) a middle-type man.

**Proof of Proposition 2:** In the PSEI, we can rewrite $R_{Mm}^p$ in (4) as

$$R_{Mm}^p = \frac{\alpha(\mu + \gamma \phi) \lambda_M^m x_M}{(\tau + \alpha(\mu + \gamma \phi) \lambda_M^m)} \quad (38)$$

by replacing $\eta$ and $\zeta$ by $(\mu + \gamma \phi)$ and $(\psi + \kappa)$, respectively, from Figure 1.

From (7) - (11), we obtain

$$\theta = \gamma = \frac{\lambda_M^m}{\lambda_H^m + \lambda_M^m},$$

$$\mu = \frac{(\lambda_M^m)^2}{(\lambda_H^m + \lambda_M^m)^2}, \quad \phi = \psi = \frac{\lambda_M^m}{(\lambda_H^m + \lambda_M^m)} \quad (40)$$

$$\nu = \frac{\lambda_M^m}{(\lambda_L^m + \lambda_M^m)}, \quad \kappa = \frac{(\lambda_M^m)^2}{(\lambda_L^m + \lambda_M^m)^2} \quad (41)$$

The proposition follows immediately by substituting (39)-(41) into (5), (6), and (38).

**Proof of Lemma 3:** Since a middle-type man accepts a low-type woman, an $M_{ML}$-type woman and an $L_{ML}$-type woman face the same problem. They decide whether to accept or reject low-type men. Therefore, if an $l_{ML}$-type woman rejects a low-type man, her value function is

$$r V_{l_{ML}} = \alpha \lambda_M^m \left( \frac{x_M}{r} - V_{l_{ML}}^a \right). \quad (42)$$

When she accepts a low-type man, her value function is

$$r V_{l_{ML}} = \alpha \lambda_M^m \left( \frac{x_M}{r} - V_{l_{ML}}^a \right) + \alpha \lambda_L^m \left( \frac{x_L}{r} - V_{l_{ML}}^a \right). \quad (43)$$

Therefore, the reservation level of an $l_{ML}$-type woman for a low-type man is

$$R_{l_{ML}}^{l_{1}} = \frac{\alpha \lambda_M^m x_M}{r + \alpha \lambda_M^m} = R_{M}^L.$$

As $x_L < R_{M}^L$, $x_L < R_{l_{ML}}^L$ holds. Hence, an $l_{ML}$-type woman turns down a low-type man.

Given the behaviors of all agents except a $k_0$-type woman, we can obtain the lifetime utility of a $k_0$-type ($k = H, M, L$) woman. Since a middle-type man accepts a low-type woman, an
M_0-type woman and an L_0-type woman face the same problem. Then, if a k_0-type woman rejects a middle-type man, her value function is

$$r\hat{V}^r = p_{M_0}^{T_1} \alpha \lambda_H^m \left( \frac{x_H}{r} - V_H^r \right) + \left( p_{M_0}^{T_1} + p_{L_0}^{T_1} \right) \alpha \lambda_M^m \left( \hat{V}_{M_1}^r - V_M^r \right).$$

(44)

where $\hat{V}^r \equiv p_{M_0}^{T_1} V_H^r + (p_{M_0}^{T_1} + p_{L_0}^{T_1}) V_M^r$, $p_{M_0}^{T_1} \equiv \frac{\lambda_H^m}{\lambda_H^m + (1-\tau)\lambda_M^m + (1-\omega)\lambda_L^m}$, $p_{M_0}^{T_1} \equiv \frac{(1-\tau)\lambda_M^m}{\lambda_H^m + (1-\tau)\lambda_M^m + (1-\omega)\lambda_L^m}$, and $p_{L_0}^{T_1} \equiv \frac{(1-\omega)\lambda_L^m}{\lambda_H^m + (1-\tau)\lambda_M^m + (1-\omega)\lambda_L^m}$. The second term in Equation (44) means that, if the actual type of a k_0-type woman is the middle- or low-type, she becomes an l_{ML}-type woman after meeting a high-type man. She then changes her optimal strategy to $R_{l_{ML}}^T > x_L$.

If a k_0-type woman accepts a middle-type man, her value function becomes

$$r\hat{V}^a = p_{M_0}^{T_1} \alpha \lambda_M^m \left( \frac{x_H}{r} - V_H^a \right) + \left( p_{M_0}^{T_1} + p_{L_0}^{T_1} \right) \alpha \lambda_M^m \left( \hat{V}_{M_1}^a - V_M^a \right).$$

(44)

where $\hat{V}^a \equiv p_{M_0}^{T_1} V_H^a + (p_{M_0}^{T_1} + p_{L_0}^{T_1}) V_M^a$. Therefore, the reservation level of a k_0-type woman for a middle-type man is

$$R_{k_0}^{T_1} = \frac{\lambda_H^m \alpha x_H (r+\alpha \lambda_M^m) p_{M_0}^{T_1}}{r(r+\alpha \lambda_H^m + \alpha^2 \lambda_M^m \lambda_L^m) p_{H_0}^{T_1}}.$$

Compared to the benchmark case, we have

$$R_{k_0}^{T_1} - R_H^* = -\frac{(r+\alpha \lambda_H^m) \alpha x_H (r+\alpha \lambda_M^m + \alpha \lambda_L^m)(1-p_{H_0}^{T_1})}{(r+\alpha \lambda_H^m) (r^2 + r \alpha \lambda_H^m + r \alpha \lambda_M^m + \alpha^2 \lambda_M^m \lambda_L^m) p_{H_0}^{T_1}} < 0.$$

Proof of Proposition 3: In the T1E, we can rewrite $R_{l_{mn}}^{T_1}$ as

$$R_{l_{mn}}^{T_1} = \frac{\alpha \lambda_H^m}{r(r+\alpha \lambda_H^m + \alpha^2 \lambda_M^m \lambda_L^m)} < R_M^*$$

(45)

by replacing $\eta$ and $\zeta$ by $\tau$ and $\omega$, respectively (see Figure 2).

From (13)-(14), we obtain $\tau = \omega = \frac{\lambda_H^m}{\lambda_H^m + \lambda_M^m}$. The proposition follows immediately by substituting $\tau = \omega = \frac{\lambda_H^m}{\lambda_H^m + \lambda_M^m}$ into (45) and (12).

Proof of Lemma 4: Let us assume that, since there are enough apparently underconfident $M_{ML}$-type women for a low-type man, low-type men reject low-type women. However, given these strategies of low-type men, the decision of an $l_{ML}$-type man is as follows: by meeting a high-type man, an $l_{ML}$-type woman thinks that her actual type is the middle-type with probability $p_M^l$ and the low-type with probability $p_L^l$, where $p_M^l + p_L^l = 1$. Now, the option of an $l_{ML}$-type woman is to marry or to reject a low-type man.
If she accepts a low-type man, her value function is

$$\hat{V}_{iML}^a = p_M' \left[ \alpha \lambda_M^a \left( \frac{x_M}{r} - V_{MML}^a \right) + \alpha \lambda_L^a \left( \frac{x_L}{r} - V_{MML}^a \right) \right] + p_L' \left[ \alpha (\lambda_M^m + \lambda_L^m) \left( 0 - V_{LML}^a \right) \right].$$

where $\hat{V}_{iML}^a \equiv p_M' V_{MML}^a + p_L' V_{LML}^a$. On the other hand, if she turns down a low-type man

$$r \hat{V}_{iML}^a = p_M' \left[ \alpha \lambda_M^a \left( \frac{x_M}{r} - V_{MML}^a \right) + \alpha \lambda_L^a \left( V_{MML} - V_{MML}^a \right) \right] + p_L' \left[ \alpha (\lambda_M^m + \lambda_L^m) \left( 0 - V_{LML}^a \right) \right],$$

$$r V_{MML} = \alpha \lambda_M^a \left( \frac{x_M}{r} - V_{MML} \right),$$

$$r V_{LML} = \alpha \lambda_M^a \left( \frac{x_M}{r} - V_{MML}^a \right) + \alpha \lambda_L^a \left( V_{MML} - V_{MML}^a \right) + p_L' \left[ \alpha (\lambda_M^m + \lambda_L^m) \left( 0 - V_{LML}^a \right) \right],$$

where $\hat{V}_{iML}^a < p_M' V_{MML}^a + p_L' V_{LML}^a$.

Here, $\hat{V}_{iML}^a < V_{iML}^a$ holds, as $R_{M}^a > x_L$. That is, an $l_{ML}$-type ($l = M, L$) woman rejects a low-type man when low-type men reject low-type women. This contradicts the assumption that an $M_{ML}$-type woman accepts a low-type man. Therefore, even if there are enough $M_{ML}$-type women who accept low-type men, a low-type man always accepts a low-type woman. 

**Proof of Lemma 5:** As $R_{Mm}^T > x_L \geq R_{Lm}^T$, the information about types of women from the proposals by men and the learning processes of women are the same as those in the PSEI. Hence, the reservation levels of women are obtained in the same manner as those in the PSEI:

$$R_{1ML}^T = \frac{\lambda_M^m \alpha x_M (r + a \lambda_L^m) p_{1ML}^T}{r (r + a + a \lambda_L^m + a \lambda_L^m + a \lambda_L^m + \lambda_L^m) p_{1ML}^T} < R_M^*,$$

$$R_{2HM}^T = \frac{\lambda_M^m \alpha x_M (r + a \lambda_L^m) p_{2HM}^T}{r (r + a + a \lambda_L^m + a \lambda_L^m + a \lambda_L^m + \lambda_L^m) p_{2HM}^T} < R_H^*,$$

where $p_{1ML}^T = \frac{\mu_2 \lambda_M^m}{\mu_2 \lambda_M^m + \psi_2 (1 - \mu_2) \lambda_L^m}$ and $p_{2HM}^T = \frac{\theta_2 \lambda_M^m}{\theta_2 \lambda_M^m + \gamma_2 (1 - \phi_2) \lambda_L^m}$ from Figure 3.

**Proof of Proposition 4:** In the T2E, we can rewrite $R_{Mm}^T$ and $R_{Lm}^T$ as

$$R_{Mm}^T = \frac{\alpha (\mu_2 + \gamma_2 \phi_2) \lambda_M^m x_M}{(r + \alpha (\mu_2 + \gamma_2 \phi_2) \lambda_M^m)} (\geq x_L),$$

$$R_{Lm}^T = \frac{\alpha \psi_2 \lambda_M^m x_M}{r + \alpha \psi_2 \lambda_M^m} (\leq x_L),$$

by replacing $\mu$ and $\varphi$ by $(\mu_2 + \gamma_2 \phi_2)$ and $\mu_2$, respectively (see Figure 3).

From (17) - (22), we obtain

$$\theta_2 = \frac{\lambda_M^m}{\lambda_H^m + \lambda_M^m},$$

$$\mu_2 = \frac{(\lambda_M^m)^2}{\lambda_H^m + (\lambda_M^m + \lambda_L^m)^2},$$

$$\gamma_2 = \frac{(\lambda_M^m + \lambda_L^m) (\lambda_H^m + \lambda_L^m)}{(\lambda_H^m + (\lambda_M^m + \lambda_L^m) (\lambda_L^m + \lambda_L^m))},$$

$$\psi_2 = \frac{\lambda_M^m}{\lambda_H^m + \lambda_L^m},$$

$$\nu_2 = \frac{\lambda_M^m}{\lambda_H^m + \lambda_L^m},$$

$$\kappa_2 = \lambda_M^m.$$
The proposition follows immediately by substituting (49)-(52) into (15), (16) (47) and (48).■

Proof of Proposition 5: From (23) and (24),

\[ T^* - T^p = N\alpha ((1 - (\mu + \gamma \phi)) \lambda_M w_M + (1 - (\kappa + v\psi)) \lambda_L w_L) > 0 \]

holds. From (27) and (28), we also have

\[ W^p - W^* = -\lambda_L w_L x_L (1 - (\kappa + v\psi)) - \lambda_M w_M x_M (1 - (\mu + \gamma \phi)) < 0. \]

■

Appendix B

In this appendix, we show that the apparent underconfidence never has an indirect externality in steady state. To see this, we consider the following cases in which indirect externality of apparent underconfidence occurs (i.e., the men who now are accepted by apparently underconfident women reject the women whom these men accept when all agents know their own types): (I) low-type men reject low-type women, (II) middle-type men reject middle-type women, and low-type men reject low-type women, (III) middle-type men reject middle-type women and low-type men accept low-type women, (IV) middle- and low-type men reject middle- (and low-) type women.

First, we investigate the case of (I): low-type men reject low-type women. This case requires that \( l_{ML} \)-type women accept low-type men. This case is already shown in Lemma 4.

Next, we investigate the case of (II). The case of (II) requires that (i) \( H_{HM} \)-type women accept middle-type men or (ii) \( H_0 \)-type women accept middle-type men. First, let us show that, even when there are enough \( H_{HM} \)-type women who accept middle-type men for middle-type men to refuse middle-type women, middle-type men always accept middle-type women. To see this, let us assume that, since there are enough \( H_{HM} \)-type women for middle-type men, middle-type men reject middle-type women, and low-type men reject low-type women. Therefore, the means of the proposal and rejection by a high- and a middle-type man are the same as those by a high-type man in the PSEI. Moreover, a rejection from a low-type man means that the woman who is rejected by him is a low-type. An offer from a low-type man means that the woman who is accepted by him is either a high- or a middle-type. Hence, there are at least \( k_0 \)-type women \((k = H, M, L)\), \( l_{ML} \)-type women \((l = M, L)\), and \( j_{HM} \)-type women \((j = H, M)\) in the market.

However, when middle-type men reject middle-type women and low-type men reject low-type women, the optimal strategies of a \( j_{HM} \)-type and an \( l_{ML} \)-type woman are obtained in the next lemma.

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38The existence of \( j_{HM} \)-type women requires that low-type men reject low-type women.
Lemma 6 Let us assume that \( x_M < R_H^r \) and \( x_L < R_M^r \) and that middle-type men reject middle-type women and low-type men reject low-type women. In this case, an \( l_{ML}(l = M, L) \)-type woman accepts a low-type man. A \( j_{HM}(j = H, M) \)-type woman rejects a middle-type man.

Proof. By meeting a high- or a middle-type man, an \( l_{ML} \)-type woman thinks that her actual type is the middle-type with probability \( p_{M1} \) and the low-type with probability \( p_{L1} \). If an \( l_{ML} \)-type woman turns down a low-type man, her expected discounted lifetime utility becomes zero. Therefore, an \( l_{ML} \)-type woman always proposes to a low-type man with

\[
r_{V_{l_{ML}}} = p_{M1} \left[ \alpha \lambda^r_H \left( \frac{x_L}{r} - V_{M_{ML}} \right) \right] + p_{L1} \left[ \alpha \lambda^r_L \left( 0 - V_{L_{ML}} \right) \right],
\]

where \( r_{V_{l_{ML}}} = p_{M1} r_{V_{M_{ML}}} + p_{L1} r_{V_{L_{ML}}} \) and \( p_{M1} + p_{L1} = 1 \).

The option of a \( j_{HM} \)-type woman is to marry or to reject a middle-type man. Let us assume that a \( j_{HM} \)-type woman thinks that her actual type is the high-type with probability \( p_H2 \) and the middle-type with probability \( p_M2 \). If she accepts a middle-type man, her value function \( \hat{V}^a_{j_{HM}} \) becomes

\[
r_{\hat{V}^a_{j_{HM}}} = p_H2 \left[ \alpha \lambda^a_H \left( \frac{x_H}{r} - V^a_{H_{HM}} \right) \right] + p_M2 \left[ \alpha \lambda^a_M \left( x_M - V^a_{M_{HM}} \right) \right], \tag{53}
\]

where \( r_{\hat{V}^a_{j_{HM}}} = p_H2 r_{V^a_{H_{HM}}} + p_M2 r_{V^a_{M_{HM}}} \) and \( p_H2 + p_M2 = 1 \).

If a \( j_{HM} \)-type woman rejects a middle-type man, her value function \( \hat{V}^r_{j_{HM}} \) becomes

\[
r_{\hat{V}^r_{j_{HM}}} = p_H2 \left[ \alpha \lambda^r_H \left( \frac{x_H}{r} - V^r_{H_{HM}} \right) \right] + p_M2 \left[ \alpha \lambda^r_M \left( V_{H_H} - V^r_{M_{HM}} \right) \right], \tag{55}
\]

where \( r_{\hat{V}^r_{j_{HM}}} = p_H2 r_{V^r_{H_{HM}}} + p_M2 r_{V^r_{M_{HM}}} \).

From Equations (53)-(55) and \( x_M < R_H^r, \hat{V}^a_{j_{HM}} < \hat{V}^r_{j_{HM}} \) holds. Therefore, a \( j_{HM} \)-type woman always rejects a middle-type man. \( \blacksquare \)

From this lemma, a \( j_{HM} \)-type woman always rejects a middle-type man when middle-type men reject middle-type women and low-type men reject low-type women. This contradicts the assumption that, since there are enough \( H_{HM} \)-type women who accept a middle-type man, middle-type men reject middle-type women. Therefore, even if there are enough \( H_{HM} \)-type women who accept middle-type men, a middle-type man always accepts a middle-type woman.

Next, let us show that, even when there are enough \( H_0 \)-type women who accept middle-type men, middle-type men always accept middle-type women. To see this, let us assume that, since there are enough \( H_0 \)-type women who accept middle-type men for these men to
refuse middle-type women, middle-type men reject middle-type women. However, at this
time, a contradiction is generated.

**Lemma 7** Let us assume that \( x_M < R^*_H \) and \( x_L < R^*_M \) and that middle-type men reject
middle-type women and low-type men reject low-type women. In this case, a \( k_0 \) \((k = H, M, L)\)-
type woman rejects a middle-type man.

**Proof.** From Lemma 6, when middle-type men reject middle-type women and low-type men
reject low-type women, \( l_{ML} \)-type women accept low-type men, and \( j_{HM} \)-type women reject
middle-type men. Thus, we obtain the discounted lifetime utility of a \( k_0 \)-type woman. Here, let \( p_{H3} \) denote the probability that the actual type of a \( k_0 \)-type woman is the high-type, \( p_{M3} \) denote the probability that the actual type of a \( k_0 \)-type woman is the middle-type, and \( p_{L3} \) denote the probability that the actual type of a \( k_0 \)-type woman is the low-type. If she accepts
a middle-type man, her value function becomes

\[
r\hat{V}^a = p_{H3} \left[ \alpha \lambda_H^m \left( \frac{x_H}{r} - V_H^a \right) + \alpha \lambda_M^m \left( \frac{x_M}{r} - V_M^a \right) + \alpha \lambda_L^m \left( \hat{V}_{jHM}^r - V_H^a \right) \right] + p_{M3} \left[ \alpha (\lambda_H^m + \lambda_M^m) \left( \hat{V}_{jML}^r - V_M^a \right) + \alpha \lambda_L^m \left( \hat{V}_{jHM}^r - V_M^a \right) \right] + p_{L3} \left[ \alpha (\lambda_H^m + \lambda_M^m) \left( \hat{V}_{jML}^r - V_L^a \right) + \alpha \lambda_L^m (0 - V_L^a) \right],
\]

where \( r\hat{V}^a = p_{H3} rV_H^a + p_{M3} rV_M^a + p_{L3} rV_L^a \), and \( p_{H3} + p_{M3} + p_{L3} = 1 \).

If she rejects a middle-type man, her value function becomes

\[
r\hat{V}^r = p_{H3} \left[ \alpha \lambda_H^m \left( \frac{x_H}{r} - V_H^r \right) + \alpha \lambda_M^m (V_H^r - V_H^r) + \alpha \lambda_L^m \left( \hat{V}_{jHM}^r - V_H^r \right) \right] + p_{M3} \left[ \alpha (\lambda_H^m + \lambda_M^m) \left( \hat{V}_{jML}^r - V_M^r \right) + \alpha \lambda_L^m \left( \hat{V}_{jHM}^r - V_M^r \right) \right] + p_{L3} \left[ \alpha (\lambda_H^m + \lambda_M^m) \left( \hat{V}_{jML}^r - V_L^r \right) + \alpha \lambda_L^m (0 - V_L^r) \right],
\]

where \( r\hat{V}^r = p_{H3} rV_H^r + p_{M3} rV_M^r + p_{L3} rV_L^r \).

Since \( \frac{x_M}{r} < V_{HH} \) from \( x_M < R^*_H \), a \( k_0 \)-type woman rejects a middle-type man \((\hat{V}^r > \hat{V}^a)\).

Similarly to Lemma 6, from this lemma, even if there are enough \( H_0 \)-type women who
accept middle-type men, middle-type men always accept middle-type women.

Thirdly, we investigate the case of (III): middle-type men reject middle-type women and
low-type men accept low-type women. The case of (III) requires that \( H_0 \)-type women accept
middle-type men. When high- and middle-type men reject middle-type women and low-type
men accept low-type women, there are five kinds of women in the market: \( k_0 \)-type women
\((k = H, M, L)\) and \( l_{ML} \)-type women \((l = M, L)\). However, the next lemma shows that a \( k_0 \)-
type woman rejects a middle-type man when high- and middle-type men reject middle-type
women and low-type women accept low-type women.
Lemma 8 Let us assume that \( x_M < R^*_H \) and \( x_L < R^*_M \) and that middle-type men reject middle-type women and low-type men accept low-type women. In this case, a \( k_0 \)-type \((k = H, M, L)\) woman rejects a middle-type man.

Proof. Let us obtain the discounted lifetime utility of a \( k_0 \)-type woman. Let \( p_{H4} \) denotes the probability that the actual type of a \( k_0 \)-type woman is the high-type, \( p_{M4} \) denotes the probability that the actual type of a \( k_0 \)-type woman is the middle-type, and \( p_{L4} \) denotes the probability that the actual type of a \( k_0 \)-type woman is the low-type. If she accepts a middle-type man, her value function becomes,

\[
\hat{V}^a = p_{H4} \left[ \alpha \lambda^m_H \left( \frac{x_H}{r} - V_H^a \right) + \alpha \lambda^m_M \left( \frac{x_M}{r} - V_H^a \right) \right] + (p_{M4} + p_{L4}) \left[ \alpha (\lambda^m_H + \lambda^m_M) \left( \hat{V}_{ML} - V_M^a \right) \right],
\]

where \( \hat{V}^a \equiv p_{H4} V_{H}^a + (p_{M4} + p_{L4}) V_{M}^a \) and \( p_{H4} + p_{M4} + p_{L4} = 1 \).

If she rejects a middle-type man, her value function becomes,

\[
\hat{V}^r = p_{H4} \left[ \alpha \lambda^m_H \left( \frac{x_H}{r} - V_H^r \right) + \alpha \lambda^m_M (V_H^r - V_H^r) \right] + (p_{M4} + p_{L4}) \left[ \alpha (\lambda^m_H + \lambda^m_M) \left( \hat{V}_{ML} - V_M^r \right) \right],
\]

where \( \hat{V}^r \equiv p_{H4} V_{H}^r + (p_{M4} + p_{L4}) V_{M}^r \).

Since \( x_M < x_H \) from \( x_M < R^*_H \), a \( k_0 \)-type woman rejects a middle-type man.

From this lemma, if middle-type men reject middle-type women due to the expectation to marry \( H_0 \)-type women, a \( k_0 \)-type woman rejects a middle-type man. This contradicts the assumption that \( H_0 \)-type women accept middle-type men. Therefore, when there are apparently underconfident \( H_0 \)-type women who accept middle-type men, middle-type men always accept middle-type women.

Finally, we investigate the case of (IV): middle- and low-type men reject middle-type women. The case of (IV) requires that \( H_0 \)-type women accept low-type men.\(^{39} \) (When \( k_0 \)-type women accept low-type men, middle-type men and low-type men face the same problem. Then, since middle-type men reject middle-type women, low-type men also reject middle-type women.)

When high-, middle- and low-type men reject middle-type women, there are at least \( k_0 \)-type women \((k = H, M, L)\) and \( l_{ML} \)-type women \((l = M, L)\). In the next lemma, we obtain the optimal strategy of a \( k_0 \)-type woman when middle- and low-type men reject middle-type women.

\(^{39} \)We do not consider the case in which, since \( j_{HM} \)-type women accept low-type men, middle- and low-type men reject middle-type women. This is because that we now assume that \( R^*_M > x_L \) and \( R^*_H > x_M \) in this paper.
**Lemma 9** Let us assume that \(x_M < R^*_H\) and \(x_L < R^*_M\) and that middle- and low-type men reject middle-type women. In this case, a \(k_0(k = H, M, L)\)-type woman rejects a low-type man.

**Proof.** First, let \(p_{H5}\) denotes the probability that the actual type of a \(k_0\)-type woman is the high-type, \(p_{M5}\) denotes the probability that the actual type of a \(k_0\)-type woman is the middle-type, and \(p_{L5}\) denotes the probability that the actual type of a \(k_0\)-type woman is the low-type. If a \(k_0\)-type woman accepts a low-type man, her value function becomes,

\[
r^V^a = p_{H5} \left[ \alpha \lambda^a_H \left( \frac{x_H}{r} - V_H^a \right) + \alpha \lambda^m_M \left( \frac{x_M}{r} - V_M^a \right) + \alpha \lambda^m_L \left( \frac{x_L}{r} - V_L^a \right) \right] + (p_{M5} + p_{L5}) \left[ \alpha (0 - V_M^a) \right],
\]

where \(rV^a \equiv p_{H5}rV_H^a + (p_{M5} + p_{L5})rV_M^a\) and \(p_{H5} + p_{M5} + p_{L5} = 1\).

If a \(k_0\)-type woman accepts a low-type man, her value function becomes,

\[
r^V^r = p_{H5} \left[ \alpha \lambda^a_H \left( \frac{x_H}{r} - V_H^r \right) + \alpha \lambda^m_M \left( \frac{x_M}{r} - V_H^r \right) + \alpha \lambda^m_L \left( V_H^a - V_H^r \right) \right] + (p_{M5} + p_{L5}) \left[ \alpha (0 - V_M^r) \right],
\]

\[
rV_H^r = \alpha \lambda^a_H \left( \frac{x_H}{r} - V_H^r \right),
\]

where \(rV^r \equiv p_{H5}rV_H^r + (p_{M5} + p_{L5})rV_M^r\).

From these equations and \(x_L < x_M < R^*_H\), \(\dot{V}^a < \dot{V}^r\) holds. Therefore, a \(k_0\)-type woman always rejects a low-type man. ■

From this lemma, even if there are enough \(H_0\)-type women who accept middle- and low-type men, middle- and low-type men always accept middle-type women.

**Appendix C**

**Two types** In this paper, we assume not two but three types of agents in order to show the influence of the indirect externality of apparent overconfidence on the market. If we consider two types of agents in the case of apparent overconfidence, the indirect externality does not occur, and we cannot find a case in which the indirect externality of apparent overconfidence prevents the lowest type of agents from marrying. To see this, now, let us assume two types of agents: good and bad. Let us assume that, when all agents have perfect self-knowledge, the PSE occurs. To describe the apparent overconfidence, let us assume that \(i_0\)-type women \((i = g, b)\) reject bad-type men. Therefore, there are two kinds of bad-type women with respect to differences in their beliefs: bad-type women who are apparently overconfident and bad-type women who know their own types. However, good-type men do not change their reservation utility levels relative to the case of perfect self-knowledge because all women want to marry them. Then, the indirect externality of apparent overconfidence does not occur. Hence, bad-type men always marry bad-type women with perfect self-knowledge.
On the other hand, in the case of apparent underconfidence with two types of agents, we obtain, qualitatively, the same results as those of Lemma 4 and Proposition 4. That is, the indirect externality of apparent underconfidence does not occur, and, as a result, all agents can then marry.

The assumption of apparent overconfidence We adopt the assumption that a $k_0$-type woman rejects a middle-type man in the apparent overconfidence case because we focus on the case in which middle-type women are apparently overconfident in order to show that the indirect externality of apparent overconfidence affects the marriage behaviors of lower-type agents.

If $k_0$-type women accept middle-type men (or reject low-type men), all agents can marry. To see this, let us assume that $k_0$-type women accept middle-type men. At this time, $L_0$-type women are apparently overconfident, and $H_0$-type women are apparently underconfident. The apparent underconfidence of $H_0$-type women does not change the behaviors of middle-type men for the same reason as in Lemma 4. Moreover, the indirect externality of apparent overconfidence by low-type women does not arise, similarly to that in two types of agents.

Let us consider another apparent overconfidence case in which since there are enough $M_{HM}$-type women who reject middle-type men, middle-type men accept low-type women. However, this case does not arise. This is because the existence of $M_{HM}$-type women requires that a middle- or low-type man rejects a low-type woman (now, high-type men reject middle-type women).

Benchmark case In our analysis, we consider the case in which $x_M < R^*_H$ and $x_L < R^*_M$ hold as the benchmark case. If we consider the case in which $x_M \geq R^*_H$ and $x_L < R^*_M$ hold as the benchmark case, the result is the same as the case of the two types. Thus, the indirect externality of apparent overconfidence does not occur. To see this, let us define the next situation as a benchmark case: if all agents know their own types perfectly, $x_M \geq R^*_H$ and $x_L < R^*_M$ hold. Let us assume that, under $x_M \geq R^*_H$ and $x_L < R^*_M$, $k_0$-type women reject middle-type men, and then middle-type men accept low-type women due to the rejection from $M_0$-type women. That is, the indirect externality of apparent overconfidence occurs. However, the reservation level of a $k_0$-type woman is always lower than $R^*_H$ as she assigns probabilities to her own types, similarly to the case of Lemma 2. This contradicts the assumption that, under $x_M \geq R^*_H$ and $x_L < R^*_M$, $k_0$-type women reject middle-type men. Therefore, when $x_M \geq R^*_H$ and $x_L < R^*_M$, $k_0$-type women always accept middle-type men.

Under $x_M < R^*_H$ and $x_L \geq R^*_M$, the indirect externality of apparent overconfidence does not occur. When $x_M < R^*_H$ and $x_L \geq R^*_M$, there are few enough middle-type agents.

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40The assumption of $x_M \geq R^*_H$ means that there are a few high-type men and women in the market. If $x_M \geq R^*_H$ and $x_L < R^*_M$, any high-type woman accepts a middle-type man. In this case, even if high-type women with imperfect self-knowledge accept middle-type men, the behavior of these women is the same as that of the high-type with perfect self-knowledge. Then, a middle-type man does not change his behavior: he accepts a middle-type woman. If some middle-type women with imperfect self-knowledge accept low-type men under $x_M \geq R^*_H$ and $x_L < R^*_M$, the indirect externality of apparent underestimation does not occur in the steady state for the same reason as in Lemma 4 qualitatively.
Therefore, even if there are some middle-type women who reject middle-type men due to imperfect self-knowledge, middle-type men do not change their behaviors: they accept low-type women. Now, as there are few enough middle-type men ($x_L \geq R^*_M$), some low-type women (at least, the low-type women who were rejected by high-type men) always accept low-type men. Therefore, in this case, the indirect externality of apparent overconfidence does not occur.\footnote{When $x_M < R^*_H$ and $x_L \geq R^*_M$, the following cases of apparent underconfidence can be considered. If high-type women with imperfect knowledge accept middle-type men under $x_M \leq R^*_H$ and $x_L \geq R^*_M$, the indirect externality of apparent underconfidence does not occur for the same reason as Lemma 4. If middle-type women with imperfect knowledge accept low-type men under $x_M < R^*_H$ and $x_L \geq R^*_M$, low-type men do not change their behavior: they accept low-type women. Then, in this case, there is no influence of the apparent underconfidence of middle-type women.}
Figure 1: P.S.E. with imperfect knowledge
high-type women

middle-type women

low-type women

Figure 2: Type 1 equilibrium
Figure 3: Type 2 equilibrium