Abstract

This paper considers the efficiency properties of risk-neutral workers’ mobility decisions in an equilibrium model with search frictions, but no search externalities, and where matches are experience goods. It is shown that the efficiency of workers’ mobility decisions depends on the degree of enforceability of contracts: in the absence of enforceability by a third-party the equilibrium fails to be efficient, there is too little turnover, even though decisions are privately efficient. I also show that a simple firing tax can reestablish efficiency, thereby increasing mobility.

Keywords: On-the-Job Search; Bargaining; Contracts.
JEL codes: J30; J63.

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1 Introduction

It is well documented that the level of job-to-job transitions is large.\(^1\) However, although economists have devoted considerable attention to the efficiency of the labor market, they have paid limited attention to the question of the efficiency of job-to-job mobility decisions.\(^2\) There are at least two reasons to consider the efficiency in job-to-job movements. First, the work of Hornstein at al. (2011) suggests that modeling on-the-job search is crucial to explain the degree of frictional wage dispersion observed in the data. And as they underline, "it is the frequency of job-to-job transitions, besides unemployment duration, that constrains the magnitude of frictional wage dispersion." Hence, if any inefficiency hampers job-to-job mobility, then this can have a significant impact on the ability of search models to explain the degree of frictional wage dispersion. Second, job-to-job flows are large, and thus understanding whether these flows are efficient, and if not knowing what the source of the inefficiency is, is important as it can help in designing labor market policies that can improve efficiency.

The explanations for job-to-job transitions found in the literature fall into two main categories. In the first category the focus is on the fact that it is time consuming for workers (firms) to learn about employment opportunities (workers) available, i.e., there are search frictions. The Diamond-Mortensen-Pissarides model (DMP) (see Diamond, 1982; Mortensen, 1982; and Pissarides, 1984) and the Burdett-Mortensen (1998) model are the canonical equilibrium models of this branch of the literature.\(^3\) The existence of search frictions means that workers and firms typically do not wait for the perfect match to come along. Instead, a firm and a worker agree to match if its value is acceptable to both of them, which requires the value of the match to each party to be above their own reservation values. However, because firms and workers do not always enter into the best possible match, there is an incentive for them to search for another better match, although it is

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\(^1\)See Fallick and Fleischman (2004) and Nagypál (2008).
\(^2\)See below for a discussion of the related literature.
\(^3\)See Burdett (1978) for a partial equilibrium model with on-the-job search in the spirit of McCall (1970).
standard to assume that only workers search while employed. In this case, a job-to-job mobility decision is driven solely by the arrival of information about a new, more profitable match.

Another strand of the literature, following Jovanovic (1979, 1984), focuses on the information frictions that limit the ex ante knowledge that a firm and a worker have about the quality of the match, and on the fact that more information is learnt as the tenure of the worker on the job increases, i.e., jobs are experience goods. If the worker and the firm believe that the match they are in is of low quality, they might decide to sever the match, in which case the worker changes job and the firm hires a new worker. In this case the mobility decision of the worker is driven by new information about the current match.

This paper combines the two aforementioned literatures, and shows that the degree of enforceability of contracts is crucial for the efficiency of job-to-job mobility decisions of workers in an environment where there are search frictions, but no externalities, and when jobs are experience goods. Employment contracts, which can be contingent on the productivity of the match and on tenure, and can include lump-sum payments like sign-on fees, are the outcome of a bargaining game taking place before agents match. The game is chosen to ensure that mobility decisions are privately efficient. I consider two contractual environments, where the distinction lies in the degree of enforceability of labor contracts. In one case contracts can be enforced by a third party like a court of law, while in the second case contracts must be self-enforcing because workers and firms can walk away from a contract at will. While job-to-job mobility decisions are efficient when contracts are enforceable by a third party, they are not when contracts are self-enforcing (unless workers have all the bargaining power), in the latter case there is too little mobility. And the lower the workers’ bargaining power, the lower worker turnover and thus the greater the inefficiency are. I also show that, unless contracts are restricted to fixed-wage contracts, the fact that workers get paid their marginal productivity is not a sufficient condition for efficiency. Finally, it is shown that a flat

\footnote{Mortensen (1978), Diamond and Maskin (1979) and Kiyotaki and Lagos (2007) are notable exceptions.}
firing tax can deliver efficiency, thereby implying that, contrary to results from in the literature, a firing tax can increase mobility.

The reason for the inefficiency of job-to-job mobility decisions in the absence of enforceability by a third party lies in the endogenous incompleteness of contracts arising from the endogenous limits to the liabilities that agents are willing to accept before walking away from a contract. This contract incompleteness puts a current and a new potential employer in asymmetric positions. Consider a worker matched with some firm A who contacts another firm B, and suppose that both firms are willing to offer her the entire (expected) surplus of their respective matches. If the worker chooses firm A, this means she will never choose to leave firm A, implying that the surplus of the match with firm A depends exclusively on the productivity of the match. On the other hand, the surplus (over unemployment) that firm B can offer the worker implicitly depends on the share of the surplus the worker will obtain with other firms in subsequent matches if the match with B turns out to be of low quality. When contracts are self-enforcing, the value of the match to firm B cannot be negative, implying that, unless the worker has all the bargaining power, her position when bargaining with a subsequent employer is weakened by her limited value of employment with firm B if the match with B is of low quality.

When contracts are enforceable by a third party there is no limit to the level of liabilities agents can agree to, implying contracts are complete. Hence, workers and firms can design contracts that guarantee a worker is able to capture the entire surplus of all her future matches, regardless of her bargaining power, by giving a high employment value even when the productivity draw is bad. These types of contracts require more than self-enforcement because the value of a match to the firm is negative for a positive measure of values of the productivity draw, implying that the firm would like to either renegotiate or walk away from the contract. When contracts are not enforceable by a third party and workers do not have all the bargaining power a flat tax can deliver efficiency of workers’ job mobility decisions: if it is costly to severe a match, a firm can be better off continuing
a match of negative value than paying the severance tax.

Cahuc, Postel-Vinay and Robin (2005) estimate, using a French administrative data set and a variant of the Burdett-Mortensen model where wages are determined by bargaining, that the bargaining powers of all but one group of workers are no greater than 40 per cent and can be as low as 0 per cent for the lowest-skilled groups. Although Cahuc, Postel-Vinay and Robin’s model abstracts from learning, the fact that workers appear to be far from having all the bargaining power suggests that the welfare consequences of employment contracts enforceability uncovered in this paper can be important.

In addition to the papers already mentioned, this paper is related to the models that study the efficiency of worker turnover in excess of job turnover. Mortensen (1978), Diamond and Maskin (1979), and Kiyotaki and Lagos (2007) study the efficiency of match turnover in an environment where the quality of a new match is observed before the match is formed. In this paper if the quality of a match is known before it is formed, then mobility is always efficient, and thus the inefficiencies identified in these papers are thus different from the contractual inefficiency presented here. This paper also complements the work of Gautier et al. (2010) and Menzio and Shi (2011). Gautier et al. study the importance of the types of contracts firms can offer for efficiency. In their model matches are ex ante heterogeneous and firms are restricted to offer wage contracts. They identify a new form of congestion externality, which they name a “business-stealing” externality: when deciding to post a vacancy firms do not take into account the output loss they impose on other firms when they hire an employed worker, which leads to an inefficiently high level of vacancy creation when firms cannot commit to a contract. Menzio and Shi consider instead a competitive search model with on-the-job search where jobs can be inspection and/or experience goods, and they show that the unique equilibrium is efficient.

\footnote{In these papers both parties to a match can search for another match whereas here only workers search. In the present paper firms do not have to replace a worker since they have unlimited capacity.}
In models of investments that are subject to hold-up problems (e.g., Grout, 1984; Acemoğlu, 1997; Masters, 1998; Acemoğlu and Shimer, 1999, Malcomson, 1999), the inefficiency in the absence of enforceability by a third party lies in the incompleteness of contract that prevents agents from maximizing the joint surplus just like in this paper. But in these papers the inefficiency is both private and social, while in this paper mobility decisions are always privately efficient. Moreover, the nature of the economic decision in this paper is different, for instance, there is no sunk cost to be paid.

The learning literature, following Jovanovic (1979), generally abstracts from search frictions, which ensures that workers get paid their expected marginal productivity and that their job mobility decisions are efficient (see Jovanovic, 1979, and Felli and Harris, 1996). In learning models with search frictions other than Menzio and Shi (2011), either the assumption that workers are paid their marginal product is maintained (Jovanovic, 1984), or they do not consider the efficiency of mobility decisions (Moscarini, 2005; and Nagypál, 2007).

Finally, Stevens (2004) shows that, in a Burdett-Mortensen framework where firms post employment contracts rather than fixed-wage contracts, the equilibrium need not even be privately efficient: although firms can design employment contracts that ensure that separations between workers and firms are privately efficient, firms’ recruitment policies are inefficient if they are precluded from making contract postings contingent on a worker’s employment situation.

The paper is organized as follows. The setup is presented in section 2, and the efficient mobility decision is characterized in section 3. Section 4 characterizes the equilibrium mobility decision rule when contracts are enforceable by a third-party and when they are self-enforcing when contracts are restricted to be what I call Flat-Wage Contracts. The source of the inefficiency and labor market policies that yield efficiency with self-enforcing contracts are discussed in section 5. Section 6 shows that the result of section 4 generalizes to the case with general contracts, and I conclude in section 7.
2 The Setup

In this section I first present the setup before discussing the importance of the main assumptions.

2.1 Agents

Time is continuous and the horizon is infinite. There are two types of agents in the economy. There is a mass one of infinitely-lived and homogeneous workers who maximize the expected discounted sum of flow utility and who discount the future at rate $r > 0$. The flow utility of being unemployed for a worker is $b > 0$, while the flow utility of being employed at wage $w$ is $u(w) = w$. In addition there is a mass $n$ of profit-maximizing and homogeneous firms. Firms do not have any capacity constraint, in that a firm can hire as many workers as it wishes, and the productivity of a match with a worker is independent of the number of other workers the firm is matched with and of the productivities of these other matches.\footnote{In general firms make profits, but I will ignore the dividends associated with these profits. This is inconsequential for the mobility decision of workers if one assumes that each worker receives a fixed fraction of the total profits at all times.}

2.2 Search and Matching

Workers’ search is random, and both unemployed and employed workers search at no cost and with the same efficiency: workers contact firms according to a Poisson process with parameter $\alpha > 0$. Jobs are experience goods: when a worker meets a firm the productivity of the match is ex ante unknown; the match needs to be formed for the match-specific productivity index $x$ to be learnt, and $x$ is drawn from a known distribution with cdf $F$ on the support $X = [x, \overline{x}]$, $\overline{x} > b$. I further assume, so as to simplify the analysis, that the productivity draw is learnt as soon as the matched is formed, that both the worker and the firm observe this productivity perfectly, and that it is not possible to draw another productivity as long as the worker and the firm are matched.\footnote{The question of whether the productivity is observable by a third party will be addressed shortly.}
There are two types of match destructions in the model. First, matches are destroyed by some exogenous shock that follows an exponential distribution with parameter $\delta > 0$. In addition to these exogenous match destructions matches are endogenously severed when an employed worker leaves her current match for a match with a new firm.

2.3 Employment Contracts and Bargaining

Contractual Environments – I consider two types of contractual environments, where the distinction between the two is whether contracts are enforceable by a third party. Contracts are said to be enforceable by a third party if a third party, say a court of law, can force each signatory to a contract to honor its contractual obligations. If the productivity of a match is not perfectly observable, then contracts will not be perfectly enforceable by a third party. Hence, I also consider the case where contracts are self-enforcing, where by self-enforcing I mean that when a contract has been agreed upon it is not possible to amend it unless both parties agree to the change, although the lack of third-party enforceability means it is possible to walk away from the contract at will.\(^8\)

The Contract – A contract consists of a wage schedule and a schedule of one-time lump sum payments. The wage schedule specifies the wage the firm must pay the worker for the duration of the match until the worker finds another employment opportunity, at which point the worker and the firm renegotiate according to the bargaining game described below. The wage can be contingent on the productivity of the match, on the tenure of the worker on the job, and on the contract of the worker in her previous match if she is employed when hired. The schedule of lump sums lists one-time payments contingent on the formation of a match, and on the continuation or termination of an existing match. I restrict my attention to renegotiation-proof contracts, in that contracts do not need to be renegotiated unless the worker contacts another firm, and to avoid unnecessarily heavy notation I introduce the relevant formal definition of a contract when needed.

\(^8\)In Section 6.3 I also briefly consider the case where contracts are continuously renegotiated.
Contract Determination Mechanism – It is assumed that all contracts are the outcome of a strategic bargaining game between a worker, either employed or unemployed, and one or two firms, and negotiations take place before the match specific productivity of a new match is learnt. The bargaining protocol is as follows.

Bargaining Protocol:

1. In the first stage firms propose a contract to the worker.
   
   (a) If the worker accepts a contract, bargaining is over;
   
   (b) If she turns down all offers, she keeps her existing contract, if any, and the bargaining process reaches the second stage.

2. In the second stage the worker gets to propose a contract with probability $\beta \in (0,1)$ and firms get to propose with probability $1 - \beta$. In the latter case if the worker is employed then firms enter a Bertrand competition for the worker.\(^{10}\)
   
   (a) If the worker and one firm agree on a contract, this contract agreed on governs the match between the two agents.
   
   (b) If agreement is reached with two firms, then the worker picks her preferred contract.
   
   (c) Otherwise the worker keeps her previous contract, if any, or stays unemployed.

\(^{9}\)Because the probability of meeting two firms at the same time is zero, an unemployed worker will almost surely negotiate with only one firm at a time. I therefore ignore the possibility of tripartite bargaining involving an unemployed worker. However, employed workers can be in contact with other firms, in which case the bargaining will involve three parties, ignoring once again measure zero events where a worker is in contact with two or more new firms at the same time.

\(^{10}\)The result would not be changed if the stochastic processes giving the right to make an offer were uncorrelated for the two firms. This is because the second stage of the game is never reached in equilibrium and thus only the expected value of going to the second stage matters for the outcome of the game.
The equilibrium concept used to solve for the bargaining game is that of subgame perfect Nash equilibrium. Although the game is simple, I focus on the outcome of the game, and I therefore do not formally describe the strategies followed by each agent. And, in order to keep the exposition as simple as possible, I assume the following tie-breaking rules: (i) if the value of a match to a firm (worker) is zero (the value of unemployment), then it (she) accepts the offer; (ii-a) if a worker is indifferent between two contracts offered to her, she accepts the offer of the firm with which the match has the greatest (expected) joint value; (ii-b) And if the two match joint values are the same, then the worker leaves her current match for the new match.

2.4 Discussion of the Assumptions

The setup has been chosen so as to focus on the source of the inefficiency presented in this paper. This is done by ensuring that congestion and composition externalities are absent, and that the contract determination mechanism delivers privately efficient mobility decisions. In fact, it is well known that in search models where agents’ meeting probabilities depend on each others’ search decisions congestion externalities exist and that the equilibrium is generically inefficient when contracts are determined by ex post bargaining (e.g., Hosios, 1990; Gautier et al., 2010). Assuming that the rate at which workers meet firms is constant enables me to abstract from such congestion externalities.

Moreover, search and matching models are also subject to composition externalities, like in Kiyotaki and Lagos (2007): the composition of the pools of searching and matched agents matters for the formation of new matches, an effect which is not internalized by agents when bargaining over the formation of new matches. Here it is assumed that firms do not face any capacity constraints so as to assume away composition externalities: the mobility decision of a worker has no bearing on other workers, nor on the ability and willingness of firms to hire other workers, and if an employed worker does not accept to form a new match, it will never do so, so the firm turned down is treated
like all other firms.\textsuperscript{11} If, however, I were to adopt a search and matching framework à la Diamond-Mortensen-Pissarides where firms have limited capacity (and post vacancies), then composition externalities would appear.

There are typically two ways to approach bargaining in search models: adopting an axiomatic approach such as the Nash solution or choosing a specific strategic bargaining game.\textsuperscript{12} If Nash bargaining were to be used, it would not be clear in such situations what would be the outside options and threat points of the worker and of each of the two firms, and any specific choice might seem arbitrary. By having recourse to a strategic bargaining approach, although the specific game chosen can itself be deemed arbitrary, it is easier to identify the source of the inefficiency in the mobility decision of the worker, should there be any.\textsuperscript{13} In particular, I am able to focus on the social inefficiency by choosing a bargaining game that ensures that equilibrium mobility decisions are privately efficient.\textsuperscript{14} I could have chosen other bargaining protocols, but it should be clear from the analysis that the inefficiency with self-enforcing contracts would be present as long as workers do not always capture the whole surplus of a match.

\textsuperscript{11}For the reader who is not convinced composition externalities are absent, it should anyway suffice to realize that the source of inefficiency is the contractual inefficiency presented in this paper. In fact, it is easy to show that if the quality of a new match is known before a new match is formed (such an environment is essentially that of Kiyotaki and Lagos where firms have unlimited capacity, so not allowing firms to replace workers is inconsequential), the equilibrium mobility decisions in the setup of this paper are privately and socially efficient, implying that any existing composition externality is internalized.

\textsuperscript{12}Another possibility would be to use a cooperative solution like the Shapley value, but, as Kiyotaki and Lagos (2007) highlight, it can lead to surplus sharing rules that seem unintuitive, for instance because an agent might obtain a strictly positive surplus even though she is in an unfavorable situation.

\textsuperscript{13}Moreover, as Shimer (2006) has shown, the conditions of applicability of the Nash bargaining solution can be violated when on-the-job search is allowed.

\textsuperscript{14}Kiyotaki and Lagos (2007) follow the same approach in order to avoid the possibility of privately inefficient mobility decisions that Diamond and Maskin (1979) encounter when using Nash bargaining in models where single and double breaches are allowed.
There are also a number of simplifying assumptions. Workers’ search efficacy, as captured by the parameter $\alpha$, is assumed not to depend on their employment status. It is possible to introduce differentiated arrival rates of meetings for unemployed and employed workers without changing the main results. Furthermore, I did allow for $x < b$ in a previous version of the paper but the extra “risk” of unemployment associated with changing jobs in this environment actually clouds out the real source of inefficiency. It would also be possible to allow contracts to include payments to be made contingent on whether the worker meets another firm, and allow these payments to be contingent on whether the worker quits. As will appear later, the two formulations are equivalent. Finally, the resolution of uncertainty regarding the productivity of a new match is purposely kept as simple as possible by assuming that the productivity is revealed right after a match is formed.\textsuperscript{15}

The results I obtain would be left unchanged if instead it were assumed that either (i) all matches start with the same productivity but are eventually hit by a permanent productivity shock; or (ii) productivity were to be learnt gradually over the match tenure. But these alternative assumptions would complicate the algebra without bringing any additional insight.

### 3 The Efficient Mobility Decision Rule

In this section I present the constrained socially efficient allocation of workers across jobs, where the criterion used for efficiency is the maximization of the discounted sum of aggregate output, augmented by the discounted sum of the value of leisure for unemployed workers,\textsuperscript{16}

\[
\int_0^\infty e^{-rt} \left[ (1 - u_t) \times \int_{x \in X} xdG_t(x) + u_t \times b \right] dt, \tag{1}
\]

\textsuperscript{15}Wright (1986) makes the same assumption. It would be easy to extend the analysis to introduce a signal of match quality before the decision to match is made. There would then exist a threshold for the signal for each productivity: workers employed in a job with productivity $x$ would change jobs only if the signal obtained is above some value $p(x)$, with $p(x)$ increasing in $x$. See Pries and Rogerson (2005) for a model with such signals.

\textsuperscript{16}Workers are risk neutral and therefore this is what a utilitarian social planner would do.
where \( u_t \) denotes the mass of unemployed workers at time \( t \) and \( G_t \) denotes the cdf at time \( t \) of the productivity of existing matches, which is an endogenous object.

Output, augmented with leisure of unemployed workers, is maximized if and only if the decisions regarding the formation of matches are efficient. There are only two decisions in the model, whether a match that has just been formed and whose productivity has been revealed should be continued; and whether a match between a firm and a worker contacted by another firm should be severed so that the worker can join the new firm. Since it is assumed that \( \bar{x} > b \), that there is no cost of search, and that there is no difference in the efficiency of search while employed and unemployed, we have, trivially, that it is always efficient for an unemployed worker to match. Hence, the allocation of workers across firms is efficient if and only if the mobility decision between jobs is efficient. In order to maximize this objective function the social planner thus simply chooses a mobility policy \( \sigma^{sp} = (\sigma^{sp}_x)_{x \in X} \), where \( \sigma^{sp}_x \) is equal to 1 if a worker employed in a match with productivity \( x \) should form a new match if she contacts another firm, and is zero otherwise. Note that I consider only stationary policies, but this is because the optimal policy is stationary. In fact, the productivity of a match is fixed once the match is formed, the productivity of new matches is drawn from a constant distribution, and firms have unlimited capacity so a worker’s allocation does not matter for other workers. This means that if it is efficient to form a new match at some point in time, then it is efficient to do so at any time.

The demographic constraints the social planner faces are

\[
\dot{u}_t = \delta(1 - u_t) - \alpha u_t, \tag{2}
\]

and

\[
\dot{G}_t(x) = \alpha \left[ \frac{u_t}{1 - u_t} + \int_{x' \in X} \sigma^{sp}_{x'} dG_t(x') \right] F(x) - \left[ \delta G_t(x) + \alpha \int_x^x \sigma^{sp}_{x'} dG_t(x') \right]. \tag{3}
\]

Equation (2) is the law of motion for unemployment, where \( \dot{u}_t \) is the time derivative of unemployment: given that there is a mass \( 1 - u_t \) of employed workers and a mass \( u_t \) are unemployed, a
mass \( \delta(1 - u_t) \) of employed workers become unemployed because their matches are exogenously destroyed, while a mass \( \alpha u_t \) of unemployed workers meet a firm and therefore become employed. Equation (3) highlights that \( G \) is an endogenous object which depends on the mobility decisions of employed workers. The first two terms on the right-hand side capture the inflow of workers into matches with productivity no more than \( x \): employed and unemployed workers get the opportunity to form a new match at rate \( \alpha \), and if a worker forms a new match then the productivity is no more than \( x \) with probability \( F(x) \); unemployed workers always accept, while workers employed in a match with productivity \( x \) accept with probability \( \sigma^{sp}_x \). The last two terms capture the outflow: workers employed in matches with productivity no more than \( x \) see their matches being exogenously destroyed at rate \( \delta \), while they form new matches at rate \( \alpha \int_x^\infty \sigma^{sp}_{x'}dG(x') \).

Hence, the social planner chooses \( \sigma^{sp} = (\sigma^{sp}_x)_{x \in X} \) to maximize (1) subject to (2) and (3) and initial conditions for \( G \) and \( u \). If we define by \( \omega_u \) and \( \omega_x \) the shadow values of an unemployed worker and of a match with productivity \( x \) respectively, we have that\(^{17}\)

\[
r'_u = b + \alpha (\bar{\omega} - \omega_u),
\]

and

\[
r'_x = x + \alpha \sigma^{sp}_x (\bar{\omega} - \omega_x) - \delta (\omega_x - \omega_u),
\]

where

\[
\bar{\omega} \equiv \int_{x \in X} \omega_x dF(x).
\]

An unemployed worker enjoys an instantaneous flow value \( b \) and she encounters a firm at rate \( \alpha \), and the expected capital gains arising from the formation of a new match are \( \bar{\omega} - \omega_u \), and therefore \( r'_u \), the flow shadow value of an unemployed worker, is given by (4). When a worker is in a match with productivity \( x \) the flow product is \( x \); when the match is destroyed, the gross capital loss is \( \omega_x \);

\(^{17}\)See Kiyotaki and Lagos (2007) for the derivation of shadow values in a related environment with a discrete distribution of match qualities.
if the match destruction is exogenous, then the worker becomes unemployed and the net capital loss is $\omega_t - \omega_u$. If, however, the match has been destroyed because the worker leaves for another firm, the expected net capital gain is then $\tilde{\omega} - \omega_x$. Hence, the shadow value of a match with productivity $x$ is given by (5).\footnote{Note that, contrary to Kiyotaki and Lagos (2007), none of the shadow values depend on the endogenous distribution of match quality $G$, which confirms that there are no composition externalities and that the efficient mobility decision rule is stationary.} We thus have that the steady-state efficient mobility policy $\sigma^{sp}$ is such that

$$
\sigma_x^{sp} = \begin{cases} 
1 & \text{if } \tilde{\omega} > \omega_x, \\
[0,1] & \text{if } \tilde{\omega} = \omega_x, \\
0 & \text{otherwise.}
\end{cases}
$$

That is, the efficient mobility policy $\sigma^{sp}$ requires that a worker employed in a match with productivity $x$ agrees to form a new match if the expected shadow value of a new match $\tilde{\omega}$ exceeds the shadow value of the current match $\omega_x$. If the two shadow values are equal, then it does not matter what the worker does, while a worker must stay in her current match if the expected shadow value is less than the shadow value of her current match.

It is clear that $\omega_x$ is strictly increasing in $x$, and therefore the efficient mobility decision is characterized by a reservation strategy with cut-off $x^{sp}$ and the mobility decision is such that

$$
\sigma_x^{sp} = \begin{cases} 
1 & \text{for } x < x^{sp}; \\
[0,1] & \text{for } x = x^{sp}; \\
0 & \text{for } x > x^{sp}.
\end{cases}
$$

This is intuitive. If a worker is currently employed in a low productivity match, say $x = x + \epsilon$ for $\epsilon$ small, then the probability with which the worker would draw a higher productivity when changing jobs is $1 - F(x)$ which is very close to 1 for a small enough $\epsilon$. Therefore, the risk of drawing a lower productivity than her current one is small, and the expected gains from changing employer will be positive. If, on the contrary, the worker is currently employed in a match with a
high productivity, say $x = \overline{x} - \epsilon$, then it is highly likely she will be less productive in a new job. Therefore, the expected gains from changing jobs will be negative. The social planner is indifferent between $\sigma_x^{sp}$ equal to zero or one for $x = x^{sp}$, and I will, in order to facilitate the comparison with the equilibrium mobility decisions, use as a tie-breaking rule that says the worker should accept a new match if indifferent.

Equation (5) and the mobility policy (7) yield that the shadow value of a match with productivity $x$ is

$$\omega_x = \begin{cases} 
\frac{x + \delta \omega_u + \alpha \overline{\omega}}{r + \delta + \alpha}, & \text{if } x \leq x^{sp}, \text{ and} \\
\frac{x + \delta \omega_u}{r + \delta}, & \text{otherwise.}
\end{cases}$$

When a worker is in a match whose productivity $x$ is below the mobility cutoff, the match is destroyed at rate $\alpha + \delta$: New jobs arrive at rate $\alpha$, and the expected value of this new match is $\overline{\omega}$; and jobs are exogenously destroyed at rate $\delta$, in which case the new value for the worker is the value of unemployment $\omega_u$. Hence, the expected discounted value of a match with productivity $x \leq x^{sp}$ is given by $(x + \delta \omega_u + \alpha \overline{\omega}) / (r + \delta + \alpha)$. When the mobility cutoff is lower than the productivity of the match the match is still severed exogenously at rate $\delta$, but the worker never quits. Hence, the value of such a match is $(x + \delta \omega_u) / (r + \delta)$.

Solving for $\overline{\omega}$ yields

$$\overline{\omega} = \frac{\delta \omega_u}{r + \delta} + \Delta(x^{sp}; \alpha),$$

where, for $x^c = \int_{x \in X} x dF(x)$,

$$\Delta(x^{sp}; \alpha) = \frac{1}{r + \delta + \alpha} \left[ x^c + \frac{\alpha}{r + \delta} \int_{x^{sp}}^x x dF(x) \right].$$

Hence we obtain that

$$\omega_x = \begin{cases} 
\frac{x + \alpha \Delta(x^{sp}; \alpha)}{r + \delta + \alpha} + \frac{\delta \omega_u}{r + \delta}, & \text{if } x \leq x^{sp}, \text{ and} \\
\frac{x}{r + \delta} + \frac{\delta \omega_u}{r + \delta}, & \text{otherwise.}
\end{cases}$$
Since the efficient mobility cut-off is such that $\omega_{x^{sp}} = \bar{\omega}$ we have the following proposition.\footnote{Proofs not in the main text can be found in the Appendix.}

**Proposition 1** The efficient mobility cut-off $x^{sp}$ is such that

$$x^{sp} = x^e + \frac{\alpha}{r + \delta} \int_{x^{sp}}^{x} (x - x^{sp}) dF(x), \quad (9)$$

and it is unique.

It is worth noting for further reference that $x^{sp} > x^e$, and it is because the existence of on-the-job search means workers have the option of searching for another match if their current match is of poor quality. In fact, if we assume workers are restricted to only one job change in between unemployment spells, the flow value of the current job if the worker never leaves is $r\omega_x = x - \delta(\omega_x - \omega_u)$, whereas the expected value of the new job is $\bar{\omega} = \int_{x}^{x} [(x + \delta \omega_u)/(r + \delta)] dF(x)$. We therefore have that in this case $x^{sp} = x^e$. If instead workers are unrestricted in their job mobility decisions and one considers a worker currently employed in a job with productivity $x$ who changes job and draws a low level of productivity for her new job, although this decreases the aggregate production level, the worker can search for another job, and she thus has another chance at getting a highly productive match. Hence, the option of looking for another job if a productivity draw is unsatisfactory makes a new job more attractive than if on-the-job search is restricted, explaining that the social planner demands workers to change jobs even for some job productivities that are above $x^e$.

For the sake of completeness we can use (2)-(3), the mobility decision rule (41), and the tie-breaking rule mentioned above to derive the steady-state demographic characteristics of the model. We have that the steady-state unemployment rate $u$ and the steady-state distribution of match productivities among employed worker $G$ are given by

$$u = \frac{\delta}{\alpha + \delta}, \text{ and}$$

$$G(x) = \begin{cases} \frac{\delta}{\delta + (1 - \alpha)F(x)} F(x), & \text{for } x \leq x^{sp}; \text{ and} \\ F(x) + \frac{\alpha G(x^{sp})}{\delta} (1 - F(x)), & \text{for } x > x^{sp}. \end{cases}$$
4 Equilibrium Mobility Decision Rule With Flat-Wage Contracts

In this section I focus my attention of contingent fixed-wage contracts (FWCs), that is, contracts where fees of any kind are ruled out, the wage does not depend on the tenure of the worker on the job but can be contingent on the productivity draw. A FWC is thus entirely characterized by a contingent schedule of wages $w = (w_x)_{x \in X}$, where $w_x$ is the wage the firm pays the worker when the productivity draw is $x$.

Before considering the value functions a few important remarks are in order. First, in general the joint value of a match between a worker and a firm depends on the specific contract that is governing the match. In fact, the mobility decision of the worker, as well as the value of a new match should the worker contact a new firm and elect to join that firm, can depend on the contract the worker and the current firm had agreed on. This is because, for instance, different wage levels can imply different bargaining positions for the worker when bargaining takes place after she contacts another firm.

However, all contracts for a given bargaining situation observed in equilibrium must yield the same expected values of the match for the firm and the worker: it must be that any contract observed in equilibrium maximizes the expected joint value of a new match, subject to the constraints imposed by the contractual environment considered. In fact, as will be shown shortly, the bargaining game is such that the firm and the worker who form a match get respectively a fraction $1 - \beta$ and $\beta$ of the surplus of the match (over the value of unemployment when the worker was previously unemployed, and over the value of the worker’s previous match when she was previously employed). The worker and the firm therefore agree *ex ante* on the best contracts, which are the ones that maximize the expected joint value of the match because they maximize their respective expected values of the match as well.

It is well known that when workers and firms are risk-neutral contracts are generically indeter-
minate, in that for a given match there are many contracts that yield the same present value of
the match to both agents as well as the same mobility decisions. So there will typically be more
than one contract possible for each type of match, and the attention will be focused on the value
of these contracts, which is unique for each type.

4.1 The Value Functions

Since I am focusing on FWCs, the individual and joint flow values of a match do not vary with
the tenure of the match, and thus capital gains or losses can happen only when workers get the
opportunity to form a new match or when their match is exogenously severed.

Workers - Let \( W_x(w), \bar{W}(x, w), \) and \( \bar{W}(b, b) \) respectively denote the value of employment in
a match with productivity \( x \) at wage \( w \), the expected value of starting a new match for a worker
currently in a match with productivity \( x \) paid \( w \), and the expected value of starting a match for
an unemployed worker.

The value of being unemployed for a worker, \( U \), is such that

\[
 rU = b + \alpha (\bar{W}(b, b) - U),
\]

where for all \( w = (w_x)_{x \in X} \) in the equilibrium set of contracts for unemployed workers,

\[
 \int_{x \in X} W_x(w_x) dF(x) = \bar{W}(b, b).
\]

In fact, the instantaneous flow return from being unemployed is \( b \), and the worker meets other firms
at rate \( \alpha \), in which case the capital gains are \( \bar{W}(b, b) - U \).

The value of employment \( W_x(w) \) is such that

\[
 rW_x(w) = w + \alpha \sigma_x(w) \left[ \bar{W}(x, w) - W_x(w) \right]
 + \alpha (1 - \sigma_x(w)) \left[ W_x(\bar{w}_x(w)) - W_x(w) \right] - \delta [W_x(w) - U],
\]

where \( \sigma_x(w) \) is an indicator function which is equal to 1 if a worker employed in a match with
productivity \( x \) at wage \( w \) changes job when she contacts another firm, and 0 otherwise. In fact, the
instantaneous flow return from the match to the worker is $w$; the job gets exogenously destroyed at rate $\delta$, in which case the worker suffers a capital loss of $W_x(w) - U$; and the worker meets other firms at rate $\alpha$. In this latter situation if the worker changes jobs then she gains $\tilde{W}(x, w) - W_x(w)$, while if the worker stays in her current job with wage $\bar{w}_x(w)$, then her capital gains are $W_x(\bar{w}_x(w)) - W_x(w)$.

**Firms** - There is no concept of a vacant job in this environment. Moreover, the productivity of each match is independent of all other matches the firm is engaged in. Hence, the outside option of a firm to being matched with a given worker is of value zero. The value to a firm of a match with productivity $x$ when the worker is paid $w$, $J_x(w)$, is thus such that

$$rJ_x(w) = x - w - \alpha (1 - \sigma_x(w)) [J_x(\bar{w}_x(w)) - J_x(w)] - (\delta + \alpha \sigma_x(w)) J_x(w).$$  \tag{12}$$

The instantaneous flow return from the match to the firm is $x - w$, the job gets exogenously destroyed at rate $\delta$, in which case the firm suffers a capital loss of $J_x(w)$, and the worker meets other firms at rate $\alpha$. In this latter case, if the worker changes jobs then $J_x(w)$ is lost for the firm, while if the worker stays with wage $\bar{w}_x(w)$, then the firm suffers the capital loss $J_x(\bar{w}_x(w)) - J_x(w)$.

**Joint Value** - Denoting by $V$ the joint value of a match, i.e., $V = W + J$, we have from (11) and (12) that

$$rV_x(w) = x + \alpha \sigma_x(w)(\tilde{W}(x, w) - V_x(w)) - \delta (V_x(w) - U).$$  \tag{13}$$

It is worth noting that (i) the joint surplus of a match depends on the surplus $\tilde{W}(x, w) - V_x(w)$ that the worker extracts from a future employer should she be better off by accepting a new match than staying in her current match, and (ii) $V_x(w)$ is strictly increasing in $x$ provided that $\tilde{W}(x, w)$ is weakly increasing in $x$, which will be shown to be true shortly.

### 4.2 Mobility with Third-party Enforceability

When a contract can be enforced by a third-party there are no ex ante restrictions on the values a match can have for the different realizations of the productivity draw for either the firm or
the worker. In particular, contracts such that the value of a match for the firm (the worker) is $J_x(w_x) < 0$ ($W_x(w_x) < U$) for some $x$, meaning that the firm (the worker) would like to, but cannot, terminate the match if the productivity drawn is $x$ are feasible. One might object to allowing for contracts such that the worker would like to quit, but is not allowed to, as it amounts to some form of slavery and runs against labor laws. However, as I will explain later on, allowing for $W_x(w_x) < U$ for some $x$ might be necessary to obtain efficiency with third-party enforceability when contracts are restricted to FWCs, although it is possible to obtain efficiency and have $W_x(w_x) \geq U$ for all $x$ once we allow for one-time fees.

4.2.1 Optimal Contracts

The first step in determining the mobility decision rule is to characterize the set of optimal contracts, i.e., the set of contracts that deliver the highest expected joint value of a match. The next lemma gives the nature of optimal contracts for a new match between a worker and a firm given that all other firms offer optimal FWCs and the implied expected value of a new match is $\tilde{V}$.

**Lemma 1** A FWC $w = (w_x)_{x \in X}$, is an optimal FWC if and only if it is such that $W_x(w) = \tilde{V}$ for all $x \leq x^*$, where $x^* \equiv r\tilde{V} + \delta(\tilde{V} - U)$.

The intuition as to why optimal FWCs with third-party enforceability are as described in Lemma 1 is as follows. First, the mobility decision of a worker must be such that the worker accepts a new job if and only if the value of a new match for her exceeds the joint value of her current match, so that the mobility decision of a worker is privately efficient. This, together with the bargaining game, means an optimal FWC $w = (w_x)_{x \in X}$ must be such that $W_x(w_x) \leq \tilde{V}$ for all $x < x^*$.

The second condition that a contract must satisfy to be optimal is that it ensures that, should it be optimal for the worker to change job, she extracts the entire surplus of a new match from her new employer. Since it is optimal for the worker to quit her current job when the productivity is
no more than $x^*$, it follows that an optimal FWC must be such that for all productivity draws no more than $x^*$ the worker receives a wage giving her a value of employment $\tilde{V}$. In fact, this way when the worker meets another firm, that firm is obliged to bid $\tilde{V}$ for the worker in order for her to join. For productivity draws greater than $x^*$ the wage is indeterminate because the worker will never leave her current match (because the joint value of the match is then greater than $\tilde{V}$), and therefore the wage she is paid has no impact on the joint value of the match.

Given that in equilibrium all contracts observed must be optimal contracts, we have that $x^*$ is such that $V_{x^*} = \tilde{V}$, and the mobility decision rule is given by

$$
\sigma_x = \begin{cases} 
1, & \text{for } x \leq x^*, \text{ and} \\
0, & \text{otherwise.}
\end{cases}
$$

(14)

A corollary to lemma 1 is that in equilibrium all matches with the same productivity share the same joint value, and for this reason I from now on simply denote by $V_x$ the joint value of a match with productivity $x$.

### 4.2.2 Bargaining and Match Values

Having characterized optimal FWCs with third-party enforceability, we are now ready to determine the individual and joint match values. To that aim we need to consider the bargaining games between an unemployed worker and a firm, and between a worker employed in a match with productivity $x$ and her employer and a new firm. The following lemma gives the outcome of the bargaining game for either case.

**Lemma 2** (i) *When an unemployed worker meets a firm, at the first round of bargaining the firm*

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20If $x \leq x^*$, the value of employment for the worker is $\tilde{V}$, so for all these matches the expected value of a new match for a worker is the same, implying the joint value of their current match is also the same; and if $x > x^*$, then the worker never quits, so the joint value of the match depends only on $x$. 

---
offers an optimal contract $w^*(b)$ such that

$$\tilde{W}(b,b) = \int_{x \in X} W_x(w^*_x(b))dF(x) = U + \beta(\tilde{V} - U), \quad (15)$$

and the worker accepts.

(ii) When a worker employed with an optimal contract in a match with productivity $x$ at wage $w$ with joint value $V_x$, if $x \leq x^*$, so that $\tilde{V} \geq V_x$, the new firm offers the worker an optimal contract $w^*(x)$ such that

$$\tilde{W}(x,w) = \int_{x' \in X} W_{x'}(w^*_x(w))dF(x') = \tilde{V}, \quad (16)$$

and the worker accepts, while if $x > x^*$, so that $\tilde{V} < V_x$, the current employer offers the worker a contract with wage $\bar{w}(x,w)$ such that

$$W_{x}(\bar{w}(x,w)) = \max\{W_{x}(w); V_x\} + \beta \left[ \max\{W_{x}(w); V_x\} - \max\{W_{x}(w); V_x\} \right], \quad (17)$$

and the worker accepts this offer.

It follows from this lemma that (13), the joint value of a match with productivity $x$, can be expressed as

$$rV_x = x + \alpha \sigma_x(\tilde{V} - V_x) - \delta(V_x - U), \quad (18)$$

and therefore workers obtain an individual value of match greater than the joint value for productivities $x < x^*$.

The simplest examples of optimal FWCs are contracts where the worker can be paid one of two wages: if the worker is initially unemployed, she gets paid $w^*_x(b) = x^*$ for all $x \leq x^*$, and receives $w^*_x(b) = d(\tilde{V}; \beta)$ for all $x > x^*$, where $d(\tilde{V}; \beta)$ is such that

$$\tilde{V}F(x^*) + W(d(\tilde{V}; \beta)) (1 - F(x^*)) = U + \beta(\tilde{V} - U).$$

This contract is such that the wage for $x \leq x^*$ ensures the worker will be able to extract the entire surplus of a future match should the productivity draw be bad, and the wage for $x > x^*$ is chosen
so that the shares of the total expected surplus of the new match going to the worker and the firm are $\beta$ and $1 - \beta$ respectively.

For a worker employed with an optimal contract in a match with productivity $x \leq x^*$, the simplest FWCs when she starts a new match is such that she gets paid $w^*_z(x) = x^*$ for all $z \in X$. As above, this contract is such that the wage for $x \leq x^*$ ensures the worker will be able to extract the entire surplus of a future match should the productivity draw be bad. And since the worker extracts the entire (expected) value of the new match, $V = V^*_x$, the wage for $x > x^*$ is also $x^*$.

Finally, when a worker employed in a match with productivity $x > x^*$ contacts another firm, the expected value of a new match is not large enough to induce the worker to move. Then the current employer only needs to offer in the first round a contract with a wage such that the worker’s value to the match is given by $W_x(\bar{w}(x, w)) = \max\{W_x(w_x); \bar{V}\} + \beta(V - \max\{W_x(w_x); \bar{V}\})$.

### 4.2.3 Equilibrium Mobility Decision Rule

The joint private value of a match satisfies (18) with the mobility decision given by (14). Therefore, it follows that

$$V_x = \begin{cases} \frac{x+\delta U + \alpha \bar{V}}{r+\delta+\alpha} , & \text{for } x \leq x^*, \text{ and} \\ \frac{x+\delta U}{r+\delta} , & \text{otherwise.} \end{cases}$$

The interpretation of the private joint value of the match $V_x$ is similar to that for the social value, $\omega_x$. Solving for $\bar{V}$ yields

$$\bar{V} = \frac{\delta}{r+\delta} U + \Delta(x^*; \alpha),$$

---

Note that if $W_x(w_x) \in (\bar{V}, \bar{V}_x)$, the fact that the worker contacts another firm triggers renegotiations of the contract and enables the worker to obtain a better contract even though the newly contacted firm could not offer a better contract than the one the worker already had. One could modify the bargaining game to avoid such cases without having an impact on the mobility decision rule, since this happens only in cases where the worker does not change employer.
where $\Delta(x^*, \alpha)$ is defined as in (8), and hence we obtain that
\[
V_x = \begin{cases} 
\frac{x + \alpha \Delta(x^*, \alpha)}{r + \delta + \alpha} + \frac{\delta U}{r + \delta}, & \text{for } x \leq x^*, \text{ and} \\
\frac{x}{r + \delta} + \frac{\delta U}{r + \delta}, & \text{otherwise.}
\end{cases}
\]
Therefore the equilibrium mobility cut-off in this case is such that
\[
x^* = x^e + \frac{\alpha}{r + \delta} \int_{x^e}^{x^*} (x - x^*) dF(x). \tag{19}
\]
Equation (19) is identical to (9) which characterizes the efficient mobility productivity cut-off, and since this equation has a unique solution, we obtain that $x^* = x^p$. This is summarized in the following proposition.

**Proposition 2** When contracts are enforceable by a third party and restricted to FWCs, the equilibrium mobility cut-off $x^*$ is equal to the efficient mobility cut-off $x^p$ for all values of $\beta \in (0, 1]$.

It is clear from the nature of optimal contracts that enforceability of contracts is crucial, for firms would like to walk away from matches with productivity below the cutoff. It is also worth noting that with FWCs workers might also want to walk away from a match. In fact, the expected value of employment for an unemployed worker is $\tilde{W}(b, b) = U + \beta(\tilde{V} - U) < \tilde{V}$, which implies that the expected value of employment, conditional on the draw being $x > x^p$, is
\[
U + \frac{\beta - F(x^p)}{1 - F(x^p)} (\tilde{V} - U). \tag{20}
\]
This expression is strictly less than the value of unemployment $U$ if $\beta < F(x^p)$, and the lower the $\beta$ the more likely it is. If the expected value of employment conditional on the draw being $x > x^p$ is less than $U$, then the initial value of employment must be less than $U$ for a positive mass of productivities greater than $x^p$. Such FWCs might thus not be feasible in practice because labor laws tend to restrict the ability of employers to force an employee to stay on a job without any time limit.\footnote{Labor laws usually require workers to give prior notice to their employer before quitting, and thus in practice workers do stay in jobs they wish to quit, but it is only for a limited, and usually short, period.}
Demographics - The steady-state equilibrium unemployment rate \( u \) is given by (2), and since the optimality of equilibrium contracts implies that the mobility rule distribution is the same in equilibrium for all workers in matches with the same productivity, the distribution of match productivity \( G \) are as given in (3).

4.3 Mobility Decision Rule with Self-Enforcing Contracts

A FWC \( w = (w_x)_{x \in X} \) is self-enforcing if and only if \( J_x(w_x) \geq 0 \) and \( W_x(w_x) \geq U \) for all \( x \). In fact, if either of these two conditions is violated either the firm or the worker is better off by quitting the match. These restrictions put on the individual values of a match when contracts are self-enforcing imply the following lemma.

Lemma 3 A FWC \( w = (w_x)_{x \in X} \) is self-enforcing only if \( w_x \leq x \) for all \( x \).

In fact, a firm never experiences a capital gain after a match has been formed and the productivity draw is realized, which also means the wage the firm pays to the worker never goes down, and therefore when contracts are restricted to being FWCs the value of the match to the firm is positive only if \( w_x \leq x \).

The above lemma crucially means that the optimal FWCs described in lemma 1 are not self-enforcing. What does this mean for the mobility decisions of workers? Is the mobility decision rule with self-enforcing contracts efficient or not? If not, is there too little or too much turnover? The answer is not obvious because the restriction to self-enforcing contracts yields two opposite effects on workers' mobility decision. On one hand, a firm which tries to poach a worker can promise a lower expected value to a worker with self-enforcing FWCs than with an optimal FWC enforceable by a third party, which, ceteris paribus, implies that worker turnover is too low with self-enforcing contracts. On the other hand, a self-enforcing FWC cannot give a value of employment for low productivity draws as high as an optimal FWC enforceable by a third-party could. This implies,
ceteris paribus, that worker turnover is too high with self-enforcing FWCs. In what follows I derive the equilibrium match values and mobility decision rule when FWCs are self-enforcing and show that the first effect dominates, and therefore there is too little mobility.

4.3.1 Bargaining and Match Values

The next lemma characterizes the outcomes of the different possible scenarios for the bargaining game.

Lemma 4 (i) When an unemployed worker meets a firm, at the first round the firm offers a self-enforcing FWC $w(b) = (w_x(b))_{x \in X}$ such that

$$\tilde{W}(b, b) = \int_{x \in X} W_x(w_x(b)) dF(x) = U + \beta(\tilde{V} - U),$$

and the worker accepts.

(ii) When a worker employed in a match with productivity $x$ at wage $w$ with joint value $V_x$ meets another firm, if $\tilde{V} \geq V_x$ the new firm offers the worker a self-enforcing FWC $w(x)$ such that

$$\tilde{W}(x, w) = \int_{x' \in X} W_{x'}(w_{x'}(x')) dF(x') = V_x + \beta(\tilde{V} - V_x),$$

and the worker accepts, while if $\tilde{V} < V_x$ the current employer offers the worker the self-enforcing contract $\tilde{w}(x, w)$ such that

$$W_x(\tilde{w}(x, w)) = \max\{W_x(w); \tilde{V}\} + \beta[V_x - \max\{W_x(w); \tilde{V}\}],$$

and the worker accepts.

The outcome of bargaining for a worker employed with productivity $x$ who contacts another firm implies that the mobility decision of a worker, $\sigma_x(w)$, and the expected value of a new match, $\tilde{W}(x, w)$, do not depend on the wage $w$, they only depend on the productivity $x$, and

$$\sigma_x = \begin{cases} 1, & \text{for } x \leq x^*, \text{ and} \\ 0, & \text{otherwise}, \end{cases}$$

(23)
where the cutoff $x^*$ is such that $V_{x^*} = \tilde{V}$. Crucially, if $V_x < \tilde{V}$ the value of a new job as given in (21) depends on the productivity and joint value of the previous match, which is not the case when contracts are enforceable by a third-party. Using (21) and (22), (13) can thus be rewritten as

$$rV_x = x + \alpha \beta \sigma_x (\tilde{V} - V_x) - \delta (V_x - U).$$  

(24)

An example of a self-enforcing contract for an unemployed worker is such that for all $x \in X$ the worker gets a fraction $\beta$ of the surplus, that is, for all $x \in X$, $W_x(w_x) = U + \beta (V_x - U)$. This is the contract that would prevail if contracts were continuously renegotiated, as is standard in most of the search and matching literature based on the Diamond-Mortensen-Pissarides framework. For a worker employed in a match with productivity $x \leq x^*$ at wage $w_x \leq x$ meets another firm, an example of a self-enforcing FWC satisfying (21) is one where the worker receives a wage $w_{x'}$ such that $W_{x'}(w_{x'}) = V_x + \beta (V_{x'} - V_x)$ for all productivity draws $x'$. In this case the worker suffers a loss if $x' < x$, and obtain a gain if $x' > x$.

### 4.3.2 Equilibrium Mobility Decision Rule

The joint private value of a match satisfies (24) with the mobility decision given by (23). Therefore, it follows that

$$V_x = \begin{cases} 
\frac{x + \delta U + \alpha \beta \tilde{V}}{r + \delta + \alpha \beta}, & \text{for } x \leq x^*, \text{ and} \\
\frac{x + \delta U}{r + \delta}, & \text{otherwise.}
\end{cases}$$

The interpretation of the private joint value of the match $V_x$ is again similar to the one for the social value, $\omega_x$, except that in equilibrium when a worker changes jobs she captures a fraction $\beta$ of the surplus generated from this job change. Solving for $\tilde{V}$ yields

$$\tilde{V} = \frac{\delta U}{r + \delta} + \Delta(x^*; \alpha \beta),$$
where $\Delta(x^*; \alpha \beta)$ is as defined in (8), and hence we obtain that

$$V_x = \begin{cases} 
\frac{x + \Delta(x^*; \alpha \beta)}{r + \delta + \alpha} + \frac{\delta U}{r + \delta}, & \text{if } x \leq x^*, \text{ and} \\
\frac{x}{r + \delta} + \frac{\delta U}{r + \delta}, & \text{otherwise.}
\end{cases}$$

Proposition 3 says that in steady-state the equilibrium mobility cut-off $x^*$ exists and is unique, and that as long as workers do not have all the bargaining power the equilibrium mobility cutoff is strictly lower than the efficient one.

**Proposition 3** (i) The equilibrium mobility cut-off with self-enforcing FWCs, $x^*$, is such that

$$x^* = x^e + \frac{\beta \alpha}{r + \delta} \int_{x^*}^{x^p} (x - x^*) dF(x), \quad (25)$$

and it is unique.

(ii) For all $\beta \in [0, 1]$, $x^* \leq x^{sp}$, with strict inequality for $\beta < 1$, and $\partial x^*/\partial \beta > 0$.

As we will show soon this result holds even when one allows for more general contracts with tenure-dependent wages and one-time fees. In particular, the mobility cutoff is still given by (25), and the intuition is quite simple: all self-enforcing contracts are such that the value of the match to a firm is always non-negative, which also implies the value of a match to the worker can never exceed the joint value of the match. But before looking at more general contracts I discuss the source of the inefficiency when contracts are self-enforcing in the next section.

**Demographics** - The steady-state equilibrium unemployment rate $u$ is still given by (2), and the distribution of match productivity $G$ are as given in (3) with $x^*$ as given in proposition 3 replacing $x^{sp}$.
5 The Source of Inefficiency and Policy Implications

5.1 Understanding the Source of Inefficiency with Self-Enforcing Contracts

The inefficiency of the equilibrium mobility decision rule of workers when contracts are self-enforcing comes from an endogenous incompleteness of contracts, which implies there is an asymmetry in the positions an incumbent firm and a poaching firm are in. Let me first explain the asymmetry and its role. I will then explain its origins.

Consider a worker employed in a match with productivity $x$ with firm A and who has the opportunity to start a new match with firm B. If the worker chooses firm A, this means she will never choose to leave firm A, implying that the surplus of the match with firm A depends exclusively on the productivity of the match. Although Firm B can also offer the worker the entire (expected) surplus of the new match, this surplus implicitly depends on the share of the surplus the worker will obtain with other firms in subsequent matches if the match with B turns out to be of low quality. When contracts are self-enforcing the worker can only obtain a fraction $\beta$ of the surplus (over the value of the match with B) of any future match with some firm C if the match with B ends up being of low productivity, i.e., if $x \leq x^*$. Hence, if the worker decides to start a new match with B and this new match has a low productivity, the worker will not be capturing the entire surplus of the subsequent matches she is a part of she becomes unemployed.

The following example illustrates that the asymmetry between the incumbent and poaching firms is essential for the existence of the inefficiency. Assume that when an unemployed worker becomes employed she can change employer only once before her match is hit by an exogenous destruction. In this example it is clear that the efficient mobility decision is such that a worker quits her current match for a new match if her current match productivity is $x < x^e$, while she should remain in her current match if $x > x^e$, and if $x = x^e$ then it does not matter whether
she moves or not. It can be shown that in this case the equilibrium mobility decision with self-enforcing contracts is given by \( \sigma_x = 1 \) for \( x \leq x^* = x^e \), and \( \sigma_x = 0 \) otherwise, that is the equilibrium mobility cutoff is equal to the efficient mobility cutoff. In this example the equilibrium mobility decision is efficient even when contracts cannot be enforced by a third-party because, no matter whether a worker decides to quit her current job, a worker does not have to bargain with another employer before she is forced into an unemployment spell. Hence, when a worker is able to bargain for the entire expected joint value of a new match \( \tilde{V} \), the worker de facto captures the entire surplus of the match she is in until she becomes unemployed.

The asymmetry in the case where contracts are self-enforcing stems from the incompleteness of contracts resulting from the ability of agents to walk away from a contract. In fact, since agents can walk away from contracts there is an endogenous limit to the liability of each agent. In particular, a firm cannot offer to the worker more than the joint value of the match, meaning that if their match is of low quality the share of the surplus the worker can bargained for with her next employer depends on her bargaining power. When contracts are enforceable by a third party contracts can guarantee a high value of employment to a worker even if the productivity of her match is low, because the firm has to abide by the terms of the contract even if it means it makes a negative profit. And as I have shown the contract can, should, be designed to ensure the worker captures the entire surplus of the subsequent match.

It is worth nothing that the type of market incompleteness at the heart of the inefficiency bears some resemblance to the type of endogenous market incompleteness in a Walrasian general equilibrium environment where agents can default à la Kehoe-Levine (1993, 2001). In fact, in both cases agents would like ex ante to be able to commit to fulfill their promise, but the lack of a commitment device in the absence of third-party enforceability means that ex ante beneficial agreements are not observed in equilibrium.

\(^{23}\)See the Appendix for the proof.
5.2 Labor Market Policy and Efficiency with Self-Enforcing Contracts

The mobility cutoff is too low when contracts cannot be enforced by a third party, implying there is too little mobility. However, it is possible to design labor market policies that yield efficiency without relying on the productivity of a match being observable. This last feature is important because it is in practice often difficult to determine accurately the productivity of a worker on a job. Moreover, a labor market policy that would require the productivity of a match be observable for it to be effective would run into the same issues of enforceability as private contracts. In fact, the impossibility of accurately observing the productivity of a match is one of the reasons why enforceability of contracts by a third party like a court of law or a court of arbitration might not be possible.

There are more than one possible policies that can deliver efficiency, and I focus on a firing tax and show that a flat firing tax does the job. I make this choice because it is a simple policy, and it shows that an appropriately chosen firing tax increases mobility, which is in contradiction to the results obtained in the literature, and does not hold when the quality of a match is known before the match is formed. Since firms are better off, at least from an ex post point of view, severing a match whose value is negative to them, one way to get around the absence of third-party enforceability is to make it costly for firms to sever a match: if a firm’s value to a match is negative but the cost of destroying the match exceeds the loss associated with continuing the match, then firms will choose not to severe the match. Hence, in the context of this paper a firing tax can play the role of a commitment device for firms to not severe a low productivity match, thereby protecting workers against the risk of a bad productivity draw.

Consider a firing tax such that when a firm sacks a worker the firm is liable to pay to a

\[24\] See, for instance, Section V-A in Kiyotaki and Lagos (2007) for the impact of a firing tax in a model with on-the-job search. And see Boeri and van Ours (2008) for a review of the theoretical results in the absence of on-the-job search.
third-party a tax contingent on the wage the worker was paid before she got laid off. Let \( \tau = (\tau(w))_{w \in \mathbb{R}^+} \) be the firing tax schedule. Clearly, as long as \( \tau(w) \geq -J_x(w) \) a firm is better off continuing the match than breaking it. Since the efficient mobility decision is obtained if workers receive the wage \( x^{sp} \) for all productivities \( x \leq x^{sp} \) and that in such a case the value of the match to the firm is

\[
J_x(x^{sp}) = \frac{x - x^{sp}}{r + \delta + \alpha},
\]

which is negative, with strict inequality for all \( x < x^{sp} \), a firing tax yields the efficient mobility decision if it is such that

\[
\tau(x^{sp}) \geq \frac{x^{sp} - \bar{x}}{r + \delta + \alpha}, \tag{26}
\]

In fact, if (26) holds, then \( J_x(x^{sp}) \geq -\tau(x^{sp}) \) for all \( x \leq x^{sp} \), i.e., firms which have agreed to pay workers a wage \( x^{sp} \) for productivities \( x \leq x^{sp} \) are better off keeping the worker until she finds another match than firing her. Hence, the optimal FWCs with third-party enforceability become self-enforcing because the penalty to breaking the match that the government can impose on firms is severe enough to deter them from doing so. The rest of the firing tax rate schedule is indeterminate and thus a flat firing tax such that \( \tau(w) = \tau(x^{sp}) \) for all \( w \) works. This means that if firms face a large enough fixed administrative or legal cost to firing a worker, then they would retain her.

It has now been well documented that worker turnover is much lower in continental Europe than in the U.S. or the U.K., although countries from continental Europe tend to have much stronger employment protection legislations (EPL) than the U.S. or the U.K. This evidence seems at odds with the impact that a firing tax has on worker flows in this paper. However, I have ignored the possibility of productivity draws too low to make a match viable and, as Pries and Rogerson (2005) highlight in a model without on-the-job search, EPL can, especially through interaction with other

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25If the payment made by the firm were to be to the worker, then it would have no effect. This is reminiscent of Lazeard (1990). See the next section regarding the impact of lump-sum payments when contracts are self-enforcing.

26See Pries and Rogerson (2005) for a review of the evidence.
legislations like a binding minimum wage, unemployment benefits, and labor income tax, have a significant downward impact on worker turnover in such cases. Combining the two effects might explain why empirical studies have failed to reach unambiguous conclusions relative to the impact of EPL on labor turnover.\footnote{See Boeri and van Ours (2008) for a review of the evidence.}

6 Inefficiency with General Contracts without Third-Party Enforceability

So far I have restricted my attention to FWCs. I now allow for more general contracts and show that the inefficiency of the mobility decision without third-party enforceability uncovered was not due to the restriction to FWCs. In order to keep the presentation simple I first allow for one-time fees, and then I introduce tenure-contingent wages. I will then discuss briefly the case where contracts are continuously renegotiated.

6.1 One-Time Fees

When contracts are not enforceable by a third-party, quitting fees are not enforceable, for a firm or a worker that would have to pay a fee when the worker quits her job would simply choose to sever the match. Thus, I only need to consider sign-on and stay-on fees. Finally, I focus on the case where sign-on fees are decided before the match productivity is drawn,\footnote{Allowing for contingent fees would impose an additional constraint for each productivity draw: for each \( x \) the fee \( \phi_x \) must be such that neither the firm nor the worker are better off breaking up the match than paying the fee. However, the expected value of the fee is what really matters, and therefore allowing for productivity-contingent fees would not change the result.} and where stay-on fees are not part of an initial contract and instead are only determined and paid if a worker contacts another firm and a new round of bargaining takes place.\footnote{As we will see shortly this is inconsequential for the mobility decision.} Hence, a contract is a double \((\phi, w(\phi))\),
where \( \phi \) is the sign-on fee and \( w(\phi) = (w_x(\phi))_{x \in X} \) with \( w_x(\phi) \) being the wage received by the worker when the productivity draw is \( x \) given she received the sign-on fee \( \phi \).

The value of unemployment for a worker still satisfies (10), but now for all contracts \((\phi, w(\phi))\) observed in equilibrium the expected value of employment \( \bar{W}(b, b) \) is given by

\[
\bar{W}(b, b) = \phi + \int_{x \in X} W_x(w_x(\phi)) dF(x), \tag{27}
\]

where \( W_x(w) \) is net-of-fee value of employment for a worker employed in a match with productivity \( x \) at wage \( w \). The contract \((\phi, w(\phi))\) must be such that \( \bar{W}(b, b) \) is the value the worker obtains from bargaining as given by the game, and the wages must be such that \( W_x(w_x(\phi)) \geq U \) for all \( x \).

The expression for \( W_x(w) \) is now\(^{30}\)

\[
rW_x(w) = w + \alpha \sigma_x(w) \left[ \bar{W}(x, w) - W_x(w) \right] + \alpha (1 - \sigma_x(w)) \left( \bar{\phi} + W_x(\tilde{w}_x(w, \phi)) - W_x(w) \right) - \delta [W_x(w) - U], \tag{28}
\]

where \( \bar{\phi} \) is the stay-on fee the worker obtains from her employer and \( \tilde{w}_x(w, \phi) \) is the new wage she is paid given the stay-on fee she receives (and given her previous wage). And the new contract \((\tilde{\phi}, \tilde{w}_x(w, \phi))\) must be such that \( \tilde{\phi} + W_x(\tilde{w}_x(w, \phi)) \) is the value the worker obtains from bargaining as given by the game, and the wage must be such that \( W_x(\tilde{w}_x(w, \phi)) \geq U \).

Paralleling the expected value of a match for an unemployed worker, \( \bar{W}(x, w) \), the expected value of a new match for a worker currently employed in a match with productivity \( x \) at wage \( w \) is given

\[
\bar{W}(x, w) = \phi + \int_{x \in X} W_x(w_x(\phi)) dF(x), \tag{29}
\]

where we still have that \( \phi \) is the sign-on fee, \( w_x(w, \phi) \) is the wage received by the worker when the productivity draw is \( x \) given she received the sign-on fee \( \phi \). And the contract \((\phi, w(w, \phi))\) must be

---

\(^{30}\)The mobility decision, value of a new match, or the value of staying in an existing match do not depend on a stay-on fee as it is assumed stay-on fees are negotiated ex post. But it will be clear later on that this would still be true if stay-on fees are part of an initial contract.
such that $\bar{W}(x, w)$ is the value the worker obtains from bargaining as given by the game, and must be such that $W_x(w_x(\phi)) \geq U$ for all $x$.

Symmetrically for firms, the net-of-fee value of a match of productivity $x$ to a firm when the worker’s wage is $w$ is given by

$$rJ_x(w) = x - w - \alpha (1 - \sigma_x(w)) [J_x(\bar{w}_x(w, \bar{\phi})) - \bar{\phi} - J_x(w)] - (\delta + \alpha \sigma_x(w)) J_x(w),$$

where $\bar{\phi}$ is the stay-on fee the firm pays and $\bar{w}_x(w, \bar{\phi})$ is the wage paid to the worker staying given she receives the fee $\bar{\phi}$. The initial contract $(\phi, w)$ bargained must be such for each productivity the wage implies that $J_x(w) \geq 0$, and any new contract $(\bar{\phi}, \bar{w}_x(w, \bar{\phi}))$ bargained must be such that $J_x(\bar{w}_x(w, \bar{\phi})) - \bar{\phi} \geq 0$ and $J_x(\bar{w}_x(w, \bar{\phi})) \geq 0$.

As is the case without fees, the lack of third-party enforceability implies that the wage paid by a firm at any point in time cannot be larger than the productivity of the match for productivity draws lower than the mobility cutoff $x^*$. In fact, if the wage were to be larger than the productivity then the instantaneous flow return to the firm is negative, which means the firm must expect some capital gains later on for the value of the match to be non-negative to it. However, the only capital gains could come from the worker paying a quitting fee, and such fees have already been ruled out because they would require a third party to be enforced.

It follows from the facts that quitting fees are not self-enforcing and that the wage must be no more than the productivity of a match for $x \leq x^*$, that the mobility decision rule does not depend on the wage, i.e., $\sigma_x = 1$ for all $x \leq x^*$ and the joint value of such matches cannot depend on the wage. Hence, the joint value of a match of productivity $x$, $V_x$, is still given by

$$rV_x = x + \alpha \sigma_x(\bar{W}(x) - V_x) - \delta (V_x - U), \quad (30)$$

and thus for all equilibrium contracts the wage paid must be such that $W_x(w) \in [U, V_x]$ and $J_x(w) \in [U, V_x]$. These last two facts imply that the introduction of one-time fees does not change the outcomes of the game, and therefore does not alter the nature of the mobility decision problem.
When a worker and a firm draw a bad productivity, i.e., \( x \) is below the mobility cutoff \( x^* \), the wage cannot be more than \( x \). This in turn implies that the value of employment to the worker cannot exceed the joint value of the match \( V_x \), thereby preventing the worker from extracting the entire expected value of a future match: in the bargaining game the new employer would only need to offer \( V_x \) in the second stage to induce an acceptance from the worker, which means it would offer only \( V_x + \beta(\tilde{V} - V_x) < \tilde{V} \) in the first stage. The mobility decision rule is therefore still given by (23) with the mobility cutoff still satisfying (25).

Actually, it appears from (27)-(29) that allowing for one-time fees can only change the timing of payments by front-loading the transfer a firm makes to a worker. In fact, the expected value of a match to the firm and a worker are the same with and without fees. Hence, it appears from (27) and (29) that if a sign-on fee is agreed on, i.e., \( \phi > 0 \), then since \( \int_{z \in X} W_z(w) dF(z) < \tilde{W}(x) \), the wage the worker receives must be lower than without the fee for some strictly positive mass of productivities. The effect is exactly the same with stay-on fees: if a firm makes a lump-sum payment to the worker to retain her, i.e., \( \tilde{\phi} > 0 \), since the joint value of the match is the same the values of continuing the match to the worker and the firm are identical to those without fees; so it must be that \( W_x(\tilde{w}_x(w, \tilde{\phi})) < W_x(\tilde{w}_x(w, 0)) \), and thus the continuation wage must be lower than without a fee.

It is worth noting that with one-time fees the mobility decision is inefficient even if workers can be paid their marginal product at all times. In fact, firms can obtain their share of the surplus by receiving a sign-on fee from workers. For instance, when a firm meets an unemployed worker the firm can offer a contract that offers the worker her productivity for each draw \( x \) and still have an expected value of a match \( \tilde{J} = (1 - \beta)\tilde{V} \) if the worker pays a sign-on fee \( (1 - \beta)\tilde{V} \). Hence, the fact that workers get paid their marginal productivity is not a sufficient condition for efficiency.
6.2 Tenure-Contingent Wage Contracts

In this section I focus on tenure-contingent wage contracts and ignore one-time fees. A tenure-contingent wage contract is a contract \( w = (w_x)_{x \in X} \), where for each \( x \) the contract contingent on the productivity draw \( x \) specifies the payment the firm makes to the worker at any point of the worker’s tenure before she is contacted by another firm, i.e., \( w_x = (w^t_x)_{t \geq 0} \) where \( t \) indicates the tenure since the worker was first hired or contacted by another firm for the last time.\(^{31}\)

In the absence of third-party enforceability an equilibrium contract must be such that the value of a match to a firm must still be non-negative at all times, i.e., \( J_x((w^r_x)_{r \geq t}) \geq 0 \) for all \( t \). With FWCs \( J_x(w_x) \geq 0 \) implied that \( w_x \leq x \) because a firm never gets to enjoy capital gains once a match has been formed. With tenure-dependent contracts a firm can enjoy capital gains if the wage contract prescribes that the wage decreases at some point, and thus an equilibrium contract can be such that \( w^t_x > x \) for some time interval and some productivity \( x \) despite the absence of third-party enforceability. However, since the only capital gains come from changes in the wage, it follows that \( J_x((w^r_x)_{r \geq t}) \geq 0 \) at all \( t \) only if the present value of wages does not exceed the present value of the match product at all \( t \), i.e., \( J_x((w^r_x)_{r \geq t}) \geq 0 \) at all \( t \) only if \( \int_t^\infty w^r_x e^{-(r+\delta)\tau} d\tau \leq x/(r+\delta) \) for all \( t \). It follows that the value of a match to a worker is always no more than the joint value of the match. The bargaining game thus implies that neither the job mobility decision of the worker nor her expected value of a new job depend on her wage contract, or her tenure on the job, implying the joint value of a match is independent of the wage contract and the tenure of the worker on the job.

However, the value of the match to the worker and the firm when the productivity is \( x \), the

\(^{31}\)Hence, the contract does not include the payment the worker would receive if she gets the chance to form a new match, but this would not change the mobility decision rule because the continuing part of the contract has to be self-enforcing as well. See the second-next footnote.
wage contract is \( w_x \), and the tenure of the worker is \( t \) are given respectively by

\[
\begin{align*}
\dot{r}W_x(w_x^t) &= w_x^t + \alpha \sigma_x (\bar{W}(x) - W_x(w_x^t)) \\
&+ \alpha (1 - \sigma_x) (W_x(\bar{w}_x) - W_x(w_x^t)) - \delta (W_x(w_x^t) - U) + \dot{w}_x(w_x^t),
\end{align*}
\]

and

\[
\begin{align*}
\dot{r}J_x(w_x^t) &= x - w_x^t - \alpha (1 - \sigma_x) (J_x(w_x^t) - J_x(\bar{w}_x)) - (\delta + \alpha \sigma_x) J_{\bar{x}}(w_x^t) + \dot{J}_x(w_x^t),
\end{align*}
\]

where \( \dot{W}_x(w_x^t) \) and \( \dot{J}_x(w_x^t) \) are the time derivative of the value functions at tenure \( t \).\(^{32}\) The fact that the joint value of a match is independent of the wage contract and the tenure of the worker on the job implies that \( W_x(w_x^t) + J_x(w_x^t) = 0 \), and therefore

\[
\dot{r}V_x(w_x^t) = x + \alpha \sigma_x (\bar{W}(x) - V_x(w_x^t)) - \delta (V_x(w_x^t) - U).
\]

Since the expected value of a new job for a previously employed worker \( \bar{W}(x) \) depends neither on the contract \( w_x \) nor on the tenure of the worker in the match, it follows that \( V_x \) is also independent of \( w_x \) and of the tenure of the worker in the match, i.e., it also satisfies (30).\(^{33}\)

The fact that the joint value of a job is still given by (30) implies that when a worker is employed in a match with productivity \( x \leq x^\ast \) the expected values of a new match to a firm and a worker as

\(^{32}\)Note that \( \bar{w}_x \), the wage contract governing the match if the worker is contacted by another firm and elects to stay in her current match, is indicated as dependent of the tenure \( t \). In fact, if the value of employment to the worker changes with tenure and \( W_x(w_x^t) \in (\bar{V}, V_x) \) for \( t \) belonging to some interval while \( W_x(w_x^t) \leq \bar{V} \) for some other interval, then the outcome of the bargaining game is that the worker stays in her current job and the new value of employment for her is \( W_x(\bar{w}_x^t) = \beta V_x + (1 - \beta)W_x(w_x^t) \) in the first time interval while it is \( W_x(\bar{w}_x^t) = \beta V_x + (1 - \beta)\bar{V} \) in the second.

\(^{33}\)If the wage contract included the continuation wage schedule for a worker who contacts another firm and elects to stay in her current match, then this still holds. In fact, the value of this continuation contract is non-negative for all \( t \) only if the present value of wages does not exceed the present value of the match product at all \( t \), i.e., \( J_x(\bar{w}_x^t) \geq 0 \) at all \( t \) only if \( \int_t^\infty \bar{w}_x^t e^{-(r+\delta)\tau} d\tau \leq x/(r+\delta) \) for all \( t \). Hence, even if the firm can promise a wage above \( x \) from some time, the value of the continuation of the match it can offer the worker still cannot exceed the joint value of the match.
given by the outcomes of the bargaining game are unchanged, and the mobility decision of a worker is still given by (23) with the mobility cutoff $x^*$ still given by (25). In other words, the mobility decision rule is identical to the case with FWCs.

### 6.3 Continuous Renegotiations

In this section I briefly consider the case where contracts can be contingent on tenure and include event-dependent fees but are not self-enforcing, and thus can be continuously renegotiated. This case is of interest because it looks at first sight as if both a current and potential employer are in symmetric position. However, this is not the case and the job mobility decision rule is identical to that with self-enforcing contracts.

In this contractual environment it is clearly not possible to have a worker’s employment value being greater than the joint value of the match, for the firm would want to renegotiate the contract. Hence, the joint value of a job still cannot depend on the contract of the worker, and it is still given by (13), while the mobility decision and mobility cutoffs are still given by (23) and (25). The main difference with the case of FWC and one-time fees is that the wage is pinned down for each productivity level, and thus the fees are adjusted to take into account the fact that firms cannot promise to pay a high wage if it will have an incentive to renegotiate it after the match is formed.

$W^t_x$ and $J^t_x$, the worker’s and firm’s value, net of sign-on or stay-on fees, of a match with productivity $x$ after tenure $t \geq 0$, are constant in between two events, that is in between the start of the match and the moment the worker is contacted by another firm, or in between two times where the worker is contacted by other firms. Denoting simply these values net of fees by $W_x$ and $J_x$, the bargaining game yields that $W_x = U + \beta (V_x - U)$, while the value of the match to the firm is $J_x = (1 - \beta) V_x$, where the joint value of the match $V_x$ is as in (13). The expected value of a match for a worker who is unemployed is given by (27), while for a worker employed in a job with productivity $x$ it is (29). This means that when a worker bargains with a firm while unemployed
there is no sign-on fee, while for a worker employed in a match with productivity \( x \leq x^* \) the sign-on fee she receives from her new employer is \( \phi_x = (1 - \beta)(V_x - U) \), and the stay-on fee a worker receives when employed in a job with productivity \( x > x^* \) is \( \tilde{\phi}_x = (1 - \beta)(\tilde{V} - U) \). Hence, one can see that a firm currently employing a worker and a new potential employer are not in a symmetric position: the new potential employer cannot include in the value of the match it offers the worker the entire value of future matches if the productivity draw is below \( x^* \), just as in the case when contracts are self-enforcing.

7 Concluding Remarks

This paper has identified a new source of inefficiency in search models. When jobs are experience goods and employment contracts are the outcomes of bargaining games, workers’ mobility decisions fail to be efficient in the absence of third-party enforceability of contracts. This is because firms and workers cannot write and enforce contracts that would guarantee the worker can capture the entire surplus of future matches, should their match be of poor quality. It was also shown that a flat firing tax can deliver efficiency of workers’ job mobility decisions, thereby increasing mobility.

It would be interesting to quantify the inefficiency identified in this paper, and the extent to which it impacts the ability of search models to generate purely frictional wage dispersion. It would also be interesting to generalize the model to allow jobs to be both inspection and experience goods in order to quantitatively investigate the impact of EPL on job and worker turnovers. But I leave these issues for future research.

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34 In fact, since \( W_x = U + \beta(V_x - U) \), we have that the expected value of employment for an unemployed worker net of any sign-on fee is \( \int W_x dF(x) = U + \beta(\tilde{V} - U) \).

35 This is because \( \tilde{W}(x) = V_x + \beta(\tilde{V} - V_x) \), while the expected value of employment net of the sign-one fee is \( \int W_x dF(x) = U + \beta(\tilde{V} - U) \). Hence, the worker receives the sign-on fee \( \phi_x = \tilde{W}(x) - \int W_x dF(x) \).

36 In fact, \( W_x(\tilde{C}_x) = \tilde{V} + \beta(V_x - \tilde{V}) \), while the value of employment from tenure \( t > 0 \) with the new contract is \( W_x = U + \beta(V_x - U) \). Thus the worker receives the stay-on fee \( \gamma_x = W_x(\tilde{C}_x) - W_x \).
Appendix

Proof of Proposition 1: Since $x^{sp}$ is such that $\omega_{x^{sp}} = \bar{\omega}$ we have

$$\frac{x^{sp}}{r + \delta} + \frac{\delta \omega_u}{r + \delta} = \frac{\delta \omega_u}{r + \delta} + \Delta(x^{sp}; \alpha),$$

or

$$x^{sp} = \frac{r + \delta}{r + \delta + \alpha (1 - F(x^{sp}))} \left[ x^e + \frac{\alpha}{r + \delta} \int_{x^{sp}}^{\pi} x dF(x) \right],$$

which can be rewritten as

$$x^{sp} = x^e + \frac{\alpha}{r + \delta} \int_{x^{sp}}^{\pi} (x - x^{sp}) dF(x).$$

Defining $\Phi(z) = z - x^e - \alpha/(r + \delta) \int_{x^{sp}}^{\pi} (x - z) dF(x)$, we have that $\Phi(x) = x - x^e - \alpha/(r + \delta) \int_{x^{sp}}^{\pi} (x - x^{sp}) dF(x) < 0$ and $\Phi(\bar{x}) = \bar{x} - x^e > 0$ and therefore since $\Phi$ is continuous on $X$ we have proven existence by the Intermediate Value Theorem. Uniqueness if obtained from the fact that

$$\frac{\partial \Phi(z)}{\partial z} = 1 + \frac{\alpha(1 - F(z))}{r + \delta} > 0.$$

Proof of lemma 1: First, the joint value of a match for a given productivity $x$ is given by (13), and therefore the joint value of a match $V_x(w)$ is maximum if and only if (i) $\sigma_x(w) = 1$ whenever $\widetilde{W}(x, w) \geq V_x(w)$, and $\sigma_x(w) = 0$ otherwise, and (ii) $\widetilde{W}(x, w) = \bar{V}$ for all $(x, w)$ such that $\sigma_x(w) = 1$.

Given $\bar{V}$, the bargaining game rules yield that at the first round we have:

- If $\bar{V} \geq \max\{W_x(w); V_x(w)\}$ the current employer offers in the first round $V_x(w)$ (because $W_x(w) > V_x(w)$ means $J_x(w) < 0$), and the poaching firm offers $\max\{W_x(w); V_x(w)\} + \beta[\bar{V} - \max\{W_x(w); V_x(w)\}]$, and thus the worker accepts the offer from the poaching firm, i.e., $\sigma_x(w) = 1$ and thus

$$\widetilde{W}(x, w) = \max\{W_x(w); V_x(w)\} + \beta[\bar{V} - \max\{W_x(w); V_x(w)\}];$$
Clearly $\tilde{W}(x, w)$ is maximum if and only if $\max\{W_x(w); V_x(w)\} = \bar{V}$, in which case $\tilde{W}(x, w) = \bar{V}$.

- If $\bar{V} < \max\{W_x(w); V_x(w)\}$ the current employer offers $\bar{V} + \beta [V_x(w) - \tilde{V}]$, and the other firm offers $\bar{V}$, and thus the worker does not quit her job, i.e., $\sigma_x(w) = 0$ and

$$
\tilde{W}_x = \max\{W_x(w); \bar{V} + \beta [V_x(w) - \tilde{V}]\}.
$$

It follows that the expected joint value of a match is maximum if and only if $\sigma_x(w) = 1$ when $\bar{V} \geq V_x(w)$ and $\tilde{W}(x, w) = \bar{V}$, and $\sigma_x(w) = 0$ when $\bar{V} < V_x(w)$. However, if $\max\{W_x(w); V_x(w)\} = \bar{V}$ for all $x$ such that $V_x(w) \leq \bar{V}$, then for these productivity levels

$$
V_x(w) = \frac{x + \delta U + \alpha \bar{V}}{r + \delta + \alpha},
$$

while for all $x$ such that $V_x(w) > \bar{V}$ we have that

$$
V_x(w) = \frac{x + \delta U}{r + \delta}.
$$

It follows from (A4) and (A5) that $V_x(w) \leq \bar{V}$ for all $x \leq x^* = r\bar{V} + \delta(\bar{V} - U)$, and that $V_x(w) > \bar{V}$ otherwise. Hence, the maximum joint value of a match is attained if and only if the contract $w(\bar{V}; z), z \in b \cup X$, is such that $w_x = x^*$ for all $x \leq x^*$, so that $W_x(w) = \tilde{W}(x, w) = \bar{V}$. $\blacksquare$

**Proof of Lemma 2:**

**An Unemployed Worker Meets A Firm** - I solve the game by backward induction. In the second stage of the game when the worker gets to propose she proposes to the firm an optimal contract $w^* = (\bar{w}^*_x)_{x \in X}$ such that she captures the entire expected surplus of the match, i.e., $\tilde{W}(w^*) = \bar{V}$. When instead the firm gets to propose it offers an optimal contract $w^* = (\bar{w}^*_x)_{x \in X}$ such that $\tilde{W}(w^*) = U$.

The expected value of reaching the second round to the worker is thus $\beta \bar{V} + (1 - \beta) U$, and she refuses any first-stage contract offer such that the expected value of employment is less than $\beta \bar{V} + (1 - \beta) U$. In the first round the firm therefore proposes an optimal contract $w^*(b) = (w^*_x(b))_{x \in X}$
such that
\[
\tilde{W}(w^*(b)) = \int_{x \in X} W_x(w_x^*(b))dF(x) = U + \beta(\tilde{V} - U),
\]
and the worker accepts the offer.

**A Worker Employed in a Match With Productivity \( x \) at wage \( w \) Contacts Another Firm** - This case is also solved by backward induction. If the worker gets to make the offer at the second stage she offers to her current employer a contract which gives her the value of employment
\[
\max\{W_x(w)\} = \max\{W_x(w) \mid V_x\}
\]
while she proposes to the other firm an optimal contract \( \tilde{w}^* = (\tilde{w}_x^*)_{x \in X} \) such that she captures the entire expected surplus of the match, i.e., \( \tilde{W}(\tilde{w}^*) = \tilde{V} \). If the firms get to make the offers, the current employer offers a contract so that the value of employment for the worker is \( \max\{W_x(w) \mid \min\{V_x, \tilde{V}\}\} \), and the other firm offers a contract such that the (expected) value of the match to the worker is \( \min\{\max\{W_x(w); V_x\}; \tilde{V}\} \). The worker therefore knows in the first stage that the expected value for her of reaching the second stage of the bargaining game is
\[
\beta \max\{\max\{W_x(w); V_x\}; \tilde{V}\} + (1 - \beta) \max\{\min\{\max\{W_x(w); V_x\}; \tilde{V}\}; \max\{W_x(w); \min\{V_x, \tilde{V}\}\}\}
\]
and she refuses any offer in the first round yielding a lower expected value of employment. If \( V_x > \tilde{V} \), then the optimal contract governing the worker’s current match is such that \( W_x(w) \leq V_x \), and thus the current employer offers in the first round to pay the worker the wage \( \bar{w}(x, w) \) such that her new value of employment is
\[
W_x(\bar{w}(x, w)) = \max\{W_x(w); \tilde{V}\} + \beta \left[ \max\{W_x(w); V_x\} - \max\{W_x(w); \tilde{V}\} \right],
\]
while the other employer offers the optimal contract \( \tilde{w}^* = (\tilde{w}_x^*)_{x \in X} \) such that she captures the entire expected surplus of the new match \( \tilde{V} \), and the worker accepts the offer of her current employer.

If instead \( V_x \leq \tilde{V} \), then the contract under which the worker is currently employed is such that \( W_x(w) = \tilde{V} \). Hence, the worker’s current employer offers her a value of employment \( \tilde{V} \), the other firm offers an optimal contract \( \tilde{w}^*(x) = (\tilde{w}_x^*(x))_{x \in X} \) such that she captures the entire expected surplus of the new match \( \tilde{V} \), that is such that
\[
\int_{z \in X} W_x(\tilde{w}_x^*(z))dF(z) = \tilde{V},
\]
and the worker accepts the latter offer.

**Proof of Lemma 3**: A FWC \( w = (w_x)_{x \in X} \) is self-enforcing only if \( J_x(w_x) \geq 0 \) for all \( x \). And I will now prove that \( J_x(w) \geq 0 \) if and only if \( w \leq x \) with FWCs. If we assume that the joint value of a match can depend on the wage paid, then the joint value of a match with productivity \( x \) where the worker is paid \( w \) is

\[
rv_x(w) = x + \alpha (1 - \sigma_x(w)) (\tilde{V}_x(w) - V_x(w)) + \alpha \sigma_x(w)(\tilde{W}(x; w) - V_x(w)) - \delta (V_x(w) - U).
\] (A6)

1. If \( \sigma_x(w) = 0 \), then (A6) yields that \( V_x(w) = (x + \alpha \tilde{V}_x(w) + \delta U)/(r + \alpha + \delta) \), and since if \( \sigma_x(w) = 0 \) the worker never voluntarily leaves her match, it must be that \( \tilde{V}_x(w) \) is such that \( r\tilde{V}_x(w) = x - \delta [\tilde{V}_x(w) - U] \), which clearly implies that \( \tilde{V}_x(w) \) does not depend on \( w \). This is intuitive since when a worker does not leave her current match for a new one then there are no capital gains for this match, and therefore the value of the match is determined by the value of unemployment and the present value of the match product \( x/(r + \delta) \), and

\[
\tilde{V}_x(w) = \tilde{V}_x = (x + \delta U)/(r + \delta).
\]

This then implies that \( \tilde{J}_x(w) \leq J_x(w) \): since the worker cannot be made worse by having the opportunity of forming a new match, i.e., \( W_x(\tilde{w}_x(w)) \geq W_x(w) \), for the worker can always keep her previous contract, the fact that the joint value of the match is fixed when \( \sigma_x(w) = 0 \) implies that the firm cannot be made better off. We thus have from (12) that when \( \sigma_x(w) = 0 \) the value of the match to the firm is such that

\[
J_x(w) = \frac{x - w - \alpha (J_x(w) - \tilde{J}_x)}{r + \delta}.
\]

Clearly we have that if \( w > x \) then \( J_x(w) < 0 \). Furthermore, if \( J_x(w) > 0 \), then it must be that \( x > w \), while if \( J_x(w) = 0 \), then it must be that \( J_x(w) = \tilde{J}_x = 0 \) and thus that \( w = x \). Hence, we have that when \( \sigma_x(w) = 0 \), \( J_x(w) \geq 0 \) if and only if \( w \leq x \).
2. If \( \sigma_x(w) = 1 \), then (12) implies that

\[
J_x(w) = \frac{x - w}{r + \delta},
\]

and it follows straightforwardly that if \( \sigma_x(w) = 1 \), \( J_x(w) \geq 0 \) if and only if \( w \leq x \).

Hence, \( J_x(w) \geq 0 \) if and only if \( w \leq x \). \( \blacksquare \)

**Proof of Lemma 4:**

**An Unemployed Worker Meets A Firm** - I solve the game by backward induction. In the second stage of the game when the worker gets to propose she proposes to the firm the contract \( \mathbf{w} = (\mathbf{w}_x)_{x \in X} \) such that \( \mathbf{w}_x = x \) for all \( x \), which means the worker assigns to herself all of the match surplus \( V_x \) for each \( x \). When instead the firm gets to propose it offers the contract \( \mathbf{w} = (w_x)_{x \in X} \) such that \( W_x(w_x) = U \).

The expected value of reaching the second round to the worker is thus \( \beta \tilde{V} + (1 - \beta) U \), and she refuses any first-stage contract offer such that the expected value of employment is less than \( \beta \tilde{V} + (1 - \beta) U \). In the first round of the game the firm therefore proposes a self-enforcing contract \( \mathbf{w}(b) = (w_x(b))_{x \in X} \) such that

\[
\int_{x \in X} W_x(w_x(b))dF(x) = U + \beta(\tilde{V} - U),
\]

and the worker accepts the offer. This implies that \( \tilde{W}(b, b) \), the equilibrium expected value of employment for an unemployed worker, is therefore

\[
\tilde{W}(b, b) = U + \beta(\tilde{V} - U).
\]

**A Worker Employed in a Match With Productivity \( x \) at wage \( w \) Contacts Another Firm** - This game is also solved by backward induction. If the worker gets to make the offer at the second stage she offers to each firm a contract that gives her the entire value of the match, that is she offers to her current employer to be paid \( w_x = x \) while she offers to the other firm the contract
\( w = (w_x)_{x \in X} \) such that \( w_x = x \) for all \( x \). If the firms get to make the offers, the current employer offers a contract so that the value of employment for the worker is \( \max \{ W_x(w); \min \{ V_x; \bar{V} \} \} \), and the other firm offers a contract such that the (expected) value of the match to the worker is \( \min \{ V_x; \bar{V} \} \). The worker therefore knows in the first stage that the expected value for her of reaching the second stage of the bargaining game is

\[
\beta \max \{ V_x; \bar{V} \} + (1 - \beta) \max \{ \min \{ V_x; \bar{V} \}; \max \{ W_x(w); \min \{ V_x; \bar{V} \} \} ,
\]

and she refuses any offer in the first round yielding a lower expected value of employment. If \( V_x > \bar{V} \), then the current employer offers in the first round \( \bar{w}(x, w) \) such that

\[
W_x(\bar{w}(x, w)) = \max \{ W_x(w); \bar{V} \} + \beta \left[ V_x - \max \{ W_x(w); \bar{V} \} \right],
\]

while the other employer offers the contract \( w \) such that she captures the entire expected surplus of the new match \( \bar{V} \), and the worker accepts the offer of her current employer.

If instead \( V_x \leq \bar{V} \), then the worker’s current employer offers her the entire product of the match, the other firm offers a self-enforcing contract \( w(x) = (w_z(x))_{z \in X} \) such that

\[
\int_{z \in X} W_z(w_z(x))dF(z) = V_x + \beta(\bar{V} - V_x),
\]

and the worker accepts the latter offer. \( \blacksquare \)

**Proof of Proposition 3:** (i) Since \( x^* \) is such that \( V_{x^*} = \bar{V} \) we have

\[
\frac{x^*}{r + \delta} + \frac{\delta U}{r + \delta} = \frac{\delta U}{r + \delta} + \Delta(x^*; \beta \alpha),
\]

or

\[
x^* = \frac{r + \delta}{r + \delta + \beta \alpha (1 - F(x^*))} \left[ x^c + \frac{\beta \alpha}{r + \delta} \int_{x^*}^{x} x dF(x) \right],
\]

which can be rewritten as

\[
x^* = x^c + \frac{\beta \alpha}{r + \delta} \int_{x^*}^{x} (x - x^*) dF(x).
\]
Defining $\Phi(z; \beta) = z - x^e - \beta \alpha/(r + \delta) \int_z^\pi (x - z) dF(x)$, we have that $\Phi(x; \beta) = x - x^e - \beta \alpha/(r + \delta) \int_z^\pi (x - z) dF(x) < 0$ and $\Phi(\pi; \beta) = \pi - x^e > 0$ and therefore since $\Phi$ is continuous on $X$ we have proven existence by the Intermediate Value Theorem. The fact that
\[
\frac{\partial \Phi(z; \beta)}{\partial z} = 1 + \frac{\beta \alpha (1 - F(z))}{r + \delta} > 0
\]
yields uniqueness.

(ii) The results follows from the facts that $\Phi(x; \beta) \leq \Phi(x; \beta)$, $\partial \Phi(z; \beta)/\partial z \leq \partial \Phi(z)/\partial z$, with strict inequality for all $\beta < 1$, and that
\[
\frac{dx^*}{d\beta} = -\frac{\partial \Phi(x^*; \beta)/\partial \beta}{\partial \Phi(x^*; \beta)/z} > 0. \tag{1}
\]

**Proof that $x^* = x^e$ in the example of Section 5.1** - Consider the equilibrium mobility decision: when an employed worker considers whether to leave her current match to form a new one she knows this will be her last mobility decision before she is eventually hit by a shock exogenously destroying her match.\textsuperscript{37} Let $\hat{W}_x(w)$ be the value of employment for a worker in a match with productivity $x$ when the wage is $w$ and she is no longer allowed to change job, and let $\hat{V}_x$ be the associated joint value of the match. $\hat{W}_x(w)$ and $\hat{V}_x$ are such that
\[
r\hat{W}_x(w) = w - \delta(\hat{W}_x(w) - U),
\]
which is independent of $x$, and
\[
r\hat{V}_x = x - \delta(\hat{V}_x - U).
\]
Hence, the expected joint value of the new match is
\[
\hat{V} = \frac{x^e + \delta U}{r + \delta}.
\]
\textsuperscript{37}If she decides not to accept to form a new match once, she will never accept a new job. And since the value of employment is always no less than the value of unemployment a worker will never quit a job unless she has another job lined up.

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One can then follow the analysis of the bargaining game in the previous sections to show that the outcome of the bargaining game is such that the mobility decision of the worker is given by

$$\sigma_x = \begin{cases} 
1, & \text{for } x \leq x^*, \text{ and} \\
0, & \text{otherwise,} 
\end{cases}$$ (A7)

where $x^*$ is such that $V_{x^*} = \hat{V}$, and that for each $x > x^*$ the value of staying with the current employer is given by

$$W_x(\bar{w}(x, w)) = \max\{W_x(w); \hat{V}\} + \beta(V_x - \max\{W_x(w); \hat{V}\}),$$

while for all $x \leq x^*$ the expected value of changing job is given by

$$\bar{W}(x, w) = V_x + \beta(\hat{V} - V_x).$$ (A8)

Using (A8) we obtain that the value $V_x$ of a match with productivity $x$ in which the worker was recruited while unemployed is such that

$$rV_x = x + \alpha\beta \sigma_x(\hat{V} - V_x) - \delta(V_x - U),$$ (A9)

with $\sigma_x$ given by (A7). (A9) and (A7) together yield that

$$V_x = \begin{cases} 
x + \delta U + \beta_0 \hat{V} \over r + \delta + j_0 & , \text{for } x \leq x^*, \text{ and} \\
x + \delta U \over r + \delta & , \text{otherwise.}
\end{cases}$$

From there, it is straightforward to show that $x^* = x^e$.]

REFERENCES


