House Prices Booms and Current Account Deficits*

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Abstract

One of the most striking features of the period before the Great Recession of 2007-2009 is the strong positive correlation between house price appreciation and current account deficits, not only in the U.S. but also in other countries that have subsequently experienced the highest degree of financial turmoil. A progressive relaxation of credit standards can rationalize this empirical observation. Lower collateral requirements facilitate access to external funding and drive up house prices. Households increase their leverage borrowing from the rest of the world so that the current account turns negative. At the same time, however, the world real interest rate counterfactually increases. Departures of nominal interest rates from a standard monetary policy rule in leveraged economies and exchange rate pegs in saving countries are the additional ingredients which reconcile a demand-based explanation of house prices booms and current account deficits with the evidence on real interest rates and exchange rates.

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1 Introduction

During the five years before the eruption of the recent financial crisis, soaring house prices and the widening deficit on the current account were perhaps the two most discussed indicators of U.S. imbalances (figure 1).

Interestingly, the negative correlation between house price dynamics and current account balances is not a U.S. peculiarity but a robust global phenomenon, affecting both advanced and emerging market economies alike (figure 2). Countries that witnessed house price booms and substantial external deficits (e.g., Iceland, Ireland, Greece, Spain) also experienced among the highest degrees of financial turmoil during the crisis.

This paper argues that a progressive relaxation of borrowing constraints can generate a strong negative correlation between house prices and the current account. Lower collateral requirements facilitate access to external funding and drive up house prices. Households

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1Bernanke (2010) plots the cumulative change between 2001Q4 and 2006Q4 in current account balances and house prices for advanced economies. The August 2007 ECB Monthly Bulletin features a similar figure for the period 1997-2005. Figure 2 extends the sample to include emerging market economies such as China, which play a key role in financing the U.S. current account deficit.

2Similar dynamics for capital inflows and real estate prices occurred before the Asian crisis in the late 1990s (see Obstfeld and Rogoff (2010) and the references therein).
increase their leverage borrowing from the rest of the world so that the current account turns negative.

To be sure, the deterioration of lending standards does not necessarily explain all house price booms. The evidence is the strongest for the U.S. and few other countries, such as Iceland and Australia. Demand factors, rather than looser borrowing constraints, were arguably the prime driving force behind the recent housing boom in Spain. Nevertheless, inflated appraisals may have helped circumvent explicit regulations on loan-to-value (LTV) ratios (Bank of Spain, 2006). Similarly, mortgage market liberalizations were probably more relevant to explain the house price boom and bust during the 1980s and early 1990s than during the recent housing cycle in the U.K. (Duca, Muellbauer and Murphy, 2010). Yet, financial innovation, in the form of increased securitization and resort to money market funding, may have amplified house price appreciation.

The analysis relies on a two-country model with tradable consumption goods and housing. The expected value of housing represents the collateral for households' debt. This endogenous borrowing constraint is buffered by a time-varying parameter which controls the LTV threshold and constitutes the key shock in the model. An increase in this threshold, for given value of the collateral, leads households to lever up and demand more housing.

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3 This model essentially interprets the non-tradable sector in Ferrero, Gertler and Svensson (2010) as housing and introduces an endogenous borrowing constraint. From an alternative perspective, this economy translates Iacoviello (2005) into an open economy framework by adding two types of goods.
hence driving up house prices. To the extent that the relaxation of credit constraints affects the whole economy, the increase in domestic borrowing must be financed from abroad, thus generating a current account deficit.

Few empirical studies discuss the relation between house prices and external imbalances. Ahearne et al. (2005) document the co-movement between house price dynamics and current account balance since 1970. Aizenman and Jinjarak (2009) provide a precise estimate of the relation between the two variables: a one standard deviation increase in lagged current account deficits is associated with a 10% appreciation of real estate prices. Fratzscher, Juvenal and Sarno (2010) adopt the opposite perspective: according to their estimates, together with equity market shocks, house price shocks account for up to 32% of the movements in the U.S. trade balance over a 20−quarter horizon. Arguably, both house prices and the current account are endogenous variables. The key question then becomes which underlying fundamentals drive the correlation between these two variables. In terms of causality, these regressions raise an interesting issue. If current account “cause” house prices to move, presumably foreign shocks are an important determinant of domestic house prices, and vice versa.

Starting with Kiyotaki and Moore (1997), several authors have highlighted the role of endogenous borrowing constraint as a source of amplification of other types of shocks. Monacelli (2009) shows that borrowing constraints can reconcile the empirical evidence on the response of durable and non-durable spending to monetary shocks with the dynamics of an otherwise standard New Keynesian model. In a small open economy, Iacoviello and Minetti (2003) relate the strength of the impact of monetary policy shocks on house prices to the degree of financial liberalization. Recently, Eggertsson and Krugman (2010) argue that a tightening of borrowing constraints (relative to pre-crisis levels) can lead to a substantial drop in aggregate demand and potentially create depression-like scenarios.

Several recent papers investigate the connection between house prices and the current account balance in quantitative general equilibrium frameworks. The mechanism that generates the negative correlation between house prices and current account balance in this work is similar to one of the shocks in Punzi (2006), who considers a richer model with residential investment and preference heterogeneity also within countries although without nominal rigidities. Shocks to the LTV ratio provide a rationale for increased housing demand, which is the more basic type of shocks investigated in Gete (2010). These demand-driven explanations contrast with the idea that differentials in expected output growth lead to house price booms and current account deficits, as in Kole and Martin (2009).

Borrowing constraints play a key role in the sudden stop literature (e.g. Mendoza, 2010). Boz and Mendoza (2010) develop a small open economy model with Bayesian learning in which the household’s borrowing constraint follows a two-state regime-switching process. Early realizations of the high-leverage regime lead agents to overestimate the true probability

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4Kole and Martin (2009) find slightly smaller results.
5Midrigan and Philippon (2010) use credit constraint shocks in an island economy to match the distribution of house prices across U.S. counties.
of persistence of easy credit (and vice versa). Learning, therefore, amplifies the conventional
debt-deflation channel that generates sudden stop scenarios. Their calibrated model explains
more than two-thirds of the increase in net households debt and more than a half of the
increase in residential land prices in the U.S. between 1997 and 2006.\footnote{Adam, Kuang and Marcet (2011) reach a similar conclusion in a small open economy model where learning
interacts with low real interest rates.}

In a closed economy setting, Favilukis, Ludvigson and Van Nieuwerburgh (2011) develop
a rich two-sector model with heterogenous households and idiosyncratic risk. Together with
the reduction in transaction costs for housing and business cycle factors, the relaxation of
borrowing constraints accounts for the entire increase in the observed price-to-rent ratio.
Iacoviello and Neri (2010) estimate a DSGE model with housing and find that slow techno-
logical progress in the housing sector explain the long run upward trend in U.S. house prices.
Housing preference and technology shocks account for about 50% of the variance of housing
investment and prices at business cycle frequencies.

The view that reductions in collateral requirements have generated increases in house
prices and consequently external imbalances is not uncontroversial. Glaeser, Gottlieb and
Gyourko (2010) find no evidence that changes in approval rates or LTV levels explain a
large part of the increase in house price in the U.S. between 1996 and 2006. By their own
admission, however, their empirical estimates suffer from an endogeneity problem and the
quality of the data in the empirical analysis may also be questionable.

Several pieces of evidence support the role financial liberalizations in generating the in-
argues that loose monetary policy both in the U.S. and abroad is at the heart of the prob-
lem. Developments in mortgage markets and in the securitization process only contributed
to worsen the problem. Extending the model to include nominal rigidities and a role for
active monetary policy allows for a quantitative evaluation of the relative importance of this
explanation vis-a-vis the role of shocks to the LTV ratio. This extension represents perhaps
the most important original contribution of the paper. Departures from a standard monetary
policy rule in the U.S. during the period 2000-2005 explain low real interest rates and con-
tribute to amplify the boom in house prices as well as to widen the current account deficit.
Monetary policy shocks alone, however, do not account quantitatively for the correlation
between house prices and current account.

Another aspect of monetary policy studied in the paper is the choice of the exchange rate
regime in countries financing the U.S. current account deficit. Foreign exchange rate pegs to
the U.S. dollar are one of the candidates to explain the recent U.S. current account deficits.
This “Bretton Woods II” hypothesis (Dooley, Folkerts-Landau and Garber, 2008) focuses on
the interplay between managed exchanged rate regimes in Asian countries and U.S. current
account deficits. The basic idea is that emerging economies stimulate their exports (their
main source of growth) by keeping the domestic currencies artificially undervalued relative
to fundamentals. A fixed exchange rate regime essentially exports the U.S. monetary policy
to the rest of the world, preventing a real depreciation of the dollar which would contribute
to rebalancing the U.S. current account.

The explanation of house prices booms and current account deficits in this paper completely abstracts from the “saving glut” hypothesis (Bernanke, 2005). Building on their earlier work, Caballero, Fahri and Gourinchas (2008b) argue that global demand for liquid assets generated capital flows from the rest of the world toward the U.S. where asset prices, and hence house prices too, took off. The two thesis (looser credit standards in the U.S. and excess of savings in emerging markets) are not mutually exclusive. Financial flows from China and other emerging economies could complement the role of financial deregulation and expansionary monetary policy in the U.S. in accounting for the boom in house prices and the deterioration of the U.S. current account.

The rest of the paper proceeds as follows. The next section provides some evidence on the relaxation of credit standards induced by the process of financial innovation. The third section presents the general model and develops some intuition using the steady state of a tractable special case. The fourth section discusses the calibration and the basic quantitative experiment. The fifth section focuses on nominal rigidities and addresses the quantitative importance of overly-accommodative U.S. monetary policy and foreign exchange rate pegs. Finally, the sixth section concludes.

2 Evidence on the Relaxation of Collateral Constraints

The key shock that generates a house price boom and a contemporaneous current account deficit in the model below is a reduction in the parameter that measures the LTV requirement. At a broader level, however, lower collateral requirements also capture easier access to housing finance for households previously excluded from credit markets.

2.1 United States

Rajan (2010) argues that easy credit in the U.S. is the consequence of the political response to increasing income inequality.\footnote{See Kumhof and Ranciere (2010) for a formalization of this hypothesis.} According to this view, the growing role of government-sponsored enterprises (primarily Fannie Mae and Freddie Mac) and the expansion of sub-prime lending follow from the need to guarantee affordable housing to low income households who are falling behind. Favara and Imbs (2010) trace the increase in supply of mortgage credit back to the deregulation of cross-state ownership of banks that started with the Interstate Banking and Branching Efficiency Act of 1994. Loosely speaking, interstate branching makes banks more efficient. More narrowly, the mortgage deregulation process creates the possibility to offer credit products across regions with less correlated housing cycles.\footnote{A similar assumption underlay the rating agency valuation models.} While the rationale and the origins of the credit boom are certainly interesting per se and well worth investigating, the analysis below starts from the presumption that a relaxation of borrowing constraints occurred and studies the consequences on asset prices and macroeconomic
quantities.

In early 2000s, subprime lending was clearly the most significant development in mortgage finance. In practice, no legally binding definition of subprime lending exists. The Office of the Comptroller of the Currency, the Board of Governors of the Federal Reserve System, the Federal Deposit Insurance Corporation and the Office of Thrift Supervision (2001) jointly issued a document defining as ‘subprime’ those borrowers who “...display a range of credit risk characteristics that may include one or more of the following:

- Two or more 30-day delinquencies in the last 12 months, or one or more 60-day delinquencies in the last 24 months
- Judgment, foreclosure, repossession, or charge-off in the prior 24 months
- Bankruptcy in the last 5 years
- Relatively high default probability as evidenced by, for example, a credit bureau risk score (FICO) of 660 or below (depending on the product/collateral), or other bureau or proprietary scores with an equivalent default probability likelihood
- Debt service-to-income ratio of 50% or greater, or otherwise limited ability to cover family living expenses after deducting total monthly debt-service requirements from monthly income.”

Pinto (2008a,b), a former Fannie Mae’s chief credit officer, points out that official classifications severely underestimate the extent of subprime lending, which could easily include most “Alternative-to-Agency” (Alt-A) loans and “Home Equity Loans” (HEL). A large portion of Federal Housing Administration (FHA), Veteran Administration (VA) and rural housing loans also conform to subprime standards. Between 2002 and 2007, approximately 83% of FHA loans consisted of LTV ratios higher than 90% and approximately 70% had a FICO of lower than 660.9

The growth in subprime lending can thus be seen not only as a relaxation of credit constraints for households previously able to borrow but also, and perhaps more importantly, as an opportunity to participate for households previously excluded from mortgage markets.

The share of subprime lending in the U.S. mortgage market grew from 0.74% to almost 9% during the 1990s (Nichols, Pennington-Cross and Yezer, 2005). After slowing down in 2001 (7%) and 2002 (1%), subprime origination soared from 8% in 2003 to 20% in 2005 and 2006. In the same period, Alt-A mortgages and HEL also gained popularity. Table 1 shows that by 2006 the higher risk segment accounted for 48% of securitized origination (equivalent to 34% of the dollar volume).10

Table 2 presents more direct evidence on the evolution of LTV ratios, broken down by type of mortgages (Prime, Alt A and Subprime) and rate (Fixed and Adjustable). For the prime segment, the average LTV ratio increased by 10 percentage points between 2002

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9While similar data is not available for the smaller volume VA and rural housing loan programs, original LTV distributions were believed to be similar.

10Mian and Sufi (2010) show that HEL are responsible for a significant fraction of the increase in U.S. household leverage between 2002 and 2006.
and 2006, for both the fixed- and adjustable-rate types. In both categories, the fraction of mortgages with LTV ratios higher than 80% increased from less than 5% in 2002 to about 25% in 2006. Alt A fixed-rate mortgages featured a similar increase, with LTV ratios higher than 80% roughly doubling between 2002 and 2006. Over the same period, the increase in LTV ratios for Alt A adjustable-rate mortgages was smaller (of the order of 5 percentage points) but LTV ratios higher than 80% more than doubled, reaching 55% in 2006. For the subprime segment, the increase in LTV ratio was also of the order of 5 percentage points and subprime LTV ratios higher than 80% soared too, especially for the adjustable-rate type.

In addition, as mortgage products became riskier due to the increasing participation of subprime borrowers and lower LTV ratios, prices did not increase. In fact, the spreads between subprime and prime mortgages of similar maturities uniformly decreased between 2000 and 2005.

The bottom line is that financial innovation, supported by the securitization process, provided greater access to mortgage finance at affordable prices for a broader pool of households, both at the extensive (higher share of subprime mortgages) and intensive (lower LTV requirements) margin.\textsuperscript{11}

\section*{2.2 Rest of the World}

Direct evidence on the relaxation of households’ borrowing constraints for countries other than the U.S. is much more scattered.

The European Mortgage Federation provides some information on housing finance in Europe although data on LTV ratios are generally not available. One notable exception is Iceland, where LTV ratios increased from 65% to 90% in 2003 before going back to 80% in 2006. Interestingly, and perhaps not surprisingly, Iceland experienced a 60% increase in

\begin{table}
\centering
\begin{tabular}{cccccccc}
\hline
\textbf{Year} & \textbf{FHA/VA} & \textbf{Conv/Conf} & \textbf{Jumbo} & \textbf{Subprime} & \textbf{Alt A} & \textbf{HEL} \\
\hline
2001 & 8 & 57 & 20 & 7 & 2 & 5 \\
2002 & 7 & 63 & 21 & 1 & 2 & 6 \\
2003 & 6 & 62 & 16 & 8 & 2 & 6 \\
2004 & 4 & 41 & 17 & 18 & 6 & 12 \\
2005 & 3 & 35 & 18 & 20 & 12 & 12 \\
2006 & 3 & 33 & 16 & 20 & 13 & 14 \\
2007 & 4 & 48 & 14 & 8 & 11 & 15 \\
\hline
\end{tabular}
\end{table}

\textsuperscript{11}Although not explicitly modeled here, the reduction in housing transaction costs provides further evidence in support of the process of liberalization in real estate financing. See Favilukis, Ludvigson and Van Nieuwerburgh (2011) for details.
<table>
<thead>
<tr>
<th>Year</th>
<th>Fixed-Rate CLTV</th>
<th>Fixed-Rate CLTV &gt; 80%</th>
<th>Adjustable-Rate CLTV</th>
<th>Adjustable-Rate CLTV &gt; 80%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002</td>
<td>65.4</td>
<td>3.0</td>
<td>66.5</td>
<td>4.1</td>
</tr>
<tr>
<td>2003</td>
<td>63.8</td>
<td>4.4</td>
<td>68.2</td>
<td>10.1</td>
</tr>
<tr>
<td>Prime</td>
<td>2004</td>
<td>67.4</td>
<td>7.0</td>
<td>73.5</td>
</tr>
<tr>
<td>2005</td>
<td>70.9</td>
<td>13.4</td>
<td>74.1</td>
<td>21.7</td>
</tr>
<tr>
<td>2006</td>
<td>74.5</td>
<td>23.1</td>
<td>75.3</td>
<td>26.2</td>
</tr>
<tr>
<td>2002</td>
<td>74.7</td>
<td>22.0</td>
<td>74.3</td>
<td>20.8</td>
</tr>
<tr>
<td>2003</td>
<td>71.5</td>
<td>21.4</td>
<td>78.0</td>
<td>33.3</td>
</tr>
<tr>
<td>Alt A</td>
<td>2004</td>
<td>75.3</td>
<td>29.5</td>
<td>82.6</td>
</tr>
<tr>
<td>2005</td>
<td>76.2</td>
<td>31.3</td>
<td>83.5</td>
<td>49.6</td>
</tr>
<tr>
<td>2006</td>
<td>79.4</td>
<td>39.6</td>
<td>85.0</td>
<td>55.4</td>
</tr>
<tr>
<td>2002</td>
<td>77.3</td>
<td>38.0</td>
<td>81.2</td>
<td>46.8</td>
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<tr>
<td>2003</td>
<td>78.0</td>
<td>41.7</td>
<td>83.5</td>
<td>55.6</td>
</tr>
<tr>
<td>Subprime</td>
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<td>41.2</td>
<td>85.3</td>
</tr>
<tr>
<td>2005</td>
<td>78.7</td>
<td>44.5</td>
<td>86.6</td>
<td>64.4</td>
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<tr>
<td>2006</td>
<td>78.7</td>
<td>44.6</td>
<td>86.7</td>
<td>64.0</td>
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</tbody>
</table>

Table 2: Evolution of LTV ratios (in %). Source: Abraham, Pavlov and Wachter (2008). CLTV stands for combined (i.e. first and second mortgage) LTV ratio. CTLV > 80% refers to the fraction of combined LTV ratios larger than 80%.

real house prices between 2001 and 2006, together with one of the largest deteriorations of the current account (more than 20%) among Western economies.

The U.K experienced an early wave of mortgage market liberalization at the beginning of the 1980s, when down-payment requirements dropped from 25% to 15% (Ortalo-Magné and Rady, 2004). During that decade, real house prices increased by about 70%.

Outside Europe, Williams (2009) finds evidence that financial liberalizations in the 1980s and 1990s account for about half of the trend increase in real house prices in Australia over the period 1972-2006.

More indirect evidence, however, points in the direction of a large boom in housing finance in several European countries. Table 3 reports residential mortgage debt as fraction of GDP for a selected group of countries over the period 2001-2006. Iceland, the U.S. and the U.K. featured a similar pattern with mortgage debt growing from about 60 to about 80% of GDP or more. Countries like Spain and Ireland started from lower levels (approximately 30%) but roughly doubled their shares. Mortgage finance in Greece accounted for a small fraction of GDP (12%) in 2001 but reached about 30%, close to the level of France, where mortgage finance increased a more moderate 10% over the sample period.

All these examples of significant growth in mortgage debt contrast with the case of Germany, where the share of GDP remained roughly constant just above 50%. The increase in mortgage finance relative to GDP was small also in Japan, from 25% in 1990 to 36% in 2006 (IMF World Economic Outlook, 2008). Finally, while on the uprise from essentially
Table 3: Residential mortgage debt in % of GDP. Source: European Mortgage Federation Hypostat, 2008.

<table>
<thead>
<tr>
<th></th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
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<td>France</td>
<td>21.7</td>
<td>22.6</td>
<td>24.2</td>
<td>26.0</td>
<td>29.3</td>
<td>32.2</td>
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<td>Germany</td>
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<td>53.2</td>
<td>53.5</td>
<td>52.4</td>
<td>51.9</td>
<td>51.3</td>
</tr>
<tr>
<td>Greece</td>
<td>11.8</td>
<td>14.8</td>
<td>17.2</td>
<td>20.2</td>
<td>25.1</td>
<td>29.3</td>
</tr>
<tr>
<td>Iceland</td>
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<td>61.1</td>
<td>66.2</td>
<td>71.0</td>
<td>80.8</td>
<td>75.5</td>
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<tr>
<td>Ireland</td>
<td>32.8</td>
<td>36.3</td>
<td>42.7</td>
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<td>Spain</td>
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<td>45.7</td>
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<tr>
<td>UK</td>
<td>58.9</td>
<td>63.9</td>
<td>69.3</td>
<td>74.1</td>
<td>78.4</td>
<td>83.1</td>
</tr>
<tr>
<td>US</td>
<td>60.5</td>
<td>66.1</td>
<td>71.1</td>
<td>76.1</td>
<td>81.1</td>
<td>84.8</td>
</tr>
</tbody>
</table>

zero in 1998, mortgage debt was still a small 10% GDP in China as of 2004 (Jain-Chandra and Chamon, 2010).

Obviously, the boom in mortgage debt can capture several factors. In the case of Spain, for example, the common explanation for the housing boom relies on factors related to housing demand, such as strong income growth and immigration flows (Cortina, 2009). Spanish authorities explicitly limited LTV ratios for securitized mortgages. However, inflated appraisals may have contributed to circumvent these limits so that higher leverage may have amplified demand shocks (Duca, Muellbauer and Murphy, 2010).

To summarize, credit market liberalizations have greatly stimulated housing finance. The evidence is quite clear for the U.S. and is at least suggestive for several other countries that have experienced contemporaneous house prices booms and current account deficits. Conversely, countries where the process of financial innovation has been less abrupt have experienced a much lower degree of house price appreciation and often current account surpluses. The next section develops a model in which the relaxation of borrowing constraints plays a key role to account for these facts.

3 An Open Economy Model with Borrowing Constraints

Time is discrete and indexed by $t$. The world consists of two countries, Home and Foreign, of equal size. In each country, a continuum of measure one of firms produce a final tradable good using a labor aggregate as the only factor of production. The representative household in each country comprises a continuum of measure one of workers who supply differentiated labor inputs and consume a composite of the tradable goods produced in each country as well as housing services, which are assumed to be proportional to the fixed housing stock.\(^{12}\) The value of housing represents the collateral needed to obtain private credit.

\(^{12}\) Davis and Heathcote (2007) calculate the contribution of the price of structures and land to account for house prices. Land prices explain the bulk of both trend growth and cyclical house price fluctuations between 1975 and 2006.
Goods and labor markets are imperfectly competitive. Goods prices and wages are set on a staggered basis. International financial markets are incomplete. The only asset traded across countries is a one-period nominal risk-free bond denominated in the Home currency. An endogenous collateral constraint limits the maximum amount of borrowing up to a fraction of the expected value of housing.

**Household’s Preferences and Constraints**

This section presents the household objective and constraints from the perspective of the Home country. An asterisk denotes foreign variables when relevant.

The representative household maximizes

\[ U_t = E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[ \frac{X_{t+s}^{1-\sigma}}{1-\sigma} - \frac{1}{1+\nu} \int_0^1 L_{t+s}(i)^{1+\nu} di \right] \right\}. \tag{1} \]

Per-period utility depends positively on the consumption index \( X_t \) and negatively on hours worked by each member of the representative household \( L_t(i) \). The parameter \( \sigma > 0 \) is the coefficient of relative risk aversion while \( \nu > 0 \) is the inverse Frisch elasticity of supply of a specific labor input.

The index \( X_t \) combines consumption of goods \( C_t \) and housing services \( H_t \) with constant elasticity of substitution \( \varepsilon > 0 \)

\[ X_t \equiv \left[ \eta C_t^{\varepsilon^{-1}} + (1 - \eta) H_t^{\varepsilon^{-1}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2} \]

where \( \eta \in (0, 1) \) represents the share of tradable goods in total consumption.

The tradable bundle \( C_t \) combines consumption of goods produced in the Home (\( C_{ht} \)) and Foreign (\( C_{ft} \)) country with constant elasticity of substitution \( \gamma > 0 \)

\[ C_t \equiv \left[ \alpha^{\frac{1}{\gamma}} C_{ht}^{\frac{2-\varepsilon}{\gamma-1}} + (1 - \alpha) \frac{1}{\gamma} C_{ft}^{\frac{2-\varepsilon}{\gamma-1}} \right]^{\frac{\gamma}{\gamma-1}}, \tag{3} \]

where \( \alpha \in [0.5, 1) \) is the share of domestic tradable goods.\(^{13}\)

The budget constraint for the representative household is

\[ P_{ht}C_{ht} + P_{ft}C_{ft} + Q_t H_t - B_t \leq \int_0^1 W_t(i)L_t(i)di + P_t + Q_t H_{t-1} + T_t - (1 + i_{t-1})B_{t-1}, \tag{4} \]

where \( P_{hf} \) is the Home price of good \( j = \{h, f\} \), \( Q_t \) is the price of housing, \( W_t(i) \) is the nominal wage for the specific labor input supplied by the \( i^{th} \) household member, \( P_t \) are profits from ownership of intermediate goods producers, \( T_t \) are lump-sum transfers and \( i_t \) is the net nominal interest rate on an internationally-traded one-period risk-free debt instrument \( B_t \), denominated in the Home currency.

\(^{13}\)If \( \alpha > 0.5 \), preferences for tradable goods exhibit home bias. The Foreign tradable bundle places a weight \( \alpha \) on consumption of Foreign tradable goods.
Household’s members perfectly pool their consumption risk within each country. The representative household can smooth consumption intertemporally by borrowing and lending in international financial markets, subject to a collateral constraint that depends on the expected value of housing

\[(1 + \nu_t)B_t \leq \Theta_t\mathbb{E}_t(Q_{t+1}H_t),\]  

where the borrowing constraint parameter \(\Theta_t\) is an exogenous shock with mean \(\Theta\) and support over the unit interval. The idea behind the borrowing constraint is that the Foreign household can only recover a fraction \(\Theta_t\) of the collateral in case of default, possibly due to various costs associated with the bankruptcy process.\(^{14}\)

**Firms and Production**

Final goods producing firms pack intermediate goods according to a constant returns technology with elasticity of substitution \(\phi_p > 1\)

\[Y_{ht} \equiv \left[ \int_0^1 Y_t(h) \frac{\phi_p}{\phi_p - 1} dh \right]^{\frac{\phi_p}{\phi_p - 1}}.\]  

All intermediate goods producing firms have access to the same constant return technology which uses a labor aggregate \(L_t\) as the only factor of production

\[Y_t(h) = AL_t,\]  

where \(A\) is a constant productivity factor.

Perfectly competitive labor agencies combine differentiated labor inputs according to a constant returns technology with elasticity of substitution \(\phi_w > 1\)

\[L_t \equiv \left[ \int_0^1 L_t(i) \frac{\phi_w}{\phi_w - 1} di \right]^{\frac{\phi_w}{\phi_w - 1}}.\]  

Finally, the stock of housing (land) is assumed to be fixed

\[H_t = H.\]

**Monetary Policy**

The central bank sets the short-term nominal interest rate in response to deviations of inflation and output from their targets

\[(1 + i_t) = (1 + i_{t-1})^\phi \left[ (1 + i) (\frac{\Pi_t}{\Pi_t})^{\phi\gamma} (\frac{Y_{ht}}{Y_{ht}})^{\phi\gamma} \right]^{1-\rho_i} e^{\varepsilon_it},\]

\(^{14}\)See, for instance, Kiyotaki and Moore (1997) or Kocherlakota (2000).
where $\rho_i$ is the degree of interest rate smoothing, $\Pi_t \equiv P_t/P_{t-1}$ is the inflation rate of goods prices $P_t$, $\tilde{\Pi}$, and $\tilde{Y}_ht$ are the targets for inflation and output respectively and $\varepsilon_{it}$ is an i.i.d. normal innovation to the interest rate rule with mean zero and standard deviation $\sigma_i$.

### 3.1 Equilibrium and Steady State

An imperfectly competitive equilibrium for the world economy is a sequence of prices and quantities such that:

1. The representative household in each country maximizes utility subject to the budget constraint and the collateral constraint, taking prices as given. Household’s members set wages on a staggered basis, taking labor demand for their specific labor input as given.

2. Intermediate goods producing firms set prices on a staggered basis to maximize the present discounted value of profits, taking the demand for their variety as given. Final goods producing firms minimize costs, taking prices as given.

3. The housing market clears in each country. Goods and financial markets clear internationally.

Appendix A describes the details of the optimization problem of households and firms and characterizes the equilibrium under flexible (A.1) and sticky prices (A.2). In the case of nominal rigidities, workers set wages on a staggered basis taking as given the demand for their own labor input, where $\zeta_w$ is the probability of not being able to adjust the wage in each period. Similarly, intermediate goods producing firms set prices on a staggered basis taking as given the demand for their specific variety, where $\zeta_p$ is the probability of not being able to adjust the price in each period. In steady state, monetary policy is neutral. The model admits two types of steady state.

If the borrowing constraint is not binding in either country, a symmetric steady state exists in which all relative prices (those of tradable goods, the terms of trade and the real exchange rate) are equal to one and foreign debt is zero. In this steady state, each country is in autarky and the level of productivity pins down output (and hence consumption). House prices are equal to the present discounted value of the marginal utility of housing services while the real return is equal to the inverse of the discount factor. The unattractive feature of a perfectly symmetric steady state is that, up to a linear approximation, borrowing constraints are irrelevant for house prices dynamics. Real house prices $Q_t \equiv Q_t/P_t$ obey the following forward looking relation

$$Q_t = \left(1 - \frac{\eta}{\eta} \right) \left( \frac{H}{C_t} \right)^{-\frac{1}{2}} + \beta E_t \left[ \left( \frac{X_{t+1}}{X_t} \right)^{\frac{1}{2}} - \sigma \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{2}} Q_{t+1} \right] + \Xi_t \Theta_t E_t \left( Q_{t+1} \right).$$

The first two terms of the right-hand side of (11) are standard. Real house prices are equal to the the current marginal utility of housing services in units of marginal utility of consumption plus the discounted expected value of future house prices. The third term
measures the contribution of the shadow value of the borrowing constraint to current house prices. If the borrowing constraint is not binding in steady state, the multiplier is equal to zero ($\Xi = 0$). Therefore, up to a first order approximation, changes in the LTV ratio $\Theta$ would have no effects on real house prices.

An asymmetric steady state (see appendix B) resuscitates a role for borrowing constraints in affecting house prices dynamics. Even with identical preferences and technologies, simply imposing that one country’s borrowing constraint is binding is enough to generate an asymmetric steady state. However, assuming a different degree of patience across countries provides a more fundamental reason why one country’s steady state may be binding. In what follows, the Foreign country representative household is assumed to be relatively more patient ($\beta^* > \beta$). This assumption implies that the Foreign country is a net saver in international financial markets.

Interestingly, the presence of a binding borrowing constraint also solves the problem of indeterminacy of the net foreign asset position typical of open economy models with incomplete international financial markets (Schmitt-Grohé and Uribe, 2003). The borrowing constraint at equality pins down the steady state level of net foreign assets as a function of house prices and the real interest rate.

The steady state of a simplified version of the model helps develop the intuition for the main result. Suppose for a moment the Home country is a small open economy which takes the world interest rate as given. Further, abstract from nominal rigidities and assume the Home country receives a fixed endowment of a single consumption good. Finally, preferences are log-separable in consumption and housing. In steady state, the real value of the housing stock in this economy is

$$Q = \frac{(1-\eta)C}{\eta(1-\beta-\Xi\Theta)}. \quad (12)$$

The shadow value of the borrowing constraint introduces a wedge in the consumption Euler equation

$$\Xi = \frac{1-\beta R}{R}. \quad (13)$$

The resource constraint for the simplified economy is

$$C = Y - (R-1)B. \quad (14)$$

Finally, the borrowing constraint at equality requires that debt is equal to a fraction of the discounted real value of the housing stock

$$B = \frac{\Theta Q}{R}. \quad (15)$$

\[15\text{See Boz and Mendoza (2010) for a quantitative analysis of the dynamics of this economy when agents must learn the true persistence of the LTV ratio parameter, which follows a two-state Markov process.}\]
Holding consumption constant, a permanent increase in the LTV ratio \( \Theta \) increases the real value of the housing stock (equation 12). At the same time, a higher LTV ratio increases foreign debt (equation 15). The increase of real house prices amplifies this mechanism. These two effects (higher house prices and higher debt) are mitigated by the drop in consumption due to the fact that higher foreign debt must be paid back by running trade surpluses. If the positive shock to the LTV ratio is persistent but not permanent, consumption initially increases too while the mitigating effect kicks in only at a later stage. Higher debt allows agents to spend more resources both on housing and goods consumption.

The assumption that the borrowing constraint holds with equality, maintained throughout the paper, requires its shadow value to be positive. This condition is satisfied only if \( 1 - \beta R > 0 \). For the case of a small open economy, this requirement corresponds to a “low” real interest rate \( (R < 1/\beta) \). The analysis of monetary policy shocks below explores one rationale for low rates during the period 2000-2005 in a world in which the real interest rate is endogenous.

4 Quantitative Results

This section discusses the calibration of the parameters and presents the central quantitative experiment of the paper – a relaxation of the borrowing constraint parameter \( \Theta \).

4.1 Calibration

The Foreign discount factor pins down the steady state real return on the internationally traded asset. A target of 4\% for the annualized real return implies \( \beta^* = 0.99 \). The Home country is a net borrower in international financial markets because of a lower discount factor \( (\beta = 0.98) \).\(^{16}\)

The coefficient of risk aversion \( \sigma \) and the inverse Frisch elasticity of labor supply are both set equal to 2, within the range of common practice in macroeconomics (see, for instance, Hall, 2010). Also standard are the values for the elasticity of substitution among goods and labor varieties \( (\phi_p = \phi_w = 11) \), calibrated to match steady state a 10\% markup in both the goods and labor market. The price and wage stickiness parameters are chosen as to match an average duration of price and wage contracts of four quarters \( (\zeta_p = \zeta_w = 0.75) \).

The parameters of the goods consumption basket are fairly standard in the international macroeconomics literature (see, for instance, Obstfeld and Rogoff, 2007). The domestic share of tradable consumption \( \alpha \) is set to 0.7 (home bias). The elasticity of substitution between Home and Foreign tradable goods \( \gamma \) equals 2.

The intratemporal elasticity of substitution between goods consumption and housing services is the most important parameter for the dynamic response of house prices to borrowing constraint shocks. Perhaps not surprisingly, the calibration of this parameter is also highly

\(^{16}\)These values coincide with the assumed discount factors of savers and borrowers in the closed economy model of Monacelli (2009).
controversial. Davis and Ortalo-Magné (2010) provide evidence from the Decennial Census of Housing indicating that expenditure shares on housing are constant over time and across US metropolitan areas. This finding is consistent with a Cobb-Douglas specification of the aggregator $X_t$ (i.e. $\varepsilon = 1$).\(^{17}\) Piazzesi, Schneider and Tuzel (2007) argue instead that a Cobb-Douglas formulation for $X_t$ may be too restrictive. Using annual U.S. data since 1929, these authors show that the non-housing share of total consumption is not constant, although its volatility is fairly low. Their calibration focuses on values of $\varepsilon$ slightly bigger than one, consistent with the estimates in Ogaki and Reinhart (1998) that lie in the 95% confidence interval $[1.04, 1.43]$. At the opposite end of the spectrum, Lustig and Van Nieuwerburgh (2004) need a low value of the intertemporal elasticity to match the volatility of U.S. rental prices in an asset pricing model with housing collateral. These authors choose a benchmark is $\varepsilon = 0.05$ and explore values up to 0.75. Given the disagreement in the literature, this paper adopts a benchmark value for the elasticity of intratemporal substitution between consumption and housing equal to one and provides several robustness checks in the interval $(0, 2)$.

Conditional on the elasticity of substitution, the parameter $\eta$ is chosen to match a consumption share of total expenditure of about 80%, which is in line with the average for the U.S. from 1929 to 2001 (Piazzesi, Schneider and Tuzel, 2007). The steady state consumption share of total expenditure in the Home country is\(^{18}\)

$$
\left(1 + \frac{QH}{C}\right)^{-1} = \left[1 + \left(\frac{1-\eta}{\eta}\right) \left(H/C\right)^{1-\frac{1}{\gamma}} \left(1-\beta - \Xi^{*}\right)\right]^{-1}
$$

In the Cobb-Douglas case, the mapping between $\eta$ and the consumption share of total expenditure is independent of the stock of housing and the steady state level of consumption (except for the small indirect effect via the steady state Lagrange multiplier $\Theta$). The relative stock of housing is adjusted so that in steady state the level of house prices in the two countries is the same.

Another important parameter to calibrate is the steady state value of $\Theta$. Based on the evidence in section 2, a LTV ratio ratio of 70% seems to characterize quite appropriately the period before credit market deregulation. The stochastic process for the borrowing constraint is assumed to follow an AR(1) process with persistence close to one (0.9999) and i.i.d. innovations $\sim \mathcal{N}(0,1)$.\(^{19}\)

For simplicity, the steady state value of the terms of trade (and hence of the real exchange rate and the relative prices of Home and Foreign tradable goods) are normalized to one by appropriately picking the steady state relative level of productivity.

\footnote{The Cobb-Douglas specification is the baseline case also in Fernandez-Villaverde and Kruger (2001), who study life-cycle consumption and portfolio decisions in a quantitative general equilibrium model with borrowing constraints.}

\footnote{A similar expression holds for the Foreign country, with the difference that $\Xi^{*} = 0$.}

\footnote{The reason for a very high persistence parameter is to capture, in a reduced form, the “regime switch” effect emphasized in Boz and Mendoza (2010).}
Finally, the targets and parameters of the monetary policy rule take fairly conventional values (e.g. Galí and Gertler, 2007). The inflation target is normalized to zero and the target for output is its steady state value. The interest rate smoothing parameter $\rho_i$ is set equal to 0.7. The response to inflation $\psi_\pi$ equals 1.5 while the response to output is mute.

4.2 The Effects of Relaxing the Borrowing Constraint

The model is approximated up to the first order about the asymmetric steady state described in section 3.1. Appendix C lists the system of log-linear equations that characterize the equilibrium.

The main experiment consists of shocking the collateral constraint parameter $\Theta$. In particular, financial innovation corresponds to a higher LTV ratio. Households can borrow a higher fraction of the expected value of their house. To illustrate the basic mechanism, this section focuses on the dynamic response of the endogenous variables to a 1% positive innovation to $\Theta$ (figure 3).

A higher value of $\Theta$ corresponds to a higher LTV ratio in the Home country. In response to the shock, households in the Home country can now borrow more. By construction, borrowing occurs in international financial markets only. Therefore, foreign debt increases (top-left) and the current account turns negative (middle-left). At the same time, higher leverage translates into higher demand for consumption of both goods (middle-right) and housing. Because the stock of housing is fixed, house prices absorb the adjustment in full (top-right). As resources flow into the Home country and the current account turns negative, the real exchange rate appreciates (bottom-left). External imbalances are partly mitigated by the increase in the real interest rate (bottom-right).

The model captures well the negative correlation between house prices and the current account balance. Interestingly, the impulse responses are also broadly consistent with two other features of the data before the recent crisis. First, the boom in consumption. Second, the appreciation of the real dollar against a basket of currency of U.S. trading partner.

The one counterfactual feature of the impulse response functions in figure 3 is the behavior of the real interest rate. In the model, the real interest rate increases because the shock to the LTV ratio relaxes the collateral constraints, hence stimulating aggregate demand. Conversely, several empirical measures point to a secular decline in real interest rates since the mid 1980s.

In the literature on global imbalances (Bernanke, 2005; Caballero, Fahri and Gourinchas, 2008a), the persistent drop in the real interest rate is a consequence of the “saving glut” that originated in Asian economies after the financial crisis of the late 1990s. Recent papers that investigate the role of lower collateral requirements for the negative correlation between house prices and current account have picked upon this idea. Favilukis, Ludvigson and Van Nieuwerburgh (2011) explicitly combine shocks that generate this excess of savings with a

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20 The near unit root of the borrowing constraint process make the shock look permanent.
21 Both the “broad” and “major currency” real exchange rate index exhibit similar patterns.
Figure 3: Impulse response function (selected variables) to a 1% increase in the LTV parameter $\Theta$. 
relaxation of borrowing constraints and lower transaction costs. Boz and Mendoza (2011) and Adam, Kuang and Marcet (2011) justify the use of a small open economy model based on the idea that saving glut shocks in the rest of the world balance the upward pressure on the real interest rate deriving from financial deregulation. The next section explores a different (and potentially complementary) rationale for low real interests at the world level.

5 The Role of Monetary Policy in the U.S. and Abroad

This section investigates the role of monetary policy in explaining the correlation between house prices and the current account while keeping world real rates low. The first question is whether monetary policy shocks have contributed to stimulate demand beyond what would be normally considered appropriate according to a standard interest rate rule. A second question is whether foreign exchange rate pegs have exacerbated the magnitude of the adjustment due to domestic (U.S. monetary policy and regulatory) factors. In both cases, if inflation expectations are well-anchored, low nominal rates translate in low real rates.

5.1 Overly-Accomodative U.S. Monetary Policy

Figure 4 presents the departures of the effective Federal Funds Rate (FFR) from two versions of a standard interest rate rule (Taylor, 1993), similar to the linearized version of equation

![Figure 4: Departures of effective Federal Funds Rate (blue line) from interest rate predicted by baseline rule rule (red line) and from rule using expected core PCE inflation (green line).](image-url)
where \( i_t \) is the effective FFR, \( \pi_t \) is the four-quarter moving average of core CPI inflation and \( y_t \) is the deviation of real GDP from potential output as measured by the CBO. In the version of the rule labeled ‘no smoothing’, the parameter \( \rho_i \) is set equal to zero. In the version labeled ‘smoothing’, \( \rho_i = 0.7 \) as in the baseline calibration. The figure captures the essence of the criticism in Taylor (2008). Between 2001 and 2005, U.S. monetary policy was excessively accommodating compared to the prescriptions of an interest rate rule that worked well as a characterization of monetary policy in the previous 15 years. According to this view, overly-accommodative monetary policy is a primary suspect for the house price boom.

The U.S. is not the only country with significant deviations from a standard monetary policy rule. Taylor (2008) presents evidence on the correlation between housing investment and deviations from a Taylor rule among European countries. Countries that have experienced the largest deviations have also the highest changes in housing investment as percentage of GDP. These countries are also the very same with a high correlation between house price and current account changes during the period 2001-2006.

One limitation of this argument is that the correlation between the departures from a standard Taylor rule and house price appreciation is much weaker (Bernanke, 2010). While this evidence may question the importance of overly-accomodative monetary policy in causing the housing boom, low interest rates may still play an important role as an amplification mechanism (Adam, Kuang and Marcet, 2011). Furthermore, combining a relaxation of borrowing constraints with an overly-accomodative monetary policy stance allows for a quantitative evaluation of the relative importance of these two potential explanations for the house prices boom and current account deficit.

Figure 5 presents a simulation of the model that combines a 35% increase in the borrowing constraint (at a rate of 7% per year, or 1.75% per quarter) with the monetary policy shocks calculated as departures of the effective FFR from the prescriptions of (16). The sequence of shocks to the borrowing constraint is consistent with an increase of the LTV ratio from 70% to 95% between the beginning of 2001 and the end of 2005. This value is certainly in line with the various evidence presented in section 2. The series of monetary policy shocks is calculated using (16) with \( \rho_i = 0.7 \) as this version of the interest rate rule provides the best fit of the observed FFR.

The simulation accounts for 66% of the observed increase in house prices, as measured by the real Case-Shiller price index, and for 25% of the deterioration of the U.S. current account between 2001q1 and 2005q4. The negative correlation between house prices and current account is quite evident. At the same time, consumption experiences an initial

\(^{22}\)Deviations from the Taylor rule differ among Euro countries because inflation and output gaps differ across countries.
Figure 5: Baseline simulation combining a 35% increase in the LTV ratio (at a rate of 1.75% per quarter) with deviations of FFR from 16 over the period 2001q1-2005q4 (blue continuous line) versus LTV ratio shocks only (red dashed line).
boom before retrenching given the need to repay the outstanding net foreign debt. The real interest rate turns negative because of the negative innovations to the FFR. In the absence of monetary policy shocks, the real interest rate increases (as discussed in the previous section), which damps the response of consumption.

Monetary policy shocks only are also able to generate and increase in house prices and a deterioration of the current account as observed in the data. The magnitudes, however, are substantially lower than in the previous case. Figure 5 shows that in this case house prices increase by 5% at their peak after five quarters and start decreasing thereafter. The current account is negative also for a short lapse of time (again about five quarters) and the maximum current account deficit is about half of the baseline simulation.

Another limitation of considering only monetary policy shocks as the main driver of the adjustment process is the unequivocal depreciation of the real exchange rate. This feature characterizes, except for the first couple of periods, also the the baseline simulation. The intuition is simple. As monetary policy becomes excessively accommodative in the U.S., the real value of the dollar tends to depreciate due to the uncovered exchange rate parity condition. To the extent that in the data such a relation is systematically violated, this model confronts the same issue as the vast majority of open economy macroeconomic frameworks. This problem will not be present in the next section where the Foreign country is assumed to peg its exchange rate to the Home currency.

5.2 Foreign Exchange Rate Pegs

One important feature of the international monetary system since the early 2000s is the fact that many emerging economies, mostly in East Asia (and most notably China) have pegged their exchange rate to the U.S. dollar. Dooley, Folkerts-Landau and Garber (2008) have called this extensive peg arrangement “Bretton Woods II”. In their view, pegged exchange rates among fast-growing, export-oriented economies are responsible for the large U.S. current account deficits. The intuition is that pegged exchange rates have kept foreign currencies significantly below their true market value, hence stimulating exports and growth abroad. From the perspective of the emerging economies, the peg made sense. The consequence for the U.S. however has been a series of widening current account deficits. For the purpose of this paper, the question is how much foreign exchange rate peg have contributed to exacerbate the boom in house prices, driven by financial liberalization.

Figure 7 repeats the simulation in the previous section assuming that the monetary authority of country F follows an exchange rate peg \( \xi_t = \xi \). In this case, the dynamics of house prices and the current account are similar to the baseline case. The real interest rate, however, convincingly drops by about 2% at the through. Similarly, the real exchange rate significantly depreciates, roughly by 5%. The appreciations of the real exchange rate, however, limits the increase in consumption both in terms of magnitudes and duration (about one year).

All in all, assuming the Foreign country pegs its nominal exchange rate to the Home
Figure 6: Baseline simulation (blue continuous line) versus deviations of Fed Funds Rate Taylor rule only (red dashed line).
Figure 7: Baseline simulation with flexible exchange rate (blue continuous line) versus Foreign nominal exchange rate peg (red continuous line).
currency augments the realism of the quantitative experiment.

5.3 Alternative Monetary Policy Formulations

6 Robustness

6.1 Goods and Housing: Complements or Substitutes?

6.2 Other Parameters

Elasticity of substitution between Home and Foreign goods

Home bias parameter

6.3 The Role of Nominal Rigidities

Absent nominal rigidities, higher volatility in prices and wages reduces the need for changes in real quantities. Contrary to the data, consumption essentially does not move, while house prices increase about a half than in the baseline case. Nevertheless, the shock has roughly the same impact on foreign debt. Therefore, the Home country experiences a similar current account deficit.

7 Conclusions

A relaxation of borrowing constraints can explain a significant fraction of the increase in U.S. house prices and, at the same, give rise to external imbalances. Departures of the nominal interest rate from a conventional monetary policy rule amplify the effects of the adjustment by lowering the real interest rate. This effect is magnified if the Foreign country pegs its nominal exchange rate to the Home currency.

Except for the role of Foreign exchange rate pegs, this explanation of house prices booms and current account deficits has its origins in U.S. policies. This approach contrasts with recent explanations based on the idea of a Foreign saving glut. The two theories are not mutually exclusive. A simple, and perhaps simplistic, interpretation of a Foreign saving glut as a preference shock (more patient Foreign households) would have the effect of further depressing the real interest rate, thus strengthening the mechanism at play in this paper. A more structural interpretation of the Foreign saving glut phenomenon would require explicit modeling of the securitization process that generates safe assets in the U.S. but not elsewhere. Nevertheless, even in this case, the effects of financial flows from the rest of world would likely amplify the consequences of looser borrowing constraints and monetary policies in the Home country.
A Optimal Price Conditions for Households and Firms

This section presents the optimality conditions for households and firms of the Home country. Given the assumption of a representative household in each country, borrowing and lending occurs in equilibrium only at the international level. In what follows, the borrowing constraint is always assumed to bind for the Home economy and never for the Foreign economy.

Cost Minimization

Expenditure minimization determines the allocation of total consumption between Home and Foreign tradable goods as a function of their relative prices and total demand

\[ C_{ht} = \alpha \left( \frac{P_{ht}}{P_t} \right)^{-\gamma} C_t \quad \text{and} \quad C_{ft} = (1 - \alpha) \left( \frac{P_{ft}}{P_t} \right)^{-\gamma} C_t, \]

where the price of the aggregate consumption bundle \( P_t \) is

\[ P_t = \left[ \alpha P_{ht}^{1-\gamma} + (1 - \alpha) P_{ft}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}. \]

Expenditure minimization also implies

\[ P_{ht}C_{ht} + P_{ft}C_{ft} = P_tC_t. \]

Final goods producers are perfectly competitive. Their cost minimization problem generates the demand for intermediate goods according to

\[ Y_t(h) = \left[ \frac{P_t(h)}{P_{ht}} \right]^{-\phi_p} Y_{ht}, \]

where the price index of the tradable bundle \( P_{ht} \) is

\[ P_{ht} = \left[ \int_0^1 P_t(h)^{1-\phi_p} dh \right]^{\frac{1}{1-\phi_p}}. \]

Intermediate goods producing firms minimize labor costs and demand the generic labor input \( L_t(i) \) according to

\[ L_t(i) = \left[ \frac{W_t(i)}{W_t} \right]^{-\phi_w} L_t, \]

where \( W_t \) is the aggregate wage index

\[ W_t = \left[ \int_0^1 W_t(i)^{1-\phi_w} di \right]^{\frac{1}{1-\phi_w}}. \]
Utility Maximization

The representative household maximizes utility \((1)\) subject to the budget constraint \((4)\) and the borrowing constraint \((5)\). Let \(\lambda_t\) and \(\lambda_t \Xi_t\) be the lagrange multipliers on the two constraints. Workers operate in monopolistic competition taking the demand for their generic labor input as given. Therefore, equation \((22)\) becomes an additional constraint for the household problem.

The first order condition for consumption is

\[
\eta e^{\varsigma t} X_t^{1/\sigma} C_t^{1/2} - \lambda_t P_t = 0. \tag{24}
\]

The first order condition for housing services is

\[
(1 - \eta) e^{\varsigma t} X_t^{1/\sigma} H_t^{1/2} - \lambda_t Q_t + \beta E_t(\lambda_{t+1} Q_{t+1}) + \lambda_t \Xi_t \Theta_t E_t(Q_{t+1}) = 0. \tag{25}
\]

The first order condition for debt is

\[
\lambda_t - \beta(1 + i_t) E_t(\lambda_{t+1}) - \lambda_t \Xi_t(1 + i_t) = 0. \tag{26}
\]

Wages are set on a staggered basis (Calvo, 1983). The probability of not being able to adjust the wage is \(\zeta_w\). The optimality condition for a worker who is able to adjust the wage at time \(t\) is

\[
\mathbb{E}_t \left\{ \sum_{s=0}^{\infty} (\beta \zeta_w)^s L_{t+s}(i) \left[ \lambda_{t+s} W_t(i) - \frac{\phi_w}{\phi_w - 1} e^{\varsigma t+s} L_{t+s}(i) \nu_t \right] \right\} = 0. \tag{27}
\]

Using the labor demand equation \((48)\) and the expression for the marginal utility of consumption \((24)\) into the previous expression yields

\[
\mathbb{E}_t \left\{ \sum_{s=0}^{\infty} (\beta \zeta_w)^s \left[ \eta X_t^{1/\sigma} C_t^{1/2} \frac{W_t(i)}{W_{t+s}} - \frac{\phi_w}{\phi_w - 1} e^{\varsigma t+s} \frac{W_t(i) L_{t+s}}{P_{t+s}} - \frac{\phi_w}{\phi_w - 1} \left( \frac{W_t(i)}{W_{t+s}} \right)^{\phi_w} L_{t+s}^{\nu} \right] \right\} = 0. \tag{28}
\]

Equation \((28)\) can be rearranged as to express the relative wage of type \(i\) as a function of the ratio between the present discounted value of the marginal disutility of labor and the present discounted value of the real wage in units of marginal utility of consumption

\[
\left[ \frac{W_t(i)}{W_t} \right]^{1 - \phi_w(1 - \nu)} = \frac{K_{wt}}{F_{wt}}. \tag{29}
\]

The terms on the right-hand side of the last expression can be written recursively as

\[
K_{wt} = \frac{\phi_w}{\phi_w - 1} e^{\varsigma t} L_t^{\nu} + \beta \zeta_w \mathbb{E}_t \left[ (\Pi_{wt+1})^{\phi_w} K_{wt+1} \right]. \tag{30}
\]
and
\[ F_{\nu t} = \eta e^{\varsigma X_t^{1-\sigma}} P_t^{1-\frac{1}{2}} W_t L_t + \beta \varsigma_w E_t \left[ (\Pi_{\nu t+1})^{\phi_{\nu t}-1} F_{\nu t+1} \right], \] (31)

where \( \Pi_{\nu t} \equiv W_t/W_{t-1} \) represents wage inflation. Expressions (29)-(31) show that the optimal choice of household members who optimally reset their wage in any given period is a function of aggregate variables only. Finally, the aggregate wage index (23) can be rewritten as to link the relative wage of type \( i \) to wage inflation
\[ \varsigma_w (\Pi_{\nu t})^{\phi_{\nu t}-1} + (1 - \varsigma_w) \left[ \frac{W_t(i)}{W_t} \right]^{1-\phi_{\nu t}} = 1. \] (32)

Using the first order condition for consumption (24), the first order conditions for housing services (25) becomes
\[ Q_t = \tilde{\eta} \left( \frac{H_t}{C_t} \right)^{-\frac{\varsigma}{\alpha}} + \beta E_t \left[ e^{\varsigma t+1-\varsigma} \left( \frac{X_{t+1}}{X_t} \right)^{\frac{1}{2}-\sigma} \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{2}} Q_{t+1} \right] + \Xi_t \Theta_t E_t \left( \Pi_{t+1} Q_{t+1} \right), \] (33)

where \( Q_t \equiv Q_t/P_t \) defines real house prices and \( \tilde{\eta} \equiv \eta^{-1} - 1 \). Equation (33) consists of a standard part, according to which real house prices are equal to the marginal utility of housing services in units of marginal utility of consumption plus expected discounted future house prices, and a second part which measures the contribution of the borrowing constraint via the shadow price \( \Xi_t \).

Similarly, using again the first order condition for consumption (24), the first order condition for debt (26) becomes
\[ (1 + i_t) \Xi_t = 1 - \beta (1 + i_t) E_t \left[ e^{\varsigma t+1-\varsigma} \left( \frac{X_{t+1}}{X_t} \right)^{\frac{1}{2}-\sigma} \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{2}} \frac{1}{\Pi_{t+1}} \right]. \] (34)

Equation (34) shows that the shadow price \( \Xi_t \) represents a wedge in the standard consumption Euler equation due to the borrowing constraint.

No Arbitrage

The representative household in the Foreign country solves the same maximization problem with one substantial difference. While the Foreign representative household can purchase Home debt, Foreign debt only circulates domestically. No arbitrage then implies the consumption-based uncovered interest parity condition
\[ E_t \left\{ e^{\varsigma t+1-\varsigma} \left( \frac{X_{t+1}^*}{X_t^*} \right)^{\frac{1}{2}-\sigma} \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\frac{1}{2}} \frac{1}{\Pi_{t+1}^*} \left[ (1 + i_t^*) - (1 + i_t) \frac{E_t}{E_{t+1}} \right] \right\} = 0. \] (35)

Because of the representative household assumption, Foreign debt is in zero net supply in equilibrium. Additionally, the Foreign country is assumed to be a net saver in international
financial markets so that the Foreign borrowing constraint to never bind \((\Xi_t^* = 0, \forall t)\).

**Profit Maximization**

Intermediate goods producers set prices on a staggered basis taking the demand for their product \((20)\) as given and subject to the technology constraint \((7)\). The probability of not being able to adjust the price is \(\zeta_p\). The optimality condition for a firm able to adjust its price at time \(t\) is\(^{23}\)

\[
E_t \left\{ \sum_{s=0}^{\infty} (\beta \zeta_p)^s \lambda_{t+s} Y_{t+s}(h) \left[ P_t(h) - \left( \frac{\phi_p}{\phi_p - 1} \right) \frac{W_{t+s}}{A} \right] \right\} = 0. \tag{36}
\]

As for the wage, using the demand for intermediate goods \((20)\) and the expression for the marginal utility of consumption \((24)\) into the previous expression yields

\[
E_t \left\{ \sum_{s=0}^{\infty} (\beta \zeta_p)^s e^{s \iota} X_{t+s}^{\frac{1}{2}} C_{t+s}^{-\frac{1}{2}} \left[ \frac{P_t(h)}{P_{ht+s}} \right]^{-\phi_p} Y_{ht+s} \left[ P_t(h) - \left( \frac{\phi_p}{\phi_p - 1} \right) \frac{W_{t+s}}{A} \right] \right\} = 0. \tag{37}
\]

In this case, equation \((37)\) can be rearranged as to express the relative price of variety \(h\) as a function of the ratio between the present discounted value of the real marginal cost and the present discounted value of the real marginal revenues

\[
\left[ \frac{P_t(h)}{P_{ht}} \right] = \frac{K_{pt}}{F_{pt}}. \tag{38}
\]

The terms on the right-hand side of the last expression can be written recursively as

\[
K_{pt} = \frac{\phi_p}{\phi_p - 1} e^{s \iota} X_{t}^{\frac{1}{2} - \sigma} C_{t}^{-\frac{1}{2}} W_t Y_{ht} \frac{P_{ht}}{AP_t} + \beta \zeta_p E_t \left[ (\Pi_{ht+1})^{\phi_p} K_{pt+1} \right] \tag{39}
\]

and

\[
F_{pt} = e^{s \iota} X_{t}^{\frac{1}{2} - \sigma} C_{t}^{-\frac{1}{2}} \frac{P_{ht} Y_{ht}}{P_t} + \beta \zeta_p E_t \left[ (\Pi_{ht+1})^{\phi_p-1} F_{pt+1} \right]. \tag{40}
\]

Expressions \((38)-(40)\) show that the optimal choice of firms who reset their price in any given period is a function of aggregate variables only. Finally, the aggregate price index \((21)\) can be rewritten as to link the relative price of variety \(h\) to price inflation

\[
\zeta_p (\Pi_{ht})^{\phi_p-1} + (1 - \zeta_p) \left[ \frac{P_t(h)}{P_{ht}} \right]^{1-\phi_p} = 1, \tag{41}
\]

where \(\Pi_{ht} \equiv P_{ht}/P_{ht-1}\) represents domestic inflation.

**Market Clearing**

\(^{23}\)The representative household in each country owns the domestic firms. Therefore, the lagrange multiplier on the budget constraint is the appropriate measure to convert the value of future profits in units of current consumption.
The law of one price holds for tradable goods

\[ P_{ht} = \mathcal{E}_t P^*_t, \]  

where \( \mathcal{E}_t \) is nominal exchange rate (the price of the Foreign currency in terms of the Home currency). Home bias, however, implies that purchasing power parity does not hold (i.e. \( P_t \neq \mathcal{E}_t P^*_t \)).

Final goods producing firms sell their products in the Home and Foreign market. Market clearing requires

\[ Y_{ht} = C_{ht} + C^*_{ht} = \alpha \left( \frac{P_{ht}}{P_t} \right)^{-\gamma} C_t + (1 - \alpha) \left( \frac{P^*_ht}{P^*_t} \right)^{-\gamma} C^*_t, \]  

where the second part of (43) uses (17) and its Foreign country counterpart. The Home tradable goods market equilibrium (43) can alternatively be expressed in terms of the Home relative price and the real exchange rate \( S_t \equiv \mathcal{E}_t P^*_t / P_t \)

\[ Y_{ht} = \left( \frac{P_{ht}}{P_t} \right)^{-\gamma} \left[ \alpha C_t + (1 - \alpha) S^*_t C^*_t \right]. \]  

As mentioned, the housing stock is fixed in both countries

\[ H_t = H \quad \text{and} \quad H^*_t = H^*. \]  

Market clearing for financial assets requires

\[ B_t + B^*_t = 0, \]  

where \( B^*_t \) represents Foreign country holdings of international debt.

A.1 Equilibrium with Flexible Prices and Wages

Consider first a version of the model in which prices and wages are flexible (\( \zeta_k \to 0 \) for \( k = \{p, w\} \)).

In the absence of nominal rigidities, the optimal labor supply decision implies that the real wage equals the marginal disutility of labor in units of marginal utility of consumption

\[ \frac{W_t}{P_t} = \left( \frac{\phi_w}{\phi_w - 1} \right) \frac{L^*_t}{\eta X^*_t^{\frac{1}{\gamma}} C^*_t^{\frac{1}{\gamma}}}. \]  

On the firm side, optimality implies

\[ P_{ht} = \left( \frac{\phi_p}{\phi_p - 1} \right) \frac{W_t}{A}. \]
Therefore, equilibrium in the labor market requires

\[ \frac{P_{ht}}{P_t} = \frac{\Phi L_t^\nu}{\eta A X_t^{\frac{1}{\sigma}} C_t^{-\frac{1}{\sigma}}}, \]  

(49)

or

\[ \frac{P_{ht}}{P_t} = \frac{\Phi Y_t^\nu}{\eta A X_t^{\frac{1}{\sigma}} C_t^{-\frac{1}{\sigma}}}, \]  

(50)

where the last equation makes use of the production function to eliminate labor and

\[ \Phi \equiv \left( \frac{\phi_w}{\phi_w - 1} \right) \left( \frac{\phi_p}{\phi_p - 1} \right). \]  

(51)

Similarly, in the Foreign country

\[ \frac{P_{ft}^*}{P_t^*} = \frac{\Phi Y_{ft}^\nu}{\eta A X_t^{\frac{1}{\sigma}} C_t^{-\frac{1}{\sigma}}}, \]  

(52)

Next, the goods market equilibrium pins down Home and Foreign consumption as a function of the terms of trade and the real exchange rate

\[ Y_{ht} = \left( \frac{P_{ht}}{P_t} \right)^{-\gamma} \left[ \alpha C_t + (1 - \alpha) S_t^r C_t^r \right]. \]  

(53)

The Foreign country counterpart of the last equation is

\[ Y_{ft} = \left( \frac{P_{ft}^*}{P_t^*} \right)^{-\gamma} \left[ (1 - \alpha) S_t^r C_t^r + \alpha C_t^r \right]. \]  

(54)

Real house prices are

\[ Q_t = \tilde{\eta} \left( \frac{H_t}{C_t} \right)^{-\frac{1}{2}} + \beta E_t \left[ e^{\xi_t+1 - \xi_t} \left( \frac{X_{t+1}}{X_t} \right)^{\frac{1}{2} - \sigma} \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{2}} Q_{t+1} \right] + \Xi_t \Theta_t E_t \left( \Pi_{t+1} Q_{t+1} \right), \]  

(55)

The Foreign counterpart of equation (55) is

\[ Q_{t}^* = \tilde{\eta} \left( \frac{H_{t}^*}{C_{t}^*} \right)^{-\frac{1}{2}} + \beta^* E_t \left[ e^{\xi^*_{t+1} - \xi^*_t} \left( \frac{X_{t+1}^*}{X_t^*} \right)^{\frac{1}{2} - \sigma} \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\frac{1}{2}} Q_{t+1}^* \right]. \]  

(56)

Differently from the Home economy, the borrowing constraint never binds in the Foreign country, therefore \( \Xi_t^* = 0 \) at all times.

The borrowing constraint (5) pins down the stock of internationally-traded real debt \( B_t \equiv B_t/P_t \)

\[ (1 + i_t)B_t = \Theta_t E_t \left( Q_{t+1}^r H \Pi_{t+1} \right). \]  

(57)
The shadow price of the borrowing constraint is
\[
(1 + i_t)\Xi = 1 - \beta (1 + i_t) E_t \left[ e^{\xi t+1 - \xi_t} \left( \frac{X_{t+1}}{X_t} \right)^{\frac{1}{1-\sigma}} \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\nu}} \right].
\] (58)

No arbitrage pins down the return in international financial markets
\[
E_t \left[ e^{\xi t+1 - \xi_t} \left( \frac{X_{t+1}}{X_t} \right)^{\frac{1}{1-\sigma}} \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\nu}} \left( 1 + i_t \right) \left( 1 + i_t \right) \right] = 0,
\] (59)

while the Euler equation for the Foreign country pins down the return in the Foreign country
\[
1 = \beta^* (1 + i_t^*) E_t \left[ e^{\xi t+1 - \xi_t^*} \left( \frac{X_{t+1}^*}{X_t^*} \right)^{\frac{1}{1-\sigma}} \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\frac{1}{\nu}} \right].
\] (60)

In each country, the central bank determines the inflation rate (monetary policy is neutral) via the interest rate rule
\[
(1 + i_t) = \left( 1 + i_{t-1} \right)^{\rho i} \left( 1 + i_t \right) \left( \frac{\Pi_t}{\Pi_t^*} \right) \left( \frac{Y_t}{Y_t^*} \right) Y_t - C_t.
\] (61)

(1 + i_t^*) = \left( 1 + i_{t-1}^* \right)^{\rho i^*} \left( 1 + i_t^* \right) \left( \frac{\Pi_t^*}{\Pi_t^*} \right) \left( \frac{Y_t^*}{Y_t^*} \right) Y_t^* - C_t^*.
\] (62)

The law of motion of foreign debt (from the resource constraint) pins down the relative price
\[
- B_t = - \frac{(1 + i_t)B_{t-1}}{\Pi_t} + \left( \frac{P_{ht}}{P_t} \right) Y_{ht} - C_t.
\] (63)

The world resource constraint pins down the real exchange rate
\[
\left( \frac{P_{ht}}{P_t} \right) Y_{ht} + \left( \frac{P_{ft}^*}{P_t^*} \right) S_t Y_{ft} = C_t + S_tC_t^*.
\] (64)

Equations (50) to (64) characterize the equilibrium in terms of domestic relative prices and the real exchange rate. The terms of trade (\( T_t = P_{ft}/P_{ht} = P_{ft}^*/P_{ht}^* \)) link domestic relative prices in the two countries
\[
\left( \frac{P_{ht}}{P_t} \right)^{(1-\gamma)} = \alpha + (1 - \alpha)T_t^{1-\gamma} \quad \text{and} \quad \left( \frac{P_{ft}^*}{P_t^*} \right)^{(1-\gamma)} = \alpha + (1 - \alpha)T_t^{-1-\gamma}.
\] (65)

Finally, first-differencing the definition of the real exchange rate allows to pin down the
nominal exchange rate

\[ \frac{S_t}{S_{t-1}} = \frac{\xi_t}{\xi_{t-1}} \frac{\Pi_t}{\Pi_t} \]  

(66)

### A.2 Equilibrium with Nominal Rigidities

The key difference compared to the previous section is the determination of the supply side of the economy. Additionally, the effects of real interest rates on asset prices depend on the nominal interest rate set by the monetary authority in each country and the expected inflation rate.

The wage determination process yields a non-linear wage Phillips curve, which combines the optimal choice of household members who reset their wage in any given period and their mass with the aggregate wage index

\[ \left[ 1 - \zeta_w \Pi_{wt}^{\phi_w - 1} \right]^{\frac{1 - \phi_w (1 - \nu)}{1 - \phi_w}} = K_{wt} \frac{F_{wt}}{W_{wt}}. \]  

(67)

According to expression (67), wage inflation \( \Pi_{wt} = W_t/W_{t-1} \) is a non-linear function of the present discounted value of the marginal disutility of labor \( K_{wt} \)

\[ K_{wt} = \frac{\phi_w}{\phi_w - 1} e^{\varsigma t} L_t^\nu + \beta \zeta_w \xi_t \left[ (\Pi_{wt+1})^{\phi_w} K_{wt+1} \right] \]  

(68)

and of the present discounted value of the real wage in units of marginal utility of consumption \( F_{wt} \)

\[ F_{wt} = \eta e^{\varsigma t} X_t^{\frac{1}{\epsilon} - \sigma} C_t^{\frac{1}{\epsilon} - \nu} W_t L_t \frac{P_t}{P_{t-1}} + \beta \zeta_w \xi_t \left[ (\Pi_{wt+1})^{\phi_w} F_{wt+1} \right]. \]  

(69)

Price setting decisions yield a non-linear price Phillips curve, which combine the optimal choice of firms who reset their price in any given period and their mass with the price index for domestic tradable goods

\[ \left[ 1 - \zeta_p \Pi_{ht}^{\phi_p - 1} \right]^{\frac{1 - \phi_p}{1 - \phi_p}} = K_{pt} \frac{F_{pt}}{P_{pt}}. \]  

(70)

According to expression (70), inflation in the domestic tradable good sector \( \Pi_{ht} = P_{ht}/P_{ht-1} \) is a non-linear function of the present discounted value of real marginal costs \( K_{pt} \)

\[ K_{pt} = \frac{\phi_p}{\phi_p - 1} e^{\varsigma t} X_t^{\frac{1}{\epsilon} - \sigma} C_t^{\frac{1}{\epsilon} - \nu} Y_{ht} A P_t + \beta \zeta_p \xi_t \left[ (\Pi_{ht+1})^{\phi_p} K_{pt+1} \right] \]  

(71)

and of the present discounted value of real marginal revenues

\[ F_{pt} = e^{\varsigma t} X_t^{\frac{1}{\epsilon} - \sigma} C_t^{\frac{1}{\epsilon} - \nu} P_{ht} Y_{ht} \frac{P_{ht}}{P_t} + \beta \zeta_p \xi_t \left[ (\Pi_{ht+1})^{\phi_p} F_{pt+1} \right]. \]  

(72)
Equations (67) and (70) (and their Foreign country counterpart) replace (47) and (48) in determining the equilibrium in the labor market.

B Asymmetric Steady State

To build an asymmetric steady state in which country H is a net borrower but relative prices, terms of trade and real exchange rate are still equal to one, start with the assumption that the Home country representative household is relative more impatient ($\beta < \beta^\ast$). Assume that the borrowing constraint is binding for country H but not for country F ($\Xi > 0$ and $\Xi^\ast = 0$).

The Home country labor market equilibrium is

$$1 = \frac{\Phi Y_h'^\nu}{A^{1+\nu}X^\frac{1}{\nu}C^{-\frac{1}{\nu}}}.$$

Equation in the market for goods produced in the Home country is

$$Y_h = \alpha C + (1 - \alpha)C^\ast.$$

These two equations, together with their Foreign country counterpart, pin down $C$, $C^\ast$, $Y_h$ and $Y_f$ as a function of productivity and the housing stock (through $X$ and $X^\ast$). The appropriate choice of $A$ and $A^\ast$, conditional on the housing stock, ensures that in steady state relative prices are equal to one. Obviously, in this asymmetric steady state, trade is not balanced ($Y_h \neq C$ and $Y_f \neq C^\ast$). From the perspective of country H, the steady state trade balance must be in surplus to repay the positive stock of foreign debt.

No arbitrage implies

$$R = R^\ast = \frac{1}{\beta^\ast}.$$

Since the borrowing constraint is binding for country H, debt is equal to

$$B = \Theta \beta^\ast QH.$$

The house price equation yields

$$Q = \left(\frac{1 - \eta}{\eta}\right) \frac{(H/C)^{-\frac{1}{\eta}}}{1 - \beta - \Xi^\theta}.$$

Holding consumption constant, higher LTV ratios increase house prices and debt, both directly and indirectly. In the Foreign country, the borrowing constraint is not binding, thus house prices are

$$Q^\ast = \left(\frac{1 - \eta}{\eta}\right) \frac{C^\ast/H^\ast}{1 - \beta^\ast}.$$

34
The ratio between the housing stocks in the two countries can be chosen so that the steady state house prices are the same.

C Log-Linear Approximation of the Model

Unless otherwise noted, for any given variable \( Z_t \) define \( z_t \equiv \log(Z_t/Z) \approx (Z_t - Z)/Z \), where \( Z \) is the steady state of \( Z_t \).

C.1 Flexible Prices and Wages

A first order approximation to labor market equilibrium conditions (50) and (52) is

\[
p_{ht} = \nu y_{ht} + \frac{1}{\varepsilon} c_t - \left( \frac{1}{\varepsilon} - \sigma \right) x_t \quad \text{and} \quad p_{ft}^* = \nu y_{ft} + \frac{1}{\varepsilon} c_t^* - \left( \frac{1}{\varepsilon} - \sigma \right) x_t^*,
\]

where \( p_{ht} \equiv \log((P_{ht}/P_t)/(P_{ht}/P)) \) and similarly for \( p_{ft}^* \).

Equilibrium in goods markets can be approximated as

\[
y_{ht} = -\gamma p_{ht} + s_H \left[ \alpha c_t + (1 - \alpha) c_{R_t}^{-1} (\gamma s_t + c_t^*) \right]
\]

and

\[
y_{ft} = -\gamma p_{ft}^* + s_F \left[ (1 - \alpha) c_{R_t} (c_t - \gamma s_t) + \alpha c_t^* \right].
\]

where \( s_i \equiv C_i/Y_i \) is the steady state consumption share of output in country \( i = \{H, F\} \) and \( c_R \equiv C/C^* \) is relative consumption across countries.

Because monetary policy is neutral in the absence of nominal rigidities, the equilibrium can be characterized in terms of real interest rates

\[
r_t \equiv i_t - \mathbb{E}_t \pi_{t+1} \quad \text{and} \quad r_t^* \equiv i_t^* - \mathbb{E}_t \pi_{t+1}^*.
\]

In each country, the central bank determines inflation via a standard interest rate rule

\[
i_t = \rho_i i_{t-1} + (1 - \rho_i) (\psi_{\pi} \pi_t + \psi_{y} y_{ht}) + \varepsilon_{it}
\]

and

\[
i_t^* = \rho_i i_{t-1}^* + (1 - \rho_i) (\psi_{\pi} \pi_t^* + \psi_{y} y_{ft}) + \varepsilon_{it}^*.
\]

Next, the approximation of the house price equations (55) yields

\[
q_t = (1 - \beta - \Xi \Theta) c_t + \beta \left[ \gamma \left( \frac{1}{\varepsilon} - \sigma \right) (\mathbb{E}_t x_{t+1} - x_t) - \frac{1}{\varepsilon} (\mathbb{E}_t c_{t+1} - c_t) \right] + \Xi \Theta (\xi_t + \theta_t) + (\beta + \Xi \Theta) \mathbb{E}_t q_{t+1} + \beta (\mathbb{E}_t s_{t+1} - \xi_t).
\]
The lagrange multiplier on the borrowing constraint introduces a wedge in the Home country Euler equation. A first order approximation of equation (58) gives

\[ r_t + \beta R \left( (E_t \varsigma_{t+1} - \varsigma_t) + \frac{1}{\varepsilon} - \sigma \left( E_t x_{t+1} - x_t \right) - \frac{1}{\varepsilon} (E_t c_{t+1} - c_t) \right) + (1 - \beta R) \xi_t = 0. \] (86)

In the Foreign country, the slack borrowing constraint implies that equation (56) becomes

\[ q_t^* = (1 - \beta^*) c_t^* + \beta^* \left( (E_t \varsigma_{t+1}^* - \varsigma_t^*) + \frac{1}{\varepsilon} - \sigma \left( E_t x_{t+1}^* - x_t^* \right) - \frac{1}{\varepsilon} (E_t c_{t+1}^* - c_t^*) \right) + \beta^* E_t q_t^* + 1. \] (87)

The approximation of the borrowing constraint (57) is

\[ r_t + b_t = \theta_t + E_t q_{t+1}. \] (88)

A first order approximation to country F Euler equation (60) gives

\[ r_t + \left( \frac{1}{\varepsilon} - \sigma \right) (E_t x_{t+1}^* - x_t^*) - \frac{1}{\varepsilon} (E_t c_{t+1}^* - c_t^*) - (E_t s_{t+1} - s_t) + (E_t \varsigma_{t+1}^* - \varsigma_t^*) = 0. \] (89)

Up to the first order, the no-arbitrage relation (59) can be written as

\[ r_t = r_t^* + E_t s_{t+1} - s_t. \] (90)

The log-linear approximation of the index (2) for the Home and Foreign country gives

\[ x_t = \eta \left( \frac{C}{X} \right)^{\varepsilon-1} c_t \quad \text{and} \quad x_t^* = \eta \left( \frac{C^*}{X^*} \right)^{\varepsilon-1} c_t^*. \] (91)

The dynamics of debt (63) can be approximated as

\[-b_t = -R (r_{t-1} + b_{t-1}) + b_y^{-1} (p_{ht} + y_{ht} - s_H c_t), \] (92)

where \( b_y \equiv B/Y_H \) is the steady state ratio between net foreign debt and GDP for the Home country.

Up to a first order approximation, the world resource constraint (64) gives

\[ \frac{1}{s_H} (p_{ht} + y_{ht}) + \frac{1}{c_R s_F} (p_{ft}^* + s_t + y_{ft}) = c_t + \frac{1}{c_R} (s_t + c_t^*). \] (93)

The approximation of equations (65) that link the terms of trade to domestic relative prices is

\[ p_{ht} = -(1 - \alpha) \tau_t \quad \text{and} \quad p_{ft}^* = (1 - \alpha) \tau_t. \] (94)
C.2 Nominal Rigidities

A first order approximation of the relative wage equation (29) gives

$$[1 - \phi_w(1 - \nu)]\omega_t(i) = k_{wt} - f_{wt},$$

(95)

where $\omega_t(i)$ is the log-deviation of the relative wage of type $i$ from steady state (in which $W(i) = W \Rightarrow \omega(i) = 1$). A first order approximation of the present discounted value of the marginal disutility of labor (30) and the real wage in units of marginal utility of consumption (31) yields

$$k_{wt} = (1 - \beta\zeta_w)\nu\ell_t + \beta\zeta_w\mathbb{E}_t(\phi_w\nu\pi_{wt+1} + k_{wt+1})$$

(96)

and

$$f_{wt} = (1 - \beta\zeta_w)\left[w_t + \left(\frac{1}{\varepsilon} - \sigma\right)x_t - \frac{1}{\varepsilon}c_t + \ell_t\right] + \beta\zeta_w\mathbb{E}_t[(\phi_w - 1)\pi_{wt+1} + f_{wt+1}],$$

(97)

where $w_t \equiv \log[(W_t/P_t)/(W_t/P_t)]$ stands for the log-deviation of the real wage from its steady state value. A first order approximation of the wage index equation (32) is

$$\omega_t(i) = \frac{\zeta_w}{1 - \zeta_w}\pi_{wt}.$$  

(98)

Combining the last four expressions gives a standard forward looking wage Phillips curve

$$\pi_{wt} = \kappa_w \left[(v - 1)\nu\ell_t - w_t - \left(\frac{1}{\varepsilon} - \sigma\right)x_t + \frac{1}{\varepsilon}c_t\right] + \beta\zeta_w\mathbb{E}_t(\pi_{wt+1}),$$

(99)

where $\kappa_w \equiv (1 - \beta\zeta_w)(1 - \zeta_w)/\{\zeta_w[1 - \phi_w(1 - \nu)]\}$.

For prices, first order approximation of equation (38) gives

$$p_t(h) = k_{pt} - f_{pt},$$

(100)

where $p_t(h)$ is the log-deviation of the relative price of variety $h$ from steady state (in which $P(h) = P \Rightarrow p(h) = 1$). A first order approximation of the present discounted value of marginal costs (39) and marginal revenues (40) yields

$$k_{pt} = (1 - \beta\zeta_p)\left[\left(\frac{1}{\varepsilon} - \sigma\right)x_t - \frac{1}{\varepsilon}c_t + w_t + y_{ht}\right] + \beta\zeta_p\mathbb{E}_t(\phi_p\pi_{ht+1} + k_{pt+1})$$

(101)

and

$$f_{pt} = (1 - \beta\zeta_p)\left[\left(\frac{1}{\varepsilon} - \sigma\right)x_t - \frac{1}{\varepsilon}c_t + p_{ht} + y_{ht}\right] + \beta\zeta_p\mathbb{E}_t[(\phi_p - 1)\pi_{ht+1} + f_{pt+1}]$$

(102)
A first order approximation of the price index equation (41) is

\[ p_t(h) = \frac{\zeta_p}{1 - \zeta_p} \pi_{ht}. \]  

(103)

Combining the last four expressions gives a standard forward looking price Phillips curve

\[ \pi_{ht} = \kappa_p(w_t - p_{ht}) + \beta E_t(\pi_{ht+1}), \]  

(104)

where \( \kappa_p \equiv (1 - \beta \zeta_p)(1 - \zeta_p)/\zeta_p. \)
References


