Trade Structure and Belief-Driven Fluctuations in a Global Economy*  

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July 23, 2012  

Abstract  

This paper examine a dynamic two-country model with country-specific production externalities and inspect the presence of multiple converging paths under alternative trades structures. It is shown the presence of sunspot-driven fluctuations is closely related to the specified trade structure. If investment goods are not internationally traded and financial capital mobility is possible, then indeterminacy arises in a wider set of parameter space than in the corresponding closed economy. By contrast, either if both consumption and investment goods are traded in the absence of international lending or borrowing or if only investment goods are traded with financial capital mobility, then the indeterminacy conditions are the same those for the closed economy counterpart.  

Keywords: two-country model, non-traded goods, equilibrium indeterminacy, social constant returns  

JEL classification: F43, O41  

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*We thank Costas Azariadis, Jun-ichi Fujimoto, Shin-ichi Fukuda, Koici Futagami, Volker Böhm, Masaya Sakuragawa, Noritaka Kudoh, Esturo Shioji, Makoto Saito, Akihisa Shibata, Yi-Chan Tsai and Ping Wang for their helpful comments on earlier versions of this paper. Our research has been financially supported by the Grant-in-Aid for Scientific Research.  
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1 Introduction

The central concern of this paper is to explore whether globalization of an economy enhances belief-driven fluctuations. We examine a dynamic two-country model with country-specific production externalities and inspect the conditions under which multiple converging paths exist. In the presence of multiple equilibria, non-fundamental shocks (sunspots) affect expectations of agents, which gives rise to belief-driven economic fluctuations. Therefore, if an open economy yields equilibrium indeterminacy in a wider parameter space than in the closed economy counterpart, then we may conclude that globalization of an economy increases the possibility of economic fluctuations in the sense that not only shocks to the fundamentals but also extrinsic uncertainty can generate business cycles.

As for our question, the foregoing literature has provided us with two contrasting answers. On the one hand, Meng (2003), Meng and Velasco (2003 and 2004) and Weder (2001) show that small-open economies with production externalities yield multiplicity of equilibrium under weaker conditions than in the corresponding closed economy models.¹ Hence, according to these studies, globalization destabilizes an economy. Nishimura and Shimomura (2002), on the other hand, examine a dynamic Heckscher-Ohlin model of the two-country world in which there are country-specific production externalities. They show that the world economy has the same conditions for equilibrium (in)determinacy as those for a closed economy counterpart. In addition, Sim and Ho (2007) find that if one of the two countries has no production externalities in Nishimura and Shimomura’s model, then the equilibrium path of the world economy would be determinate even though the country with production externalities exhibits autarkic indeterminacy. These studies indicate that globalization does not necessarily enhance the possibility of belief-driven fluctuations.

At the first sight, the opposite results mentioned above seem to stem from the difference in the modelling method used by the existing studies. The small-open economy models are based on the partial equilibrium analysis in which behavior of the rest of the world is exogenously given. In contrast, the models of world economy employ the general equilibrium

¹Lahiri (2001) also examines indeterminacy in a small-open economy model. Since he uses a framework different from the one used by Meng (2003) and others, his model needs a relatively high degree of external increasing returns to yield indeterminacy. Yong and Meng (2004) and Zhang (2008) also discuss equilibrium indeterminacy in small-open economies.
approach that treats the world economic system as a closed economy consisting of multiple countries. Thus one may think that the behavior of an integrated world economy is close to the behavior of a single, closed economy. Such a conjecture is, however, misleading. We demonstrate that the key to the relation between globalization and belief-driven fluctuations is the specification of trade structure rather than the difference in modeling strategy, that is, partial versus general equilibrium analyses. In the foregoing investigations, the papers on small-open economies such as Meng and Velasco (2003, 2004) and Weder (2001) assume that investment goods are not internationally traded, while consumption goods are traded and financial capital mobility is allowed. By contrast, Nishimura and Shimomura (2002) follow the Heckscher-Ohlin tradition where both consumption and investment goods are traded and financial capital mobility is not possible. We show that, as well as in the small-open economy models, if investment goods are traded in the domestic market alone, then the world economy model exhibits equilibrium indeterminacy under weaker conditions than those for the closed economy model.

More specifically, we construct a $2 \times 2 \times 2$ model of the world economy in which each country produces both investment and consumption goods under social constant returns. It is assumed that both countries have identical technologies and preferences. We inspect the equilibrium indeterminacy conditions of the world economy under alternative trade structures. If we assume that both investment and consumption goods are tradable and international lending and borrowing are not allowed, then our model is identical to Nishimura and Shimomura (2002), so that opening up international trade does not affect the indeterminacy conditions. If investment goods are nontradables and financial capital mobility is possible, then the world economy exhibits indeterminacy in a wider range of parameter space than in the corresponding closed economy. In this case it is revealed that the indeterminacy conditions for the two-country world are close to those for indeterminacy in the small-open economy examined by Meng and Velasco (2004).\footnote{Meng and Velasco (2003) also utilize the same forms of production function but they allow labor-leisure choice. They show that the conclusion is essentially the same in the presence of endogenous labor supply.} Finally, if consumption goods are not traded but investment goods are tradable, then it is shown that the indeterminacy conditions are the same as those for the closed economy.

As suggested above, this paper is closely related to Meng and Velasco (2004) and Nishimura...
and Shimomura (2002). Both papers are based on Benhabib and Nishimura (1998) who investigate indeterminacy conditions in a closed, two-sector growth model with sector-specific production externalities and social constant returns. The main finding of Benhabib and Nishimura (1998) is that (i) if the consumption good sector is more capital intensive than the investment good sector from the private perspective but it is less capital intensive from the social perspective; and (ii) if the elasticity of intertemporal substitution in consumption of the representative family is sufficiently large, then there is a continuum of converging equilibrium paths around the steady state. Since the integrated world economy discussed by Nishimura and Shimomura (2002) behaves like a single, closed economy, the indeterminacy conditions for their model is the same as those shown by Benhabib and Nishimura (1998). Meng and Velasco (2004) find that in a small-open economy model with non-traded investment goods and financial capital mobility, only condition (i) is necessary for establishing indeterminacy: the shape of utility function does not affect uniqueness of equilibrium.³ Our paper uses Nishimura and Shimomura’s setting as the base model and introduces nontraded goods and intertemporal trade. The case where investment goods are not traded is, therefore, a two-(large) country version of Meng and Velasco (2004).⁴

The roles of nontraded goods have been extensively discussed in the literature. The static trade theory has focused on the effects of nontraded goods on trade patterns, terms of trade and resource allocation: see, for example, Komiya (1967), Either (1972) and Jones (1974). Also, there is a vast literature on this topic in international macroeconomics and finance. Those macroeconomic studies have been concerned with how the presence of nontraded goods affects real exchange rates, current accounts, asset positions, policy impacts and international business cycles caused by the fundamental shocks.⁵ Turnovsky (1997, Chapter

³ In the two-sector endogenous growth model of a closed economy where each sector employs physical and human capital under social constant returns, the condition (ii) is not needed for holding indeterminacy: see Benhabib et al. (2000) and Mino (2001).

⁴ Weder (2001) examines an open economy version of a two-sector closed economy model studied by Benhabib and Farmer (1996). In Weder’s model the production technology of each sector exhibits constant returns from the private perspective, while it satisfies increasing (or decreasing) returns from the social perspective. It is also assumed that labor supply is endogenous and private factor intensity is identical in both sectors. Weder (2001) also considers the case where the home country is not small so that the world interest rate depends on the asset holding of the home country. Despite those differences from Meng and Velasco (2003), Weder (2001) also finds that the open economy yields indeterminacy under weaker restrictions than the closed economy.

among others, point out that outcomes may critically depend on which goods are not internationally traded. The foregoing contributions in most cases explore models with equilibrium determinacy. Therefore, the relation between trade structure and belief-driven business cycles has not been explored well in the foregoing studies. Our paper demonstrates that nontraded goods and trade structure also play pivotal roles as to the destabilizing effect of globalization caused by indeterminacy and sunspots. We also confirm that in the presence of equilibrium indeterminacy, the long-run distribution of wealth in the world market and the steady-state level of asset position of each country becomes indeterminate: not only the initial holding of asset of each country but also sunspot shocks may affect these long-run values. Therefore, if belief-driven economic fluctuations exist, we obtain outcomes and implications that are quite different from those obtained when the equilibrium path of the world economy is determinate.

The next section sets up the base model. Section 3 summarizes the main conclusion of Nishimura and Shimomura (2002). Section 4, the main part of our paper, explores the indeterminacy conditions in the presence of nontraded goods and intertemporal trade. Section 5 gives intuitive implication of our findings. This section also discusses empirical plausibility of the assumptions made for establishing our main results.

2 Baseline Setting

Consider a world economy consisting of two countries, home and foreign. Both countries have the same production technologies. In each country there is the representative household. Households in both countries have an identical time discount rate and the same form of instantaneous felicity function. The only difference between the two countries is the initial stock of wealth held by the households in each country. In this section we concentrate on modelling the home country. Since taste and technology are symmetric between the two countries, the following formulations are applied to the foreign country as well.

2.1 Production

The production side of our model is the same as that used by Nishimura and Shimomura (2002). The home country has two production sectors. The first sector \((i = 1)\) produces investment goods and the second sector \((i = 2)\) produces pure consumption goods. The
production function of \(i\)-th sector is specified as
\[
Y_i = A_i K_i^{a_i} L_i^{b_i} \bar{X}_i, \quad a_i > 0, \ b_i > 0, \ 0 < a_i + b_i < 1, \quad i = 1, 2,
\]
where \(Y_i\), \(K_i\) and \(L_i\) are \(i\)-th sector’s output, capital and labor input, respectively. Here, \(\bar{X}_i\) denotes the sector and country-specific production externalities.\(^6\) We define:
\[
\bar{X}_i = K_i^{a_i} L_i^{1-a_i-b_i}, \quad a_i < \alpha_i < 1, \quad \alpha_i + b_i < 1 \quad i = 1, 2.
\]
If we normalizes the number of producers to one, then it holds that \(\bar{K}_i = K_i\) and \(\bar{L}_i = L_i \ (i = 1, 2)\) in equilibrium. This means that the \(i\)-th sector’s social production technology that internalizes the external effects is:
\[
Y_i = A_i K_i^{a_i} L_i^{1-a_i}, \quad i = 1, 2. \tag{1}
\]
Hence, the social technology satisfies constant returns to scale, while the private technology exhibits decreasing returns to scale.\(^7\)

The factor and product markets are competitive, so that the private marginal product of each production factor equals its real factor price. These conditions are given by the following:
\[
r = p a_1 \frac{Y_1}{K_1} = a_2 \frac{Y_2}{K_2}, \tag{2a}
\]
\[
w = p b_1 \frac{Y_1}{L_1} = b_2 \frac{Y_2}{L_2}, \tag{2b}
\]
where \(w\) is the real wage rate, \(r\) is the rental rate of capital and \(p\) denotes the price of investment good in terms of the consumption good.

Considering that \(\bar{K}_i = K_i\), and \(\bar{L}_i = L_i\), we find that (2a) and (2b) yield:
\[
r = p a_1 A_1 k_1^{\alpha_1-1} = a_2 A_2 k_2^{\alpha_2-1}, \tag{3a}
\]
\(^6\) We shall omit time argument in each endogenous variable unless necessary.
\(^7\) This specification of production technology was first introduced by Behbhabib and Nishimura (1998) who inted to demonstrate that equilibrium indeterminacy may hold even in the absence of social increasing returns. Benhabib et al. (2000), Meng (2003), Meng and Velasco (2003, 2004), Mino (2001) and Nishimura and Shimomura (2002) utilize the same functional forms.
\[ w = pb_1 A_1 k_1^{a_1} = b_2 A_2 k_2^{a_2}, \quad (3b) \]

where \( k_i = K_i/L_i, \quad (i = 1, 2). \) By use of \((3a)\) and \((3b)\), we can express the optimal factor intensity in each production sector as a function of relative price:

\[ k_1 = \left( \frac{A_1}{A_2} \right)^{\frac{1}{a_2-a_1}} \left( \frac{a_1}{a_2} \right)^{\frac{a_2}{a_2-a_1}} \left( \frac{b_1}{b_2} \right)^{\frac{a_2-1}{a_2-a_1}} p^{\frac{1}{a_2-a_1}} = k_1(p), \]

\[ k_2 = \left( \frac{A_1}{A_2} \right)^{\frac{1}{a_2-a_1}} \left( \frac{a_1}{a_2} \right)^{\frac{a_1}{a_2-a_1}} \left( \frac{b_1}{b_2} \right)^{\frac{a_1-1}{a_1-a_2}} p^{\frac{1}{a_2-a_1}} \equiv k_2(p), \]

These expressions show that

\[ \text{sign} \left[ k_1(p) - k_2(p) \right] = \text{sign} \left( \frac{a_1}{b_1} - \frac{a_2}{b_2} \right), \quad (5) \]

\[ \text{sign} \ k_i'(p) = \text{sign} \ (\alpha_2 - \alpha_1), \quad i = 1, 2. \quad (6) \]

In the above, the sign of \( a_1/b_1 - a_2/b_2 \) represents the factor intensity ranking from the private perspective, while sign \((\alpha_1 - \alpha_2)\) expresses the factor intensity ranking from the social perspective.

We assume that production factors shiftable between the sectors, but they cannot cross the borders. Thus the full employment condition for capital and labor in the home country are respectively given by

\[ K_1 + K_2 = K, \quad L_1 + L_2 = 1. \]

where \( K \) denotes the aggregate capital in the home country. The labor supply is assumed to be constant and normalized to one. These full-employment conditions are summarized as

\[ k_1(p) L_1 + (1 - L_1) k_2(p) = K. \quad (7) \]

In this paper we restrict our attention to the interior equilibrium in which both countries produce both consumption and investment goods.\(^8\) To ensure this restriction, we assume that relative price in each country satisfies the equilibrium labor allocation to the first sector

\(^8\)See Footnote 9 on this restriction.
given by (7) satisfies the following:

\[ L_1 = \frac{K - k_2(p)}{k_1(p) - k_2(p)} \in (0, 1). \]  

(8)

The supply functions of investment and consumption goods are respectively given by

\[ y^1(K, p) = L_1 A_1 k_1(p) \alpha_1 = \frac{K - k_2(p)}{k_1(p) - k_2(p)} A_1 k_1(p)^{\alpha_1}, \]  

\[ y^2(K, p) = (1 - L_1) A_2 k_2(p) \alpha_2 = \frac{k_1(p) - K}{k_1(p) - k_2(p)} A_1 k_1(p^*)^{\alpha_2}. \]  

(9a, 9b)

It is easy to see that these supply functions satisfy:

\[ \text{sign } y^1_K(K, p) = \text{sign } \left( \frac{a_1}{b_1} - \frac{a_2}{b_2} \right), \quad \text{sign } y^1_p(K, p) = \text{sign } \left( \frac{a_1}{b_1} - \frac{a_2}{b_2} \right)(\alpha_1 - \alpha_2), \]  

(10a)

\[ \text{sign } y^2_K(K, p) = -\text{sign } \left( \frac{a_1}{b_1} - \frac{a_2}{b_2} \right), \quad \text{sign } y^2_p(K, p) = -\text{sign } \left( \frac{a_1}{b_1} - \frac{a_2}{b_2} \right)(\alpha_1 - \alpha_2). \]  

(10b)

Note that if the private and social factor-intensity rankings have opposite signs, that is, \( \left( \frac{a_1}{b_1} - \frac{a_2}{b_2} \right)(\alpha_1 - \alpha_2) < 0 \), then the duality between the Rybczynski and Stolper-Samuelson effects fails to hold.

### 2.2 Households

There is a continuum of identical households with a unit mass. Each household supply one unit of labor in each moment. The objective functional of the representative household is given by

\[ U = \int_{t_0}^{\infty} \frac{C^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \quad \sigma > 0, \quad \rho > 0, \]

where \( C \) is consumption and \( \rho \) denotes a given time discount rate. When \( \sigma = 1 \), then the instantaneous felicity function is \( \log C \).

(i) Financial Autarky

If the households cannot access to the international financial market, then the aggregate asset of the home country equals the aggregate capital stock held by the domestic households.
Thus their flow budget constraint is

\[ \dot{K} = \left( \frac{r}{p} - \delta \right) K + \frac{1}{p} (w + \pi_1 + \pi_2 - C), \]  

(11)

where $\delta \in [0, 1)$ is the rate of capital depreciation and $\pi_i$ denotes the excess profits in the $i$-th sector\(^9\). The households maximize $U$ subject to (11) and the initial holding of capital, $K_0$. When solving the optimization problem, the households take the sequences of $\{r_t, w_t, \pi_{1,t}, \pi_{2,t}, p_t\}_{t=0}^{\infty}$ as given. Letting $q$ be the implicit price of capital, the necessary conditions for an optimum include the following:

\[ C^{-\sigma} = q/p, \]  

(12a)

\[ \dot{q} = q (\rho + \delta - r/p), \]  

(12b)

together with the transversality condition; \( \lim_{t \to \infty} e^{-\rho t} q K = 0 \).

(ii) International Lending and Borrowing

If the households in the home country can lend to or borrow from the foreign households, then their flow budget constraint is given by

\[ \dot{\Omega} = R\Omega + w + \pi_1 + \pi_2 - C, \]  

(13)

where $\Omega$ denotes the net wealth (evaluated in terms of consumption good):

\[ \Omega = B + pK. \]

where $B$ is the stock of foreign bonds (IOUs). We assume that bond and capital are perfect substitutes and, hence, the non-arbitrage condition between the two assets requires that the rate of return to bond equal to the net rate return to capital plus capital gain:

\[ R = \frac{r}{p} - \delta + \frac{\dot{p}}{p}. \]  

(14)

\(^9\)Remember that the private technology of each production sector exhibits decreasing returns to scale with respect to capital and labor.
Using (14) and \( \dot{\Omega} = \dot{B} + p\dot{K} + \dot{p}K \), the flow budget constraint (13) is rewritten as

\[
\dot{B} = RB + rK + w + \pi_1 + \pi_2 - C - pI,
\]

where \( I \) denote gross investment, so that

\[
\dot{K} = I - \delta K.
\]

The representative household maximizes \( U \) subject (15), (16) and the non-Ponzi-game scheme given by

\[
\lim_{t \to \infty} \exp \left(- \int_0^t R_s ds \right) B_t \geq 0.
\]

Set up the Hamiltonian function for the optimization problem:

\[
H = C \left( C^{1-\sigma} - \frac{1}{1-\sigma} \right) + \lambda \left[ RB + rK + w + \pi_1 + \pi_2 - C - pI \right] + q \left( I - \delta K \right),
\]

where \( \lambda \) and \( q \) respectively denote the implicit prices of the foreign bonds and domestic capital. Focusing on an interior solution, we see that the necessary conditions for an optimum are:

\[
C^{-\sigma} = \lambda \quad (17a)
\]

\[
p\lambda = q, \quad (17b)
\]

\[
\dot{\lambda} = \lambda (\rho - R), \quad (17c)
\]

\[
\dot{q} = q (\rho + \delta) - \lambda r = q \left( \rho + \delta - \frac{r}{p} \right). \quad (17d)
\]

The optimization conditions also involve the transversality conditions on holding bond and capital: \( \lim_{t \to \infty} \lambda e^{-\rho t} B = 0 \) and \( \lim_{t \to \infty} q e^{-\rho t} K = 0 \).

### 3 Free Trade of Commodities

We first assume that there is only intratemporal trade: both investment and consumption goods are freely traded but households in each country neither lend to nor borrow from the foreign households. This is the Heckscher-Ohlin setting employed by Nishimura and
This section summarizes the main results of their contribution in order to clarify the effects of introducing nontraded goods and financial capital mobility into the base model.

### 3.1 Dynamics of the World Economy

Under free trade of both goods, the world market equilibrium conditions for investment and consumption goods are repetitively given by

\[
Y_1 + Y_1^* = K + K^* + \delta K + \delta K^*, \tag{18}
\]

\[
Y_2 + Y_2^* = C + C^*, \tag{19}
\]

where an asterisk indicates the corresponding foreign variable. When both countries produce both goods, then all the firms in the world economy face the common world price, \( p \). Following Nishimura and Shimomura (2002), we focus on the situation where both countries are inside the diversification cone.\(^{11}\) Hence, given the assumption of symmetric technologies between the two countries, both home and foreign firms in each production sector select the same capital intensity, and thus it holds that \( k_i (p) = k_i^* (p) \) \((i = 1, 2)\) for all \( t \geq 0 \). As a result, the supply function of investment goods of the foreign country is written as

\[
y_1^1 (K^*, p) = \frac{K^* - k_2 (p)}{k_1 (p) - k_2 (p)} A_1 k_1 (p)^{\alpha_1}.
\]

\(^{10}\)If international lending and borrowing allowed in the Heckscher-Ohlin setting, the instantaneous equilibrium itself becomes indeterminate. This is a reconfirmation of Mundel’s (1957) results shown in the static Heckscher-Ohlin model. See also Cremers (1997) on this point. Note that we may allow international lending and borrowing into the Heckscher-Ohlin model, if there are financial frictions or investment adjustment costs: see Antras and Caballero (2009) and Ono and Shibata (2010).

\(^{11}\)Our discussion rely on this assumption. If at least one country completely specializes, dynamic systems of the world economy examined in Sections 3 and 4 are different from those displayed in this paper. However, as shown by Appendix 1 of the paper, provided that both countries have identical taste and technology, the steady-state equilibrium of the world economy is inside the diversification cone where both countries produce both goods. Therefore, our assumption is justified as long as we focus on the local dynamics of the world economy near the steady state. If we intend to analyze the global behavior of the model, we need to treat the model out of the diversification cone. Atkeson and Kehoe (2000) explore the dynamic behavior of a small country that specializes in producing one of the two goods. Caliendo (2011) presents a detailed analysis of dynamic behavior of a 2 \( \times 2 \) model outside the diversification cone.
Hence, from (18) capital formation of the world economy is given by

\[ \dot{K}_w = \frac{K_w - 2k_2(p)}{k_1(p) - k_2(p)} A_1 k_1(p)^{\alpha_1} - \delta K_w, \tag{20} \]

where \( K_w = K + K^* \) denotes the world level of capital stock.

In addition, since firms in both countries choose the same capital intensities, the factor prices are also equalized between the two countries, that is, \( r^*(p) = r(p) \) and \( w^*(p) = w(p) \).

In the absence of international financial capital mobility, the optimization conditions for the households are given by (12a) and (12b). Therefore, the factor price equalization means that the shadow price of capital in each country follows:

\[ \frac{\dot{q}}{q} = \frac{\dot{q}^*}{q^*} = \rho + \delta - \tilde{r}(p), \tag{21} \]

where \( \tilde{r}(p) = r/p = \alpha_1 A_1 k_1(p)^{\alpha_1-1} \). Equations in (21) mean that \( q/q^* \) stays constant over time. Condition (17a) gives \( C/C^* = (q/q^*)^{-1/\sigma} \), so that the consumption ratio between home and foreign countries also stays constant over time. Letting \( C/C^* = \bar{n}(>0) \), the world market equilibrium condition for consumption goods, equation (19), presents

\[ (1 + \bar{n}) C = \frac{2k_1(p) - K_w}{k_1(p) - k_2(p)}. \tag{22} \]

To obtain (22), we use the fact that the foreign supply function of the consumption good is

\[ Y_2^* = \frac{k_1(p) - K^*}{k_1(p) - k_2(p)}. \]

Substituting \( C = q^{-\sigma} \) into (22) and solving it with respect to \( p \), we obtain the following relation:

\[ p = \pi(K_w, q; \bar{n}). \tag{23} \]

Inserting (23) into (20) and (21) leads to a dynamic system of \( K_w \) and \( q \) that depicts the dynamics of the integrated world economy.\(^\text{12}\) This aggregate dynamic system is essentially the same as the closed economy model in Benhabib and Nishimura (1998).

\(^\text{12}\) Notice that \( \bar{n} \) is constant over time but it is an endogenous variable. Nishimura and Shimomura (2002) confirm that local dynamics of the world economy near the steady state can be examined by fixing \( \bar{n} \) at a constant level.
3.2 Equilibrium Indeterminacy and Long-Run Trade Patterns

It is easy to see that the world economy has a unique steady state. First, the condition $\dot{q} = 0$ in (21) gives $\tilde{r}(p) = \rho + \delta$, which determines a unique level of relative price in the steady state. Then $\dot{K}_w = 0$ in (20) yields a unique steady-state value of $K_w$. The steady-state level of $q$ is thus given by (23). Notice that the steady-state conditions of the world economy alone cannot determine the steady-state levels of $K$ and $K^*$ held by each country. When the dynamic system has a saddlepoint property, the world economy has a unique converging path toward the steady state. If this is the case, it can be verified that the value of $q/q^*$ (so the value of $\bar{n}$) is uniquely determined depending on the initial holding of capital in each country, $K_0$ and $K_0^*$. In addition, the steady-state level of capital holding of each country is also uniquely determined, once $K_0$ and $K_0^*$ are specified. This implies that the steady-state distribution of capital stocks between the two countries is path-dependent. Hence, the initial distribution of capital between the two countries affects the long-run patterns of trade. For example, suppose that the home country is initially more abundant in capital than the foreign country. Then as long as both countries are always in the diversification cone during the transition, the home country can maintain the comparative advantage in producing capital intensive goods in the steady state equilibrium as well. In this sense, if the equilibrium path is determinate, the Heckscher-Ohlin theorem of determination of trade pattern still holds even though capital stock in each country changes over time during the transition towards the steady state.\(^\text{13}\)

As for the indeterminacy conditions, Nishimura and Shimomura (2002) present the following results:

**Proposition 1** (Nishimura and Shimomura 2002) The steady-state equilibrium of the world economy is locally indeterminate, if $\alpha_1 - \alpha_2 < 0$ and $\alpha_1 - \alpha_2 > 0$ and (ii) $1/\sigma > \max\{1, 1/\bar{\sigma}\}$, where $\bar{\sigma}$ is a function of parameters involved in the model.

\(^{13}\)This conclusion depends on the functional forms of production and utility functions we use as well as on the fact that we restrict our attention to the model behavior near the steady state. As for more general analyses on income and wealth distribution among the countries in the Heckscher-Ohlin world, see Atkeson and Kehoe (2000) and Bajona and Kehoe (2010). Atkeson and Kehoe (2000) treat a small-country model, while Bajona and Kehoe (2010) explore a two-country model.
ployes less capital intensive technology than the consumption good sector from the private perspective, while it uses more capital intensive technology from the social perspective. The second condition requires that the elasticity of intertemporal substitution in consumption is high enough.\textsuperscript{14} Since the aggregate dynamics of the world economy is identical to dynamics of the closed economy, the indeterminacy conditions given above are the same as those found by Benhabib and Nishimura (1998).

If the conditions in Proposition 1 are met, the initial values of $q$ and $q^*$ become indeterminate, so that the value of $\bar{n}$ is not determinate either. Nishimura and Shimomura (2002) show that the steady-state levels of $K$ and $K^*$ depend on $\bar{n}$. This means that in the presence of equilibrium indeterminacy, the long-run distribution of capital stocks between the home and foreign countries cannot be uniquely determined even though the initial holding in capital of each country is specified. As a consequence, a dynamic version of the Heckscher-Ohlin theorem as to comparative advantage and trade pattern may fail to hold if the equilibrium path of the world economy is indeterminate. Namely, not only the initial holdings of factor endowments but also belief-driven fluctuations may affect long-term trade patterns of the global economy.

4 Model with Nontraded Goods

The main part of this section examines the case where investment goods are not internationally traded. We also briefly consider the opposite case where consumption goods are nontradable.

4.1 Trade Structure

We now assume that consumption goods are internationally traded and financial capital mobility is allowed, but investment goods are nontradables.\textsuperscript{15} Although such an assumption is

\textsuperscript{14}The precise expression of $1/\sigma$ in Proposition 1 is

$$
\frac{1}{\sigma} = \frac{(1 - \alpha_1) a_2 b_1 (\rho + \delta) + \alpha_1 a_1 [\rho b_2 + \delta b_1 a_2 + (1 - a_1) b_2 \delta]}{(a_2 b_1 - a_1 b_2) (\alpha_1 - \alpha_2) [\rho + \delta (1 - a_1)]}
$$

\textsuperscript{15}In the small-country setting, the trade structure assumed here is a kind of dependent economy models discussed in open-economy macroeconomics literature. Meng and Velasco (2003 and 2004) and Weder (2001)
restrictive one, it helps to elucidate the role of trade structure in a dynamic world economy. In Section 5.2 we discuss the empirical plausibility of alternative trade structures used in this paper. Since investment goods are traded in the domestic markets alone and consumption goods are internationally traded, the market equilibrium conditions for investment and consumption goods are respectively given by

\[ Y_1 = \dot{K} + \delta K, \quad Y_1^* = \dot{K}^* + \delta K^*, \tag{24} \]

\[ Y_2 + Y_2^* = C + C^*, \tag{25} \]

The equilibrium condition for the bond market is

\[ B + B^* = 0, \tag{26} \]

which means that \( \Omega + \Omega^* = pK + p^*K^* \). Bonds are IOUs between the home and foreign households and, hence, the aggregate value of bonds is zero in the world financial market at large.

### 4.2 Dynamic System

Investment goods are traded in the domestic market alone, so that the price of investment goods in each country may differ from each other. Using (24), we find that capital stock in each country changes according to

\[ \dot{K} = \frac{K - k_2(p)}{k_1(p) - k_2(p)} A_1 k_1(p)^{\alpha_1} - \delta K. \tag{27a} \]

\[ \dot{K}^* = \frac{K^* - k_2(p^*)}{k_1(p^*) - k_2(p^*)} A_2 k_1(p^*)^{\alpha_1} - \delta K^*. \tag{27b} \]

employ such a formulation. In the forgoing studies on models without externalities, Turnovsky and Sen (1995) treat a small-open economy model with non-tradable capital and Turnovsky (1997, Chapter 7) studies a neoclassical two-country, two-sector model in which capital goods are not traded. Mino (2008) also discusses the similar two-country model with external increasing returns. See also Chapter 5 in Turnousky (2009) for a brief literature review.
Dynamics of the shadow values of capital are:

\[ \dot{q} = q[\rho + \delta - \tilde{r}(p)], \quad (28a) \]
\[ \dot{q}^* = q^*[\rho + \delta - \tilde{r}(p^*)], \quad (28b) \]

Here, \( p \) does not equal \( p^* \) during the transition. Therefore, unlike the model in the previous section, the relative shadow value of capital, \( q/q^* \), does not stay constant out of the steady state. Dynamic equations \((27a), (27b), (28a)\) and \((28b)\) depict behaviors of capital stocks and their implicit prices in the home and foreign countries.

To obtain a complete dynamic system, we should relate \( p \) and \( p^* \) to the state variables, \( K, K^*, q \) and \( q^* \). The foreign country’s optimization conditions corresponding to \((17a)\) and \((17c)\) are respectively given by \( C^{* - \sigma} = \lambda^* \) and \( \dot{\lambda}^*/\lambda^* = \rho - R \). Therefore, both \( \lambda^* / \lambda \) and \( C^*/C \) stay constant over time. Let us denote \( C^*/C = (\lambda^* / \lambda)^{-1/\sigma} = \bar{m} (> 0) \). Then the world market equilibrium condition for consumption \((25)\) is expressed as

\[ (1 + \bar{m}) \lambda^{-\frac{1}{\sigma}} = y^2(K, p) + y^2(K^*, p^*), \quad (29) \]

where \( y^2(K, p) \) is defined by \((9b)\) and \( y^2(K^*, p^*) \) is given by

\[ y^2(K^*, p^*) = \frac{k_1(p^*) - K^*}{k_1(p^*) - k_2(p^*)} A_2 k_2(p^*)^{\alpha_2}. \]

In view of \((29)\), we see that \( \lambda \) is expressed as a function of capital stocks, prices and \( \bar{m} : \)

\[ \lambda = (1 + \bar{m})^\sigma [y^2(K, p) + y^2(K^*, p^*)]^{-\sigma} = \lambda(K, K^*, p, p^*; \bar{m}). \quad (30) \]

Thus optimization conditions \((17b)\) and \( q^* = \lambda^* p^* \) give

\[ p = \frac{q}{\lambda(K, K^*, p, p^*; \bar{m})}, \quad \frac{q^*}{\bar{m}^{-\sigma} \lambda(K, K^*, p, p^*; \bar{m})}. \]
Solving these equations with respect to \( p \) and \( p^* \) presents the following expressions:
\[
p = \pi (K, K^*, q, q^*; \bar{m}), \quad p^* = \pi^*(K, K^*, q, q^*; \bar{m}). \tag{31}
\]

Substituting (31) into (27a), (27b), (28a) and (28b), we obtain a dynamic system of \( K, K^*, q \) and \( q^* \) under a given level of \( \bar{m} \).

Alternatively, we can obtain a dynamic system of \( K, K^*, p \) and \( p^* \) in the following manner. Differentiate both sides of (30) logarithmically with respect to time, which yields
\[
\dot{\lambda} = -\sigma \left[ \frac{Y^2_K K \dot{K}}{Y^2} + \frac{Y^2_K K^* \dot{K}^*}{Y^2} + \frac{Y^2_p p \dot{p}}{Y^2} + \frac{Y^2 p^* \dot{p}^*}{Y^2} \right], \tag{32}
\]
where \( Y^2 \equiv y^2(K, p) + y^2(K^*, p^*) \) denotes the aggregate supply of consumption goods in the world market. Note that from (17b), (17c), (17d) we obtain:
\[
\dot{\frac{p}{p}} = \dot{\frac{q}{q}} - \frac{\dot{\lambda}}{\lambda} = R + \delta - \tilde{r}(p), \tag{33a}
\]
\[
\dot{\frac{p^*}{p^*}} = \dot{\frac{q^*}{q^*}} - \frac{\dot{\lambda}^*}{\lambda^*} = R + \delta - \tilde{r}(p^*). \tag{33b}
\]
Substituting (27a), (27b), (33a), and (33b) into (32) yields the following:
\[
\rho - R = -\sigma \left[ \frac{Y^2_K K}{Y^2} \left( \frac{y^1(K, p) - \delta K}{K} \right) + \frac{Y^2_K K^*}{Y^2} \left( \frac{y^2(K^*, p) - \delta K^*}{K^*} \right) \right.
\]
\[
\left. + \frac{Y^2_p p}{Y^2} (R + \delta - \tilde{r}(p)) + \frac{Y^2 p^*}{Y^2} (R + \delta - \tilde{r}(p^*)) \right].
\]
Observe that each side of the above equation does not involve \( \bar{m} \). Solving the above with respect to \( R \), we find that the equilibrium level of the world interest rate can be expressed as a function of \( K, K^*, p \) and \( p^* \):
\[
R = R(K, K^*, p, p^*). \tag{34}
\]
Consequently, by use of (27a), (27b), (33a), (33b) and (34), we obtain the dynamic
system with respect to \((K, K^*, p, p^*)\) in such a way that

\[
\begin{align*}
\dot{K} &= y^1(K, p) - \delta K, \\
\dot{K}^* &= y^1(K^*, p^*) - \delta K^*, \\
\dot{p} &= p[R(K, K^*, p, p^*) + \delta - \tilde{r}(p)], \\
\dot{p}^* &= p^*[R(K, K^*, p, p^*) + \delta - \tilde{r}(p^*)].
\end{align*}
\] (35)

### 4.3 Steady-State of the World Economy

We first characterize the stationary equilibrium of the world economy. In the steady state, all of \(K, K^*, p, p^*, B, B^*, q, q^*\) and \(\lambda\) stay constant over time. Inspecting the steady state conditions, we obtain the following:

**Proposition 2** There is a unique, feasible steady-state equilibrium where the steady-state levels of capital and relative price in each country satisfy \(K = K^*\) and \(p = p^*\).

**Proof.** See Appendix 1. ☐

It is to be noted that while the steady-state levels of \(K (= K^*)\) and \(p (= p^*)\) are uniquely determined by the parameters involved in the model, the steady-state values of implicit prices of capital, \(q\) and \(q^*\), cannot be determined by the parameter values alone. To see this, notice that from the optimization condition (17b), in the steady state it holds that

\[
\dot{p} = \hat{\lambda} \hat{q}, \quad \ddot{p} = \tilde{m}^{\sigma} \hat{\lambda} \hat{q}^*,
\]

where a variable with a 'hat' denotes the steady-state value of the corresponding variable. From (30) in the steady state the implicit price of bond held in the home country, \(\lambda\), is given by

\[
\hat{\lambda} = (1 + \tilde{m})^{\sigma} [2y^2 \left( \dot{K}, \dot{p} \right)]^{-\sigma}.
\]

Since \(\hat{\lambda}\) depends on \(\tilde{m}\), we should know the value of \(\tilde{m}\) to determine \(\hat{\lambda}\) as well as \(\hat{q}\) and \(\hat{q}^*\). To find the value of \(\tilde{m}\), consider the current account of each country. Considering the market equilibrium condition for the investment goods in (24) and the factor income distribution relation such that \(pY_1 + Y_2 = rpK + w + \pi_1 + \pi_2\) and \(p^*Y_1^* + Y_2^* = r^*p^*K^* + w^* + \pi_1^* + \pi_2^*\),
we see that the dynamic equation of foreign bonds are expressed as

\[
\dot{B} = RB + Y_2 - C, \quad \dot{B}^* = RB^* + Y_2^* - C^*.
\]

These equations represent the current accounts of both countries. In view of the no-Ponzi game and the transversality conditions, the intertemporal constraint for the current account of each country is respectively given by the following:

\[
\int_0^\infty \exp \left( -\int_0^t R_s ds \right) C_t \, dt = \int_0^\infty \exp \left( -\int_0^t R_s ds \right) y^2 (K_t, p_t) \, dt + B_0,
\]

\[
\int_0^\infty \exp \left( -\int_0^t R_s ds \right) C_t^* \, dt = \int_0^\infty \exp \left( -\int_0^t R_s ds \right) y^2 (K_t^*, p_t^*) \, dt + B_0^*.
\]

Since it holds that \( C_t^* = \bar{m} C_t \) for all \( t \geq 0 \), the above equations yield

\[
\bar{m} = \frac{\int_0^\infty \exp \left( -\int_0^t R_s ds \right) y^2 (K_t^*, p_t^*) \, dt + B_0^*}{\int_0^\infty \exp \left( -\int_0^t R_s ds \right) y^2 (K_t, p_t) \, dt + B_0}.
\]  

Equation (36) demonstrates that \( \bar{m} \) depends on the initial holdings of bonds, \( B_0 \) and \( B_0^* \), as well as on the discounted present value of consumption goods produced in each country.\(^{16}\)

As a consequence, although \( \dot{\rho} \) depends only on the parameter values involved in the model, the steady state levels of \( \bar{q} \left( = \bar{\lambda} \dot{\rho} \right) \) and \( \bar{q}_* \left( = \bar{m}^{-\sigma} \bar{\lambda} \dot{\rho}_* \right) \) cannot be determined without specifying the initial holdings of bonds and the discounted present value of consumption goods in each country.

### 4.4 Indeterminacy Conditions

We now examine the local dynamics of the world economy around the steady state. A set of sufficient conditions for equilibrium indeterminacy for the model with nontraded investment goods is as follows:

**Proposition 3** If the investment good sector is more capital intensive than the consumption good sector from the social perspective but it is less capital intensive from the private perspec-

\(^{16}\)Remeber that the equilibrium paths of \( \{K_t, K_t^*, p_t, p_t^*, R_t\} \) determined by (35) do not depend on \( \bar{m} \).
positive, that is, \( \frac{a_2}{b_2} - \frac{a_1}{b_1} > 0 \) and \( \alpha_2 - \alpha_1 < 0 \), then the steady state of the world economy where investment goods are nontradable exhibits local indeterminacy.

**Proof.** See Appendix 2. ■

Proposition 3 claims that in our model equilibrium indeterminacy may emerge regardless of the magnitude of \( \sigma \). This is in contrast to Proposition 1 for the indeterminacy conditions for the case of free trade of both consumption and investment goods. When both investment and consumption goods are freely traded, in addition to the factor-intensity ranking conditions, the intertemporal elasticity in consumption \( (1/\sigma) \) should be sufficiently high to hold indeterminacy. Since the closed economy version of our model is the same as the integrated world economy model discussed by Nishimura and Shimomura (2002), we need the same condition for holding indeterminacy if our model economy is closed. Hence, our result shows that the financially integrated world with non-tradable capital goods may produce indeterminacy under a wider range of parameter spaces than in the closed economy counterpart. In this sense, our model indicates that globalization may enhance the possibility of belief-driven economic fluctuations, if investment goods are nontradables. In Section 5.1 we present an intuitive implication of the difference in the indeterminacy conditions in Propositions 1 and 3.

### 4.5 Long-Run Wealth Distribution

In dynamic system (35) if the steady state is locally determinate (i.e. the linearized dynamic system has two stable roots), then the equilibrium path of \( p_t \) and \( p^*_t \) are uniquely expressed as functions of \( K_t \) and \( K^*_t \) on the two-dimensional stable manifold. When we denote the relation between the relative prices and capital stocks on the stable saddle path as \( p = \phi(K, K^*) \) and \( p^* = \phi^*(K, K^*) \), the behaviors of capital stocks on the saddle path are expressed as

\[
\dot{K} = y^1(K, \phi(K, K^*)) - \delta K, \\
\dot{K}^* = y^1(K^*, \phi^*(K, K^*)) - \delta K^*.
\]

These differential equations show that once the initial capital stocks, \( K_0 \) and \( K^*_0 \), are specified, the paths of \( \{K_t, K^*_t\}_{t=0}^{\infty} \) are uniquely determined. As a result, the paths of \( \{p_t, p^*_t, R_t\}_{t=0}^{\infty} \) are also uniquely given under the specified levels of \( K_0 \) and \( K^*_0 \). This means that when...
equilibrium determinacy holds, the left hand side of (36) that depends on the entire sequences of \( \{p_t, p_t^*, K_t, K^*_t\}_{t=0}^{\infty} \) is also determinate, so that \( \bar{m} \) has a unique value under the given initial levels of \( K_0, K^*_0, B_0 \) and \( B^*_0 \).

By contrast, if the converging path of (35) is indeterminate (that is, the linearly approximated dynamic system of (35) has three or four stable roots), then the given initial levels of \( K_0 \) and \( K^*_0 \) alone cannot pin down the equilibrium paths of \( p_t \) and \( p_t^* \). Therefore, the level of \( \bar{m} \) given by (36) becomes indeterminate as well. In this situation, an extrinsic shock that affects expectations of agents in the world market may alter the equilibrium path and, therefore, it changes the level of \( \bar{m} \).

Note that in the steady state it holds that \( \dot{B} = \dot{B}^* = 0 \) and \( R = \rho \). Remembering that \( C + C^* = 2y^2 (\hat{K}, \hat{p}) \) and \( C^* = \bar{m}C \), we find that the steady-state level of bond holdings in the home and foreign countries are respectively given by

\[
\hat{B} = \frac{C - y^2 (\hat{K}, \hat{p})}{\rho} = \frac{1 - \bar{m}}{\rho(1 + \bar{m})} y^2 (\hat{K}, \hat{p}), \tag{37a}
\]

\[
\hat{B}^* = \frac{C^* - y^2 (\hat{K}, \hat{p})}{\rho} = \frac{\bar{m} - 1}{\rho(1 + \bar{m})} y^2 (\hat{K}, \hat{p}). \tag{37b}
\]

The above expressions show that when \( \bar{m} \) is selected, the long-run asset position of each country is also determined. In the steady state the asset holdings in both countries are:

\[
\Omega = \hat{B} + \hat{p}\hat{K}, \quad \Omega^* = \hat{B}^* + \hat{p}\hat{K}.
\]

Thus the long-run wealth distribution between the two countries depends on \( \hat{B} \) and \( \hat{B}^* \). It is obvious that whether the home country becomes a creditor or a debtor in the long run depends solely on whether or not \( \bar{m} \) exceeds one. As (36) demonstrates, if the equilibrium path is determinate and if the initial stocks of capital and bonds held by the home households are relatively large, then the home country tends to be a creditor in the long-run equilibrium. However, if there is a continuum of covering path around the steady state, the value of \( \bar{m} \) is affected by the expectations formation of agents. This implies that in the presence of equilibrium indeterminacy, the initial holding of wealth in each country alone does not determine the asset position of each country in the long-run equilibrium.
To sum up, we have shown:

**Proposition 4** If the steady-state equilibrium of the world economy is locally determinate (indeterminate), then the steady-state level of asset position of each country is determinate (indeterminate).

### 4.6 Non-Tradable Consumption Goods

Now consider the opposite situation where the consumption goods are not internationally traded, but the investment goods are tradable and financial capital mobility is possible. In this case the commodity market equilibrium conditions are given by

\[ I + I^* = Y_1 + Y_1^*, \quad C = Y_2, \quad C^* = Y_2^*. \]  \hspace{1cm} (38)

We take the tradable investment good as a numeraire. Then the net wealth (in terms of investment good) held by the households in the home country is \( \Omega = B + K \) and the flow budget constraint is written as

\[ \dot{B} = R (B + K) + w + \pi_1 + \pi_2 - \tilde{p}C - I. \]

where \( \tilde{p} (= 1/p) \) denotes the domestic price of consumption good in terms of tradable investment good. The Hamiltonian function for the households in the home country is given by

\[ H = C^{1-\sigma} - 1 \frac{1}{1 - \sigma} + \lambda \left[ RB + rK + w + \pi_1 + \pi_2 - \tilde{p}C - I \right] + q (I - \delta K) \]

and the key first-order conditions for an optimum are:

\[ C^{-\sigma} = \lambda \tilde{p}, \]  \hspace{1cm} (39a)

\[ \lambda = q \]  \hspace{1cm} (39b)

\[ \dot{\lambda} = \lambda (\rho - R), \]  \hspace{1cm} (39c)

\[ \dot{q} = q (\rho + \delta - r). \]  \hspace{1cm} (39d)

Conditions (39b), (39c) and (39d) lead to \( R = r - \delta \).
Since households in both country face the common interest rate, $R$ in the international bond market, the rate of return to capital in both countries satisfy

$$r^* = R + \delta = r \quad (40)$$

Therefore, $r(1/p) = r(1/p^*)$ holds in each moment, implying that $\tilde{p}$ always equals $p^{*17}$. Therefore, it holds that $k_i(1/p) = k_i(1/p^*) \quad (i = 1, 2, \ldots)$, implying that the world-market equilibrium condition of investment good yields the dynamic equation of the aggregate capital exactly the same as (20). In addition, from the equilibrium condition for consumption goods in each country in (38) we obtain

$$C = y^2(K, \tilde{p}), \quad C^* = y^2(K^*, \tilde{p}).$$

In this case it holds that $\lambda^*/\lambda = q^*/q^*$ stay constant over time. Therefore, we obtain $C^* = \bar{s}C$, where $\bar{s} = (\lambda^*/\lambda)^{-1/\sigma}$. This leads to

$$(1 + \bar{s}) C = y^2(K, \tilde{p}) + y^2(K^*, \tilde{p}).$$

Consequently, the dynamic system of the world economy is the same as that of the Nishimura-Shimomura model.

**Proposition 5** If consumption goods are not traded and financial capital mobility is possible, the indeterminacy conditions are the same as those for the case where both goods are traded without financial capital mobility.

Therefore, in this case opening up international trade does not enhance the possibility of belief-driven business cycles. An intuitive implication of this result is as follows. If only investment goods are tradable, holding of a unit of bond is equivalent to holding a claim to the future capital good. Since bonds and capital are perfect substitutes, holding a unit of bond should yield the same rate of return a unit of capital yields. Thus the interest rate of foreign bond equals the net rate of return to capital. The interest rate in the integrated

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17Since $\tilde{p} = 1/p$, the precise expression of $\bar{r}(1/p)$ is $\bar{r}(1p) = r / p = p a_1 A_1 k_1 \left( \frac{1}{\bar{p}} \right)^{\alpha-1}$. 

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financial market is common for both countries, which means that the rate of return to capital in both country is the same. Because of symmetric technology, this means that the relative price each country is also the same, so that the integrated world economy behaves exactly the same manner as that of the economy in the Heckscher-Ohlin environment.

When only consumption goods are internationally traded, one unit of bond is a claim to the future consumption good. Hence, the non-arbitrage condition between holding of bond and capital shows that the rate of return to capital diverges from the world interest rate when the relative price between consumption and investment changes. Hence, the factor prices (so that relative price between two goods) in each country are not identical during the transition. The failure of factor-price equalization makes the system with non-traded investment goods diverge form the Heckscher-Ohlin setting.

5 Discussion

5.1 Implication of the Indeterminacy Conditions

Intuition behind the difference in indeterminacy condition between Propositions 1 and 3 is as follows:

(i) Free Trade of Commodities

First consider the case where both consumption and investment goods are traded in the absence of international lending and borrowing. Suppose that a positive sunspot shock hits the world economy and all the households in the world expect that the rate of return to their capital will rise in the future. If the intertemporal elasticity of substitution in consumption, $1/\sigma$, is sufficiently high, such an impact makes the households reduce their current consumption and invest more. In addition, a high elasticity of intertemporal substitution effect means that there is a large increase in the future consumption. Meanwhile, the households expand their current investment and the world-wide capital stock will rise. Since we have assumed that consumption good sectors are more capital intensive than the investment good sectors in both countries, a higher $K_w$ will expand the consumption production in both countries through the Rybczynski effect. However, the strong intertemporal substitution effect yields
a large increase in consumption and, hence, the relative price $p$ must increase equilibrate the world-wide consumption good market. This leads to a further expansion of consumption good production. (Remember that from (10b) under our assumptions of $\frac{\alpha_1}{b_1} - \frac{\alpha_2}{b_2} < 0$ and $\alpha_1 - \alpha_2 > 0$, a higher $p$ increases $Y_2$ and $Y_2^*$.) Noting that a rise in $p$ increases $\tilde{r}(p)$ under $\alpha_1 - \alpha_2 > 0$, a higher $p$ actually raises the rate of return to capital, so that the initial expectations can be self-fulfilled.

By contrast, if $1/\sigma$ is not high enough, the above mechanism of adjustment will not work. If $1/\sigma$ is small, the intertemporal substitution effect is weak, so that expected rise in the future rate of return produce a relatively small amount of increase in the future consumption. If this is the case, an increase in consumption good production generated by a rise in $K$ through the Rybczynski effect may exceed the increase in consumption demand. As a result, the relative price will decline to curtail the production level of consumption goods to meet the relatively small increase in demand. Hence, in contrast to the case with a high $1/\sigma$, a lower $p$ reduces $\tilde{r}(p)$. This means that the initial expectations are not self fulfilled, and thus the equilibrium path of the world economy is determinate.

(ii) Nontradable Investment Goods

Next, consider the case where only consumption goods are traded and international lending and borrowing are allowed. In this case, the relative price in each country is not the same during the transition. Suppose that households in the home country expect that the rate of return to their capital will rise. As before, the households intend to raise their saving to invest more. In the Heckscher-Ohlin environment, this requires that households reduce their current consumption, and thus the magnitude of $\sigma$ plays an important role. However, in the presence of international financial market, the households in the home country may increase their investment by borrowing from foreign households rather than by lowering their current consumption. Hence, investment demand demand will increase even if $\sigma$ is not small. Then the households in the home country pay their debt by exporting consumption goods to the foreign country. Hence, the consumption good production in the home country will expand. This means that the relative price $p$ may increase to complement the positive effect of a higher $K$ on consumption good production. If this is the case, the rate of return to capital in the home country actually rise to fulfill the initial expectations of the households.
The above intuition is confirmed more clearly in the small-open economy model explored by Meng and Velasco (2004) who also assume that investment goods are not traded but international lending and borrowing are possible. Since the world interest rate is given for a small country, the dynamic behavior of the small-open economy are described by

\[
\begin{align*}
\dot{K} &= y^1(K, p) - \delta K, \\
\dot{q} &= q \left[ \bar{R} + \delta - \tilde{r}(p) \right],
\end{align*}
\]

(41)

where \( \bar{R} \) denotes a given world interest rate. Since the shadow value of bond follows \( \lambda = \lambda (\delta + \rho - \bar{R}) \), it is assumed that \( \delta + \rho = \bar{R} \) to keep \( \lambda \) at a finite level. As a result, \( \lambda \) is constant over time and thus from \( q = \lambda p \), it holds that \( \dot{q}/q = \dot{\lambda}/\lambda \) for all \( t \geq 0 \). This means that the second differential equation in (41) can be replaced with

\[
\dot{p} = p \left[ \bar{R} + \delta - \tilde{r}(p) \right].
\]

Since the behavior of consumption does not affect dynamics of \( K \) and \( p \) given above, in this case there is no direct link between the current levels of savings and consumption, so that the dynamic behavior of the economy is independent of the intertemporal substitutability in consumption. Meng and Velasco (2003 and 2004.) show that the factor-ranking conditions given in Proposition 3 are the necessary and sufficient conditions for indeterminacy. In our model of the global economy, the world interest rate is an endogenous variable. This is why the factor-ranking conditions are sufficient but not necessary for indeterminacy in our model. Despite such a difference, since we have focused on the local behavior of the world economy around the symmetric steady state where both countries hold the same level of capital, our indeterminacy conditions are closed to those for the small-open economy with non-tradable investment goods.

5.2 Empirical Plausibility of the Basic Assumptions

(i) Distinction between Traded and Nontraded Goods

In this paper we have considered three types of trade structures: (i) both investment and consumption goods are internationally traded; (ii) only consumption goods are traded, and; (iii) only investment goods are traded. In reality considerable portions of both consumption
and investment goods are traded in the domestic markets alone. For example, Ceurdacier (2009) claims that more than 50% of US consumption goods are not traded in the international markets, because value added of services, which is a primary item in the nontradable consumption goods, shares 55% of the aggregate value of consumption goods. Similarly, Baxter et al. (1998), Jin (2001), and Stockman and Tesar (1995) point out that more than 50% of consumption goods are not internationally traded in the US.

As for investment goods, Bems (2008) finds that the share of investment expenditure on nontraded goods is about 60% and that this figure has been considerably stable over the last 50 years both in developed and developing countries. Since construction and structures share a large part of investment goods, Bems’ finding seems to be a plausible one.

Judging from those empirical facts, the traditional assumption of free trade of all commodities (trade structure (i)) is far from the reality. At the same time, it is rather hard to determine which of trade structures (ii) or (iii) is more realistic. Probably, it is safe to conclude that both (ii) and (iii) have roughly the same distance from the reality. However, from the theoretical viewpoint, the key condition for the relation between openness of an economy and belief-driven fluctuations is whether or not investment goods are freely traded. As we have seen in Section 4.6, if investment goods are tradables, the indeterminacy conditions do not diverge from those for the case of free intratemporal trade of both commodities. It is worth pointing out that such a divergence may emerge even though only a part of investment goods are not internationally traded. In fact, Meng and Velasco (2004) extend their base model to consider the situation in which there are traded as well as nontraded investment goods. They find that their main conclusion still holds in this generalized framework. In view of their analysis, the main conclusion of our paper would be established, even if a part of investment goods are traded in the international market.

(ii) Externalities and Factor Intensity Ranking

The indeterminacy conditions in Proposition 1 and 3 mean that constant returns prevail in each production sector at the social level and that the external effects associated with capital would be relatively larger in the investment good sector than in the consumption good sector.\footnote{In our notation, external effects associated with capital and labor in the \( i \)-th sector are respectively given by}
conditions are empirically plausible ones. For example, the well-cited study by Basu and Fernald (1997) finds that most industries in the US approximately exhibit constant returns to scale, which may support our assumption of social constant returns. Using the US data, Harrison (2003) claims that returns to scale of the consumption goods sector are close to be constant, while the investment goods sector exhibits weak increasing returns. In addition, she reveals that external effects may be larger in the investment good sector than in the consumption good sector.

As to the factor-intensity ranking between the two sectors, a recent study by Takahashi et al. (2012) find that in most of the OECD countries, the consumption good sector uses a more capital intensive technology than the investment good sector.\textsuperscript{19} They also show that the gap in capital intensities between the two production sectors is generally small. If there is a large difference in factor intensities between the two sectors, it is hard to hold the factor-intensity reversal between the social and private technologies, unless there are large degree of external effects in the investment good sector. Hence, according to Takahashi et al. (2012), the factor-intensity ranking of private technologies can be different from that of social technologies even in the presence of small degrees of externalities. Although the empirical studies cited above do not directly support our assumptions, they indicate that the indeterminacy conditions given in Proposition 3 can hold under a set of empirically plausible magnitudes of parameter values involved in our model.

6 Final Remarks

The world economy as a whole is a closed economy in which there are multiple countries. Therefore, its model structure is similar to that of a closed, single economy model with heterogeneous agents. In particular, if consumption and saving decisions are made by the \varepsilon_i = \alpha_i - \alpha_i and \eta_i = 1 - \alpha_i - b_i \ (i = 1, 2). The factor-intensity ranking conditions in Proposition 3 mean that

\[
\frac{\alpha_1}{b_1} < \frac{\alpha_2}{b_2} \text{ and } \frac{\alpha_1 + \varepsilon_1}{b_1 + \eta_1} > \frac{\alpha_2 + \varepsilon_2}{b_2 + \eta_2}.
\]

Thus, provided that the sectoral difference in labor externalities are relatively small (so \eta_1 - \eta_2 is small), the above conditions requires that \varepsilon_1 is higher than \varepsilon_2.

\textsuperscript{19}In their estimation, Takahashi et al. (2012) do not assume the presence of production externalities. This means that their finding would support our assumption in Proposition 3, that is, the consumption good sector employs more capital intensive technology than the investment good sector from the private perspective.
representative household in each country, the world economy model is closely connected to the closed economy model with heterogeneous households. There is, however, an important difference between the world economy models and the single country setting: when dealing with the world economy model, we should specify the trade structure between the countries. This paper has revealed that the specification of trade structure plays an important role as to the presence of equilibrium indeterminacy, even if there is no international heterogeneity in technologies and preferences.

Recently, several authors have explored how the presence of heterogeneous preferences and technologies alter the determinacy/indeterminacy conditions in the equilibrium business cycle models with market distortions. These studies have shown that the heterogeneity in preferences and technologies often affects stability condition in a critical manner. In a similar vein, Sim and Ho (2007) find that introducing technological heterogeneity into the Nishimura-Shimomura model may produce a substantial change in equilibrium indeterminacy results. In addition, even if taste and technologies are identical in both countries, introducing financial frictions, policy distortions and adjustment costs of investment also breaks the symmetry between the home and foreign countries at least during the transition process. It is worth extending our model by considering further heterogeneity between the countries.

Appendices

Appendix 1: Proof of Proposition 1.

When \( \dot{q} = \dot{q}^* = 0 \) in (28a) and (28b), it holds that

\[
a_1 A_1 k_1 (p)^{\alpha_1 - 1} = a_1 A_1 k_1 (p^*)^{\alpha_1 - 1} = \rho + \delta.
\]

Thus by use of (3a) we find:

\[
p = p^* = \left( \frac{A_2}{A_1} \right) \left( \frac{a_2}{a_1} \right)^{\alpha_2} \left( \frac{b_2}{b_1} \right)^{1-\alpha_2} \left( \frac{\rho + \delta}{a_1 A_1} \right)^{\frac{\alpha_2 - \alpha_1}{\alpha_1 - 1}}.
\]

\[\text{20} \text{See, for example, Ghiglino and Olszak-Duquenne (2005).}\]

\[\text{21} \text{Antras and Caballero (2009) introduce financial frictions into the two-county Heckscher-Ohlin model. Ono and Shibata (2010) and Jin (2011) introduce adjustment costs of investment into } 2 \times 2 \times 2 \text{ models.}\]
These conditions show that the steady-state levels of $p$ and $p^*$ are uniquely given and it holds that $p = p^*$ in the steady state. The steady-state levels of capital stocks satisfying $\dot{K} = \dot{K}^* = 0$ in (27a) and (27b) are determined by the following conditions:

$$
\frac{K - k_2(p)}{k_1(p) - k_2(p)} A_1 k_1(p)^{\alpha_1} = \delta K,
$$

$$
\frac{K^* - k_2(p^*)}{k_1(p^*) - k_2(p^*)} A_1 k_1(p^*)^{\alpha_1} = \delta K^*.
$$

Using the conditions for $\dot{p} = \dot{p}^* = 0$ and the fact that $p = p^*$ holds in the steady state, we confirm that the steady-state level of capital stock in each county has the same value, which is given by

$$
K = K^* = \frac{(aA_1)^{1-\alpha_1} (\rho + \delta)^{\alpha_1-1}}{\rho + \delta (1 - \delta + \frac{a_2b_1}{a_1b_2})} \left( \frac{a_2b_1}{a_1b_2} \right),
$$

which has a positive value. We also find that the steady-state values of labor allocation to the investment good sector are:

$$
L_1 = L_1^* = \frac{a_1 \delta \left( \frac{a_2b_1}{a_1b_2} \right)}{\rho + (1 - a_1) \delta + a_1 \delta \left( \frac{a_2b_1}{a_1b_2} \right)} \in (0, 1).
$$

Hence, (8) is fulfilled so that both countries imperfectly specialize.

**Appendix 2: Proof of Proposition 2**

Since the functional form of $R(K, K^*, p, p^*)$ in (35) is complicated, it is simpler to treat a dynamic system with respect to $K, K^*, q$ and $q^*$ displayed in Section 4.1. We thus focus on the dynamics system consisting of (27a), (27b), (28a) and (28b) with $p = \pi(K, K^*, q, q^*; \bar{m})$ and $p^* = \pi^*(K, K^*, q, q^*; \bar{m})$, where $\bar{m}$ is fixed.\(^22\)

To prove Proposition 2, the following facts are useful:

**Lemma 1** *In the symmetric steady state where $K = K^*$ and $q = q^*$, the following relations hold.*

\(^{22}\)When the dynamic system of $(K, K^*, q, q^*)$ satisfies equilibrium determinacy under a given level of $\bar{m}$, then the equilibrium paths of $K$ and $K^*$ are uniquely determined under given levels of $K_0$ and $K_0^*$. Therefore, equilibrium path of (35) is also uniquely determined. Conversely, if the dynamic system of $(K, K^*, q, q^*)$ exhibits local indeterminacy, the equilibrium paths of $K$ and $K^*$ cannot be uniquely determined by selecting $K_0$ and $K_0^*$. This means that (35) also holds equilibrium indeterminacy.
are satisfied:

\[ y'_K(K, p) = y'_{K^*}(K^*, p^*), \quad i = 1, 2, \]
\[ y'_p(K, p) = y'_{p^*}(K^*, p^*), \quad i = 1, 2, \]

\[ \pi_K(K, K^*, q, q^*) = \pi^*_{K}(K, K^*, q, q^*) = \pi^*_{K^*}(K, K^*, q, q^*), \]
\[ \pi_q(K, K^*, q, q^*) = \pi^*_{q}(K, K^*, q, q^*), \]
\[ \pi_{q^*}(K, K^*, q, q^*) = \pi^*_{q}(K, K^*, q, q^*). \]

**Proof.** By the functional forms of \( y'_i(\cdot) \) \((i = 1, 2, j = K, K^*, p, p^*)\), it is easy to see that \( y'_K(K, p) = y'_{K^*}(K^*, p^*) \) and \( y'_p(K, p) = y'_{p^*}(K^*, p^*) \) are established when \( p = p^* \) and \( K = K^* \). As for the rest of the results, we use \( \frac{\partial p}{\partial q} = q \) and \( p^* \lambda(\cdot) \tilde{m} = q^* \) total differentiation of \( \frac{\partial p}{\partial \tilde{m}} = q \) and \( p^* \lambda(\cdot) \tilde{m} = q^* \) yields the following:

\[
\frac{\partial p}{\partial K} = \pi_K = -\frac{p\lambda_K}{\lambda + p\lambda_P + p^*\lambda_{p^*}}, \quad \frac{\partial p}{\partial K^*} = \pi_{K^*} = -\frac{p\lambda_{K^*}}{\lambda + p\lambda_P + p^*\lambda_{p^*}}, \tag{A1}
\]

\[
\frac{\partial p^*}{\partial K} = \pi^*_{K} = -\frac{p^*\lambda_K}{\lambda + p\lambda_P + p^*\lambda_{p^*}}, \quad \frac{\partial p^*}{\partial K^*} = \pi^*_{K^*} = -\frac{p^*\lambda_{K^*}}{\lambda + p\lambda_P + p^*\lambda_{p^*}}, \tag{A2}
\]

\[
\frac{\partial p}{\partial q} = \pi_q = \frac{\lambda + p^*\lambda_{p^*}}{\lambda(\lambda + p\lambda_P + p^*\lambda_{p^*})}, \quad \frac{\partial p^*}{\partial q} = \pi^*_{q} = -\frac{p\lambda_{p}}{\lambda(\lambda + p\lambda_P + p^*\lambda_{p^*})}, \tag{A3}
\]

\[
\frac{\partial p^*}{\partial q} = \pi^*_{q} = -\frac{p^*\lambda_{p^*}}{\lambda + p\lambda_P + p^*\lambda_{p^*}}, \quad \frac{\partial p^*}{\partial q^*} = \pi^*_{q^*} = \frac{\lambda + p\lambda_P}{\lambda + p\lambda_P + p^*\lambda_{p^*}}. \tag{A4}
\]

Since \( \lambda_K(\cdot) = \lambda_{K^*}(\cdot) \) and \( \lambda_p(\cdot) = \lambda_{p^*}(\cdot) \) in the steady state where \( K = K^* \) and \( p = p^* \), we obtain \( \pi_K = \pi^*_{K} = \pi_{K^*} = \pi^*_{K^*}, \pi_q = \pi^*_{q} \) and \( \pi_{q^*} = \pi^*_{q^*} \).

Under a given level of \( \tilde{m} \), let us linealize the dynamic system of (27a), (27b), (28a) and (28b) at the steady state. The coefficient matrix of the linealized system is given by

\[
J = \begin{bmatrix}
    y_{11}^K - \delta + y_{11}^p \pi_K & y_{11}^p \pi_{K^*} & y_{11}^1 \pi_{q} & y_{11}^1 \pi_{q^*} \\
    y_{11}^p \pi_K & y_{11}^p \pi_{K^*} - \delta + y_{11}^p \pi_{K^*} & y_{11}^1 \pi_{q} & y_{11}^1 \pi_{q^*} \\
    -q^2 \pi_K & -q^2 \pi_{K^*} & -q^2 \pi_{q} & -q^2 \pi_{q^*} \\
    -q^2 \pi_K & -q^2 \pi_{K^*} & -q^2 \pi_{q} & -q^2 \pi_{q^*}
\end{bmatrix}.
\]
By use of Lemma 1, we see that the characteristic equation of $J$ is written as

$$
\Gamma (\eta) = \det [\eta I - J] = \det \begin{bmatrix}
\eta - (y_1^1 - \delta + y_p^1 \pi K) & -y_p^1 \pi K & -y_p^1 \pi^* \\
-y_p^1 \pi K & \eta - (y_1^1 - \delta + y_p^1 \pi K) & -y_p^1 \pi^* \\
y^* & q^* & \eta + q^* \\
y^* & q^* & \eta + q^* \\
y^* & q^* & \eta + q^*
\end{bmatrix} = \det \begin{bmatrix}
\eta - (y_1^1 - \delta) & 0 & \eta & 0 \\
0 & \eta - (y_1^1 - \delta) & 0 & \eta \\
y^* & q^* & \eta + q^* & q^* \\
y^* & q^* & \eta + q^* & q^*
\end{bmatrix} = [\eta - (y_1^1 - \delta)] [\eta + q^*(\pi_q + \pi_q^*)] \xi (\eta).
$$

where $\eta$ denotes the characteristic root of $J$ and

$$\xi (\eta) \equiv \eta^2 + [q^* (\pi_q + \pi_q^*) - (y_1^1 - \delta) - 2y_p^1 \pi K] \eta - q^* (y_1^1 - \delta) (\pi_q + \pi_q^*).$$

Our assumptions mean that $\frac{a_1}{b_1} - \frac{a_2}{b_2} < 0$ and $\alpha_1 - \alpha_2 > 0$. Thus from (10a) we see that $y_1^1 - \delta < 0$. In addition, the equations in (A3) mean that $\pi_q - \pi_q^* = 1/\lambda (> 0)$. Hence, using $\hat{r} (p) \equiv a_1 A_1 k_1 (p)^{a_1-1}$, we obtain:

$$\hat{r}^* (\pi_q - \pi_q^*) = a_1 (a_1 - 1) A_1 (k_1 (p))^{a_1-2} \frac{k_1^* (p)}{\lambda} > 0.$$  

As a consequence, at least two roots of $\Gamma (\eta) = 0$ have negative real parts. The equations in (A3) also show

$$\pi_q + \pi_q^* = \frac{1}{\lambda + 2p\lambda^*_p},$$

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where

\[
\lambda_p = \frac{\partial}{\partial p} (1 + \bar{m})^{\frac{1}{2}} [y^2 (K, p) + y^2 (K^*, p^*)]^{-\frac{1}{2}} = -\frac{y_p^2}{\sigma} (1 + \bar{m})^{\frac{1}{2}} [y^2 (K, p) + y^2 (K^*, p^*)]^{-\frac{1}{2}} < 0.
\]

Therefore, in the steady state equilibrium, the following holds:

\[
\lambda + 2p\lambda_p = \frac{1}{\sigma} \left[ \sigma - \frac{py_p^2 (K, p)}{y^2 (K, p)} \right].
\]

Notice that under our assumptions, it holds that \( y_p^2 (K, p) > 0 \). Suppose that \( \sigma \) is small enough to satisfy \( \sigma < py_p^2 / y^2 \). Then \( \lambda_p + 2p\lambda_p > 0 \) so that \( \pi_q + \pi_{q^*} < 0 \), which leads to

\[-q^\prime \left( y_K^1 - \delta \right) \left( \pi_q + \pi_{q^*} \right) < 0.\]

This means that \( \xi (\eta) = 0 \) has one positive and one negative roots. As a result, \( \Gamma (\eta) = 0 \) has three stable roots. Hence, if \( \sigma \) is smaller than the price elasticity of supply function of consumption goods, then there locally exists a continuum of equilibrium paths converging to the steady state.

Now suppose that \( \sigma \) is larger than \( py_p^2 / y^2 \). Then we obtain \( \pi_q + \pi_{q^*} > 0 \). Furthermore, it holds that

\[-2y_p^1 \pi_K = -2y_p^1 \left( -\frac{p\lambda_K}{\lambda + 2p\lambda_p} \right) = -\frac{2py_p^1}{\lambda + 2p\lambda_p} y_K^2 \left[ (1 + \bar{m})^{\sigma - 1} \right] \left( 2y^2 \right)^{-\sigma - 1} > 0,\]

because \( y_p^1 < 0 \) and \( y_K^2 > 0 \) under our assumptions. Consequently, the following inequalities are established:

\[-q^\prime \left( y_K^1 - \delta \right) \left( \pi_q + \pi_{q^*} \right) > 0, \]

\[q^\prime \left( \pi_q + \pi_{q^*} \right) - \left( y_K^1 - \delta \right) - 2y_K^1 \pi_K > 0.\]

These conditions mean that \( \xi (\eta) = 0 \) has two roots with negative real parts and, hence, all
the roots of $\Gamma(\eta) = 0$ are stable ones. In sum, if $\frac{a_1}{b_1} - \frac{a_2}{b_2} < 0$ and $\alpha_1 - \alpha_2 > 0$, then the characteristic equation of the linearized system involves at least three stable roots, regardless of the value of $\sigma$. 


References


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