Cointegrated TFP Processes and International Business Cycles*

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Abstract

A puzzle in international macroeconomics is that observed real exchange rates are highly volatile. Standard international real business cycle (IRBC) models cannot reproduce this fact. We show that TFP processes for the U.S. and the "rest of the world" is characterized by a vector error correction (VECM) and that adding cointegrated technology shocks to the standard IRBC model helps explaining the observed high real exchange rate volatility. Also, we show that the observed increase of the real exchange rate volatility with respect to output in the last 20 years can be explained by changes in the parameter of the VECM.

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1. Introduction

A central puzzle in international macroeconomics is that observed real exchange rates (RERs) are highly volatile. Standard international real business cycle (IRBC) models cannot reproduce this fact when calibrated using conventional parameterizations. For instance, Heathcote and Perri (2002) simulate a two-country, two-good economy with total factor productivity (TFP) shocks and find that the model can explain only less than a fourth of the observed relative volatility of the RER with respect to output for the United States (U.S.) data. An important feature of their model, following the seminal work of Backus, Kehoe, and Kydland (1992) and Baxter and Crucini (1995), is that it considers stationary TFP shocks that follow a vector autoregression (VAR) process in levels.\footnote{Other studies that consider a VAR in levels are: Kehoe and Perri (2002), Dotsey and Duarte (2009), Corsetti, Dedola and Leduc (2008a, 2008b), and Heathcote and Perri (2009).}

In this paper we provide evidence that TFP processes for the U.S. and the “rest of the world” (R.W.) have a unit root and are cointegrated. Motivated by this empirical finding, we introduce technology shocks that follow a vector error correction model (VECM) process into an otherwise standard two-country, two-goods model. Engle and Granger (1987), Engle and Yoo (1987), and LeSage (1990) indicate that if the system under study includes integrated variables and cointegrating relationships, then this system will be more appropriately specified as a VECM rather than a VAR in levels. As Engle and Granger (1987) note, estimating a VAR in levels for cointegrated systems leads to ignoring important constraints on the coefficient matrices. Although these constraints are satisfied asymptotically, small sample improvements are likely to result from imposing them on the cointegrating relationships. Failing to impose them affects the small sample estimates and the implied dynamics.

The presence of cointegrated TFP shocks requires restrictions on preferences, production functions, and the law of motion of the shocks in order to have balanced growth. The restrictions on preferences and technology of King, Plosser, and Rebelo (1988) are sufficient for the existence of balanced growth in a closed economy. However, in a two-country model, an additional restriction on the cointegrating vector related to the TFP processes is needed. In particular, we need the cointegrating vector to be \((1, -1)\), which means the ratio of TFP processes (or, equivalently, the log difference of TFP processes) across countries is stationary. After presenting evidence supporting this additional restriction, we show that the VECM specification for TFP processes
solves a large part of the RER volatility puzzle without affecting the good match for other moments of domestic and international variables. In particular, we show that our model can generate a relative volatility of the RER more than two times larger than an equivalent model with stationary shocks calibrated as in Heathcote and Perri (2002).

Why does a model with cointegrated TFP shocks generate higher relative volatility of the RER than a model with stationary shocks? The reason is that the VECM parameter estimates imply higher persistence and lower spillovers than the traditional stationary calibrations, which translates into higher persistence of the TFP differential across countries. This is the crucial feature of the model that helps explain the relative volatility of the RER with respect to output. The mechanism works as follows. As the persistence of the TFP differential decreases, home country households know that productivity will increase sooner in their country after a shock in the foreign country has occurred. Since we consider incomplete markets, because of a wealth effect, they supply less labor and capital, output drops, and output volatility increases. Hence, a higher persistence in TFP differentials leads to lower output volatility by attenuating the mentioned wealth effect.

This same effect leads to higher RER volatility. As the persistence of TFP differentials increases, in addition to supplying less capital and labor, home country households demand more consumption goods (which need to be produced using intermediate foreign goods) after a shock in the foreign country has occurred. Provided that the elasticity of substitution between home and foreign intermediate goods is low enough, this increase in the demand for intermediate foreign goods leads to a larger increase in the foreign price level, a larger RER depreciation and higher RER volatility.

Another very well-documented empirical fact is the substantial decline in the volatility of most U.S. macroeconomic variables during the last 20 years. That change in the cyclical volatility is known as the “Great Moderation.”\textsuperscript{2} In this paper, we report that the Great Moderation has not affected the RER as strongly as it has affected output. As a result, the ratio of RER volatility to output volatility has increased. We also show that the increase in the relative volatility of the RER of the U.S. dollar coincides in time with a weakening of the cointegrating relationship of

\textsuperscript{2}Some early discussion of the Great Moderation can be found in Kim and Nelson (1999). A discussion of different interpretations for this phenomenon and some international evidence can be found in Stock and Watson (2003) and Stock and Watson (2005), respectively.
TFP shocks between the U.S. and the R.W. More important, we confirm that if we allow for a fading in the cointegrating relationship of the size estimated in the data, the model can jointly account for the observed increase in the relative volatility of the RER and the substantial decline in the volatility of output.

Baxter and Stockman (1989) showed that changes in the nominal exchange rate regime greatly affected volatility of the RER but had almost no effect on output volatility and other macroeconomic variables. This empirical fact is an important problem for IRBC models, since they assume a strong relationship between RER and output volatilities. Since our model is based on the standard IRBC framework, it suffers from the same limitation. To minimize the effect of this valid criticism, and since we cannot study the impact of changes in the nominal exchange rate regime, we focus our analysis on the post Bretton-Woods period of flexible nominal exchange rates.

Our paper relates to two important strands of the literature. On the one hand, it connects with the literature stressing the importance of stochastic trends to explain economic fluctuations. King, Plosser, Stock, and Watson (1991) find that a common stochastic trend explains the co-movements of main U.S. real macroeconomic variables. Lastrapes (1992) reports that fluctuations in real and nominal exchanges rates are due primarily to permanent real shocks. Engel and West (2005) show that RERs manifest near–random-walk behavior if TFP processes are random walks and the discount factor is near one, while Nason and Rogers (2008) generalize this hypothesis to a larger class of models. Aguiar and Gopinath (2007) show that trend shocks are the primary source of fluctuations in emerging economies. Alvarez and Jermann (2005) and Corsetti, Dedola, and Leduc (2008a) highlight the importance of persistent disturbances to explain asset prices and RER fluctuations, respectively. Also, Lubik and Schorfheide (2006) and Rabanal and Tuesta (2010) introduce random walk TFP shocks to explain international fluctuations and Justiniano and Preston (2010) suggest that, in order to explain the comovement between Canadian and U.S. main macroeconomic variables, it is important to introduce correlations between the innovations of several structural shocks. However, these papers do not formalize a VECM, test for cointegration, or estimate the cointegrating vector.

On the other hand, our paper also links to the literature analyzing different mechanisms to understand RER fluctuations. Some recent papers study the effects of monetary shocks and nominal rigidities. Chari, Kehoe, and McGrattan (2002) are able to explain RER volatility in

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3In section 4 we describe the set of countries that compose our definition of R.W.
a monetary model with sticky prices and a high degree of risk aversion. Benigno (2004) focuses on the role of interest rate inertia and asymmetric nominal rigidities across countries. Other papers use either non-traded goods, pricing to market, or some form of distribution costs (see Corsetti, Dedola, and Leduc 2008a, 2008b; Benigno and Thoenissen 2008, and Dotsey and Duarte 2009). Our model includes only tradable goods with home bias, which is the only source of RER fluctuations. Our choice is guided by evidence that the relative price of tradable goods has large and persistent fluctuations that explain most of the RER volatility (see Engel 1993 and 1999). Fluctuations of the relative price of non-traded goods accounts for, at most, one-third of the RER volatility (see Betts and Kehoe 2006, Burstein, Eichenbaum, and Rebelo 2006, and Rabanal and Tuesta 2007). In any case, this choice causes an empirical problem. Our measure of RER is based on the consumer price indices (CPIs) that include non-traded goods, while our model does not have a non-traded goods sector. To reduce the gap between model and data, we also use two alternative measures of RER. The first uses Producer Price Indices (PPIs) and the second uses export deflators. Of course, the two series still maintain some gap between theory and measurement, but the role of non-traded goods is reduced. Using these two other measures, our results do not change.

The rest of the paper is organized as follows. Section 2 documents the increase in the RER volatility with respect to output volatility for the U.S. In Section 3 we present the model with cointegrated TFP shocks. In Section 4 we report estimates for the law of motion of the (log) TFP processes of the U.S. and the R.W. In Section 5 we present the main findings from simulating the model, leaving Section 6 for concluding remarks.

2. The Great Moderation and RER Volatility

In this section, we present evidence that in the period known as “the Great Moderation,” the relative volatility of the RER (measured as the real effective exchange rate) with respect to output (measured as real GDP) has increased in the U.S.\(^4\) We construct the real effective exchange rate as a geometric average of bilateral CPI-based RERs with respect to the Euro area, Japan, Canada, the United Kingdom and Australia. We use the same weights that the Federal Reserve uses to construct the real effective exchange rate of the U.S. dollar. These countries represented 69

\(^4\)Similar behavior can be observed for the United Kingdom, Canada, and Australia. We do not present those graphs because of space considerations.
percent of the aggregate weight in 1973 and 46 percent in 2009. We picked these countries to be consistent with our definition of the R.W. later in the paper. We constructed this series from 1957:1 to 2010:1, proceeding in two steps. Between 1973:4 and 2010:1, we obtained nominal exchange rates with the U.S. dollar from the Federal Reserve and each country’s CPI through the IMF’s International Financial Statistics (IFS), except for the Euro area, where we obtain the CPI from the Area Wide Model (AWM) of the European Central Bank (ECB).

Before 1973, the Federal Reserve does not publish bilateral exchange rate data. In addition, the Federal Reserve weights start in 1973. Moreover, the AWM from which we obtain the Euro area CPI starts in 1970. Therefore, we extended the real effective exchange rate of the U.S. dollar between 1957:1 and 1973:3 as follows. We constructed a Euro area aggregate using nominal exchange rates against the U.S. dollar and CPI data for West Germany, France, Italy, Spain and the Netherlands from the IMF’s IFS. We aggregated these countries using the Federal Reserve’s weights in 1973 (the first year these weights are available). These five countries represented about 86 percent of Euro area trade with the U.S. in 1973. Then, we aggregated these countries together with the bilateral RERs of Japan, Canada, the United Kingdom and Australia using 1973 weights. We construct this series through 1973:4. Since both series overlap in 1973:4, we can use this date to normalize.

In Figure 1 we present the standard deviation of the HP-filtered output, the standard deviation of the HP-filtered RER, and the ratio of the two.\(^5\) We compute the standard deviations of rolling windows of 40 quarters using data from 1957:1 to 2010:1. Since we use rolling windows of 40 quarters, we plot data for volatilities only between 1966:4 and 2010:1. The figure shows a substantial decline in the volatility of output: from around 2 percent standard deviation until the mid 1990s to 1 percent after that date. This decline in output volatility is what is typically referred to as “the Great Moderation.” The volatility of the RER has experienced a different path: the standard deviation was at about 4 percent until the mid 1980s, it increased to values around 7 percent for the 1990s and again declined to values around 4 percent after that.

So what is the behavior of the ratio of volatilities between the two series? The ratio has increased in a non-monotonic way from around 1 percent to around 4 percent in the period we study. Hence, the volatility of the RER has multiplied by four relative to that of output. But this

\(^5\) Using the Federal Reserve’s index for the real effective exchange rate of the U.S. Dollar delivers similar results. The Fed’s measure of the RER of the U.S. Dollar is available from January 1973. The correlation between the HP-filtered RER constructed by the Fed and our measure is 0.94.
is not a formal test of a structural break. To perform such a test, we model the ratio of volatilities as an autoregressive (AR) process of order one with mean. Then, we test the stability of the mean parameter using the Quandt-Andrews unknown breakpoint test on the mean parameter (we trim 15 percent of the data). We use data on the ratio of volatilities from 1966:4 to 2010:1, since we use rolling windows of 40 quarters. The results are reported in Table 1 and clearly reject the null hypothesis of no breakpoints in the data: the exponential is well above the 5 percent critical value. Also, using the maximum likelihood ratio F-statistic, the date with the highest probability of a break is 1993:4.

<table>
<thead>
<tr>
<th>Method</th>
<th>F-statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Likelihood Ratio (1993:4)</td>
<td>15.36773</td>
<td>0.0021</td>
</tr>
<tr>
<td>Exponential Likelihood Ratio</td>
<td>4.344542</td>
<td>0.0018</td>
</tr>
</tbody>
</table>


In this paper we build a two-country, two-goods model that we calibrate using standard parameters of the IRBC literature and estimated parameters of a VECM using TFP processes for the U.S. and the R.W. After seeing the evidence presented in Figure 1, the reader may understand that we are going to match time-varying targets. That it is not the case. In Section 5 we show that if we estimate two VECMs, the first one with data until 1993:4 and the second with data after 1994:1, it is possible to explain a large fraction of the observed increase in the relative volatility of the RER with respect to output with changes in the estimated parameters of the VECM.

3. The Model

In this section, we present a standard two-country, two-goods IRBC model similar to the one described in Heathcote and Perri (2002). The main difference with respect to the standard IRBC literature is the definition of the stochastic processes for TFP. In that literature, the TFP processes of the two countries are assumed to be stationary or trend stationary in logs, and they are modelled as a VAR process. At this point it is important to mention that Baxter and Crucini (1995) was the first paper to consider permanent shocks and the possibility of cointegration in
the context of this class of models. The reason they did not pursue the VECM specification was that the evidence of cointegration was mixed for the bilateral pairs they studied. In this paper, we consider instead (log) TFP processes that are cointegrated of order C(1,1). This implies that (log) TFP processes are integrated of order one but a linear combination is stationary. According to the Granger representation theorem, our C(1,1) assumption is equivalent to defining a VECM for the law of motion of the log differences of the TFP processes. The VECM is defined in more detail in Section 3.2.3 Our cointegration assumption has strong and testable implications for the data. The empirical evidence supporting our assumption will be presented in Section 4.

In each country, a single final good is produced by a representative competitive firm that uses intermediate goods in the production process. These intermediate goods are imperfect substitutes for each other and can be purchased from representative competitive producers of intermediate goods in both countries. Intermediate goods producers use domestic capital and labor in the production process. The final good can be only domestically consumed or invested in by consumers. The stock of domestic capital can therefore be increased only by using the domestic final good, both in the home and foreign economies. Thus, all trade between countries occurs at the intermediate goods level. In addition, consumers trade across countries an uncontingent international riskless bond denominated in units of domestic intermediate goods. No other financial asset is available. In each period of time $t$, the economy experiences one of many finite events $s_t$. We denote by $s^t = (s_0, ..., s_t)$ the history of events up through period $t$. The probability, as of period 0, of any particular history $s^t$ is $\pi(s^t)$ and $s_0$ is given.

In the remainder of this section, we describe the households’ problem, the intermediate and final goods producers’ problems, and the VECM process. Then, we explain market clearing and equilibrium. Finally, we discuss the conditions for the existence of a balanced growth path and explain how to transform the variables in the model to achieve stationarity.

3.1. Households

In this subsection, we describe the decision problem faced by home-country households. The problem faced by foreign-country households is similar, and hence it is not presented. The

\[^6\text{See Engle and Granger (1987).}\]
representative household of the home country solves

$$\max_{\{C(s^t), L(s^t), X(s^t), K(s^t), D(s^t)\}} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi(s^t) \frac{C(s^t) [1 - L(s^t)]^{1-\tau}}{1 - \sigma}$$

subject to the following budget constraint

$$P(s^t) [C(s^t) + X(s^t)] + P_H(s^t) \overline{Q}(s^t) D(s^t) \leq$$

$$P(s^t) [W(s^t) L(s^t) + R(s^t) K(s^{t-1})] + P_H(s^t) \{D(s^{t-1}) - \Phi[D(s^t)]\}$$

and the law of motion for capital

$$K(s^t) = (1 - \delta) K(s^{t-1}) + X(s^t) .$$

The following notation is used: $\beta \in (0, 1)$ is the discount factor, $L(s^t) \in (0, 1)$ is the fraction of time allocated to work in the home country, $C(s^t) \geq 0$ are units of consumption of the final good, $X(s^t) \geq 0$ are units of investment, and $K(s^t) \geq 0$ is the capital level in the home country at the beginning of period $t + 1$. $P(s^t)$ is the price of the home final good, which will be defined below, $W(s^t)$ is the hourly wage in the home country, and $R(s^t)$ is the home-country rental rate of capital, where the prices of both factor inputs are measured in units of the final good. $P_H(s^t)$ is the price of the home intermediate good in the home country, $D(s^t)$ denotes the holdings of the internationally traded riskless bond that pays one unit of home intermediate good (minus a small cost of holding bonds, $\Phi(\cdot)$) in period $t + 1$ regardless of the state of nature, and $\overline{Q}(s^t)$ is its price, measured in units of the home intermediate good. Finally, the function $\Phi(\cdot)$ is the arbitrarily small cost of holding bonds measured in units of the home intermediate good.  

We assume, following the existing literature, that $\Phi(\cdot)$ takes the following functional form

$$\Phi[D(s^t)] = \frac{\phi}{2} A(s^{t-1}) \left[ \frac{D(s^t)}{A(s^{t-1})} \right]^2 .$$

Note that we need to include the level of TFP in the home country, $A(s^{t-1})$, in the adjustment

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7The $\Phi(\cdot)$ cost is introduced to ensure stationarity of the level of $D(s^t)$ in IRBC models with incomplete markets, as discussed by Heathcote and Perri (2002). We choose the cost to be numerically small, so it does not affect the dynamics of the rest of the variables.
cost function, both dividing $D(s^t)$ and multiplying $\left[ \frac{D(s^t)}{A(s^{t-1})} \right]^2$. The reason is that since $A(s^{t-1})$ is an integrated process, $D(s^t)$ will grow at the rate of growth of $A(s^{t-1})$ along the balanced growth path, making the ratio $\frac{D(s^t)}{A(s^{t-1})}$ stationary. Also, since all real variables in the home country will also grow at the rate of growth of $A(s^{t-1})$ along the balanced growth path, we need to make the adjustment cost (measured in units of the home intermediate good) also grow at the same rate in order to induce stationarity.

3.2. Firms

3.2.1. Final goods producers

The final good in the home country, $Y(s^t)$, is produced using home intermediate goods, $Y_H(s^t)$, and foreign intermediate goods, $Y_F(s^t)$, with the following technology:

$$Y(s^t) = \left[ \omega^{\frac{1}{\theta}} Y_H(s^t)^{\frac{\theta-1}{\theta}} + (1 - \omega)^{\frac{1}{\theta}} Y_F(s^t)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

(2)

where $\omega$ denotes the fraction of home intermediate goods that are used for the production of the home final good and $\theta$ controls the elasticity of substitution between home and foreign intermediate goods. Therefore, the representative final goods producer in the home country solves the following problem:

$$\max_{Y(s^t) \geq 0, Y_H(s^t) \geq 0, Y_F(s^t) \geq 0} \quad P(s^t) Y(s^t) - P_H(s^t) Y_H(s^t) - P_F(s^t) Y_F(s^t)$$

subject to the production function (2), where $P_H(s^t)$ is the price of the home intermediate good in the home country and $P_F(s^t)$ is the price of the foreign intermediate good in the home country.

3.2.2. Intermediate goods producers

The representative intermediate goods producer in the home country uses home labor and capital in order to produce home intermediate goods and sells her product to both the home and foreign final good producers. Taking prices of all goods and factor inputs as given, she maximizes profits. Hence, she solves:

$$\max_{L(s^t) \geq 0, K(s^{t-1}) \geq 0} \quad P_H(s^t) Y_H(s^t) + P_F(s^t) Y_F(s^t) - P(s^t) \left[ W(s^t) L(s^t) + R(s^t) K(s^{t-1}) \right]$$
subject to the production function

\[ Y_H (s^t) + Y'_H (s^t) = A (s^t)^{1-\alpha} K (s^{t-1})^\alpha L (s^t)^{1-\alpha} \]

where \( Y_H (s^t) \) is the amount of home intermediate goods sold to the home final goods producers, \( Y'_H (s^t) \) is the amount of home intermediate goods sold to the foreign final goods producers, \( A (s^t) \) is a stochastic process describing TFP of home intermediate goods producers, which we will characterize below, and \( P'_H (s^t) \) is the price of the home intermediate good in the foreign country.

### 3.2.3. The processes for TFP

As mentioned above, we depart from the standard assumption in the IRBC literature and consider processes for both \( \log A (s^t) \) and \( \log A^* (s^t) \) that are cointegrated of order \( C(1, 1) \). Equivalently, we specify the following VECM for the law of motion driving the log differences of TFP processes for both the home and the foreign country:

\[
\begin{pmatrix}
\Delta \log A (s^t) \\
\Delta \log A^* (s^t)
\end{pmatrix} = \begin{pmatrix}
c \\
c^*
\end{pmatrix} + \begin{pmatrix}
\kappa \\
\kappa^*
\end{pmatrix} \left[ \log A (s^{t-1}) - \gamma \log A^* (s^{t-1}) - \log \xi \right] + \begin{pmatrix}
\varepsilon^a (s^t) \\
\varepsilon^{a,*} (s^t)
\end{pmatrix}
\]

(3)

where \((1, -\gamma)\) is called the cointegrating vector, \(\xi\) is the constant in the cointegrating relationship, \(\varepsilon^a (s^t) \sim N (0, \sigma^e)\) and \(\varepsilon^* (s^t) \sim N (0, \sigma^{e,*})\), \(\varepsilon^a (s^t)\) and \(\varepsilon^* (s^t)\) can be correlated, \(\Delta\) is the first-difference operator.\(^8\)

This VECM representation implies that deviations of today’s log differences of TFP with respect to its mean value depend not only on lags of home and foreign log differences of TFP but also on a function of the ratio of lag home and foreign TFP, \(A (s^{t-1}) / [\xi A^* (s^{t-1})] \). Thus, if the ratio \(A (s^{t-1}) / [\xi A^* (s^{t-1})] \) is larger than its long-run value, then \(\kappa < 0\) and \(\kappa^* > 0\) will imply that \(\Delta \log A (s^t)\) would fall and \(\Delta \log A^* (s^t)\) would rise, driving both series toward their long-run equilibrium values. The VECM representation also implies that \(\Delta \log A (s^t)\), \(\Delta \log A^* (s^t)\), and \(\log A (s^{t-1}) - \gamma \log A^* (s^{t-1}) - \log \xi\) are stationary processes.

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\(^8\)Here we restrict ourselves to a VECM with zero lags. This assumption is motivated by the empirical results to be presented in Section 4, where no lags are significant.
3.3. Market Clearing

The model is closed with the following market clearing conditions in the final goods markets

\[ C(s^t) + X(s^t) = Y(s^t) \quad \text{and} \quad C^*(s^t) + X^*(s^t) = Y^*(s^t) \]

and in the bond market

\[ D(s^t) + D^*(s^t) = 0. \]

3.4. Equilibrium

3.4.1. Equilibrium definition

Now we are ready to define the equilibrium for this economy. Given our law of motion for (log) TFP shocks defined by (3), an equilibrium for this economy is a set of allocations for home consumers, \( C(s^t), L(s^t), X(s^t), K(s^t) \), and foreign consumers, \( C^*(s^t), L^*(s^t), X^*(s^t), K^*(s^t) \), and \( D^*(s^t) \), allocations for home and foreign intermediate goods producers, \( Y_H(s^t), Y_H^*(s^t), Y_F(s^t) \) and \( Y_F^*(s^t) \), allocations for home and foreign final goods producers, \( Y(s^t) \) and \( Y^*(s^t) \), intermediate goods prices \( P_H(s^t), P_H^*(s^t), P_F(s^t) \) and \( P_F^*(s^t) \), final goods prices \( P(s^t) \) and \( P^*(s^t) \), rental prices of labor and capital in the home and foreign country, \( W(s^t), R(s^t) \), \( W^*(s^t) \), and \( R^*(s^t) \) and the price of the bond \( Q(s^t) \) such that: (i) given prices, households’ allocations solve the households’ problem; (ii) given prices, intermediate goods producers’ allocations solve the intermediate goods producers’ problem; (iii) given prices, final goods producers allocations solve the final goods producers’ problem; (iv) and markets clear.

3.4.2. Equilibrium conditions

At this point, it is useful to define the following relative prices: \( \tilde{P}_H(s^t) = \frac{P_H(s^t)}{P(s^t)} \), \( \tilde{P}_H^*(s^t) = \frac{P_H^*(s^t)}{P^*(s^t)} \) and \( RER(s^t) = \frac{P_F^*(s^t)}{P^*(s^t)} \). Note that \( \tilde{P}_H(s^t) \) is the price of home intermediate goods in terms of home final goods, \( \tilde{P}_F^*(s^t) \) is the price of foreign intermediate goods in terms of foreign final goods, which appears in the foreign country’s budget constraint, and \( RER(s^t) \) is the RER between the home and foreign countries. We assume that the law of one price (LOP) holds; hence, we have that \( P_H(s^t) = P_H^*(s^t) \) and \( P_F(s^t) = P_F^*(s^t) \).

We now determine the equilibrium conditions implied by the first order conditions of households, intermediate and final goods producers in both countries, as well as the relevant laws of
motion, production functions, and market clearing conditions. The marginal utility of consumption and the labor supply are given by:

\[ U_C(s^t) = \lambda(s^t), \]  
\[ \frac{U_L(s^t)}{U_C(s^t)} = W(s^t), \]  

where \( U_x \) denotes the partial derivative of the utility function \( U \) with respect to variable \( x \). The first order condition with respect to capital delivers an intertemporal condition that relates the marginal rate of consumption to the rental rate of capital and the depreciation rate:

\[ \lambda(s^t) = \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t) \lambda(s^{t+1}) \left[ R(s^{t+1}) + 1 - \delta \right], \]  

where \( \pi(s^{t+1}|s^t) = \frac{\pi(s^{t+1})}{\pi(s^t)} \) is the conditional probability of \( s^{t+1} \) given \( s^t \). The law of motion of capital is:

\[ K(s^t) = (1 - \delta) K(s^{t-1}) + X(s^t). \]  

The analogous expressions for the foreign country are as follows, where all starred variables denote the foreign-country analogous to the home-country variables.

\[ U_{C*}(s^t) = \lambda^*(s^t), \]  
\[ \frac{U_{L*}(s^t)}{U_{C*}(s^t)} = W^*(s^t), \]  
\[ \lambda^*(s^t) = \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t) \lambda^*(s^{t+1}) \left[ R^*(s^{t+1}) + 1 - \delta \right], \]  

and

\[ K^*(s^t) = (1 - \delta) K^*(s^{t-1}) + X^*(s^t). \]  

The optimal choice by households of the home country delivers the following expression for the price of the riskless bond:

\[ \Omega(s^t) = \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t) \frac{\lambda(s^{t+1})}{\lambda(s^t)} \frac{\tilde{P}_H(s^{t+1})}{P_H(s^t)} - \frac{\Phi'[D(s^t)]}{\beta}. \]  

\[ \text{Page } 13 \]
The next condition equates the price of the riskless bond to the cost of adjusting bonds:

\[
\sum_{s^{t+1}} \pi (s^{t+1}|s^t) \left[ \frac{\lambda^* (s^{t+1})}{\lambda (s^t)} \tilde{P}_H (s^{t+1}) \frac{RER (s^t)}{RER (s^{t+1})} - \frac{\lambda (s^{t+1})}{\lambda (s^t)} \tilde{P}_H (s^t) \right] = - \Phi' [D (s^t)] \beta. \tag{13}
\]

From the intermediate goods producers’ maximization problems, we obtain the result that labor and capital are paid their marginal product, where the rental rate of capital and the real wage are expressed in terms of the final good in each country:

\[
W (s^t) = (1 - \alpha) \tilde{P}_H (s^t) A (s^t) ^{1-\alpha} K (s^{t-1}) ^{\alpha} L (s^t) ^{-\alpha}, \tag{14}
\]

\[
R (s^t) = \alpha \tilde{P}_H (s^t) A (s^t) ^{1-\alpha} K (s^{t-1}) ^{\alpha-1} L (s^t) ^{1-\alpha}, \tag{15}
\]

\[
W^* (s^t) = (1 - \alpha) \tilde{P}_F^* (s^t) A^* (s^t) ^{1-\alpha} K^* (s^{t-1}) ^{\alpha} L^* (s^t) ^{-\alpha}, \tag{16}
\]

and

\[
R^* (s^t) = \alpha \tilde{P}_F^* (s^t) A^* (s^t) ^{1-\alpha} K^* (s^{t-1}) ^{\alpha-1} L^* (s^t) ^{1-\alpha}. \tag{17}
\]

From the final goods producers’ maximization problem, we obtain the demands of intermediate goods, which depend on their relative price:

\[
Y_H (s^t) = \omega \tilde{P}_H (s^t) ^{-\theta} Y (s^t), \tag{18}
\]

\[
Y_F (s^t) = (1 - \omega) \left( \tilde{P}_F (s^t) RER (s^t) \right) ^{-\theta} Y (s^t), \tag{19}
\]

\[
Y_H^* (s^t) = (1 - \omega) \left( \frac{\tilde{P}_H (s^t)}{RER (s^t)} \right) ^{-\theta} Y^* (s^t), \tag{20}
\]

and

\[
Y_F^* (s^t) = \omega \tilde{P}_F^* (s^t) ^{-\theta} Y^* (s^t). \tag{21}
\]

Using the production functions of the final goods

\[
Y (s^t) = \left[ \omega \frac{1}{\theta} Y_H (s^t) ^{\frac{\theta-1}{\theta}} + (1 - \omega) \frac{1}{\theta} Y_F (s^t) ^{\frac{\theta-1}{\theta}} \right] ^{\frac{\theta}{\theta-1}}, \tag{22}
\]

\[
Y^* (s^t) = \left[ \omega \frac{1}{\theta} Y_F^* (s^t) ^{\frac{\theta-1}{\theta}} + (1 - \omega) \frac{1}{\theta} Y_H^* (s^t) ^{\frac{\theta-1}{\theta}} \right] ^{\frac{\theta}{\theta-1}}, \tag{23}
\]
and the demand equations for intermediate goods just described, we obtain the final goods deflator

\[
P^* (s^t) = \left[ \omega P_F (s^t)^{1-\theta} + (1 - \omega) P_H (s^t)^{1-\theta} \right]^{\frac{1}{1-\theta}}
\]

and

\[
P (s^t) = \left[ \omega P_H (s^t)^{1-\theta} + (1 - \omega) P_F (s^t)^{1-\theta} \right]^{\frac{1}{1-\theta}}.
\]

Hence, given that we assume that the LOP holds, the RER is equal to

\[
RER (s^t) = \frac{P^* (s^t)}{P (s^t)} = \frac{\left[ \omega P_F (s^t)^{1-\theta} + (1 - \omega) P_H (s^t)^{1-\theta} \right]^{\frac{1}{1-\theta}}}{\left[ \omega P_H (s^t)^{1-\theta} + (1 - \omega) P_F (s^t)^{1-\theta} \right]^{\frac{1}{1-\theta}}}
\]

and the only source of RER fluctuations is the presence of home bias ($\omega > 1/2$). Also, good, input, and bond markets clear. Thus:

\[
C (s^t) + X (s^t) = Y (s^t), \quad (24)
\]

\[
C^* (s^t) + X^* (s^t) = Y^* (s^t), \quad (25)
\]

\[
Y_H (s^t) + Y_H^* (s^t) = A (s^t)^{1-\alpha} K (s^t-1)^\alpha L (s^t)^{1-\alpha}, \quad (26)
\]

\[
Y_F (s^t) + Y_F^* (s^t) = A^* (s^t)^{1-\alpha} K^* (s^t-1)^\alpha L^* (s^t)^{1-\alpha}, \quad (27)
\]

and

\[
D (s^t) + D^* (s^t) = 0. \quad (28)
\]

Finally, the law of motion of the level of debt

\[
\tilde{P}_H (s^t) \bar{Q} (s^t) D (s^t) = \tilde{P}_H (s^t) Y_H^* (s^t) - \tilde{P}_F (s^t) RER (s^t) Y_F (s^t) + \tilde{P}_H (s^t) D (s^t-1) - \tilde{P}_H (s^t) \Phi [D (s^t)] \quad (29)
\]

is obtained using (1) and the fact that intermediate and final goods producers at home make zero profits. Finally, the productivity shocks follow the VECM described in Section 3.2.3.
3.5. Balanced Growth and the Restriction on the Cointegrating Vector

Equations (4) to (29) and the VECM process for (log) TFP characterize the equilibrium in this model. Since we assume that both log \( A(s_t) \) and log \( A^*(s_t) \) are integrated processes, we need to normalize the equilibrium conditions in order to obtain a stationary system more amenable to study. Following King, Plosser, and Rebelo (1988) we divide the home-country variables that have a trend by the lagged domestic level of TFP, \( A(s_{t-1}) \), and the foreign-country variables that have a trend by the lagged foreign level of TFP, \( A^*(s_{t-1}) \). In the online appendix, we detail the full set of normalized equilibrium conditions.

For the model to have balanced growth we require some restrictions on preferences, production functions, and the law of motion of productivity shocks. The restrictions on preferences and technology of King, Plosser, and Rebelo (1988) are sufficient for the existence of balanced growth in a closed economy real business cycle (RBC) model. However, in our two-country model, an additional restriction on the cointegrating vector is needed if the model is to exhibit balanced growth. In particular, we need the ratio \( A(s_{t-1}) / A^*(s_{t-1}) \) to be stationary.

In order to understand why the international dimension of the model requires this additional restriction, let us focus, for example, on the normalized demand of imported foreign-produced intermediate goods by the home country:

\[
\tilde{Y}_F(s_t) = (1 - \omega) \left[ \tilde{P}_F(s_t) RER(s_t) \right]^{-\theta} \tilde{Y}(s_t) \frac{A(s_{t-1})}{A^*(s_{t-1})}
\]

where \( \tilde{Y}_F(s_t) = Y_F(s_t) / A^*(s_{t-1}) \) while \( \tilde{Y}(s_t) = Y(s_t) / A(s_{t-1}) \). Since \( \tilde{P}_F(s_t) \) and \( RER(s_t) \) are stationary, if the ratio between \( A(s_{t-1}) \) and \( A^*(s_{t-1}) \) were to be non-stationary, the ratio between \( \tilde{Y}_F(s_t) \) and \( \tilde{Y}(s_t) \) would also be non-stationary and balanced growth would not exist. A similar argument must hold for the normalized equilibrium conditions related to: (20), (22), (23), (28), and (29).

Our VECM implies that the ratio between \( A(s_{t-1}) \) and \( A^*(s_{t-1})^\gamma \) is stationary. Therefore, a sufficient condition for balanced growth is that the parameter \( \gamma \) equals one or, equivalently, that the cointegrating vector equals \( (1, -1) \).
4. Estimation of the VECM

In this section, after describing our constructed (log) TFP series for the U.S. and the R.W., we perform three exercises. First, we show that our assumption that the (log) TFP processes are cointegrated of order C(1,1) cannot be rejected in the data. By the Granger representation theorem this implies that our VECM specification is valid. Second, we also show that the restriction imposed by balanced growth, i.e., that the parameter $\gamma$ is equal to one, cannot be rejected in the data either. Finally, we estimate the parameters driving our VECM in order to simulate our model in the next section.

4.1. Data

In order to estimate our VECM we use data for the U.S. and an aggregate for the R.W. For the U.S., we obtain quarterly real GDP data from the Bureau of Economic Analysis and hours and employment data from the Organization for Economic Cooperation and Development (OECD). Real capital stock data are also obtained from the OECD. The R.W. aggregate is the 15 countries of the Euro area, the United Kingdom, Canada, Japan, and Australia. This group accounts for about 50 percent of the basket of currencies that the Federal Reserve uses to construct the RER for the U.S. dollar. For the United Kingdom, Canada, Japan, and Australia we obtain nominal GDP, hours, employment, and real capital stock from the OECD. For the Euro area we take nominal GDP, employment, and real capital stock from the AWM. Hours are as reported in Christo¤el, Kuester and Linzert (2009). Our sample period goes from 1973:1 to 2006:4, which is when the hours series for the Euro area ends. Ideally, one would want to include additional countries that represent an important and increasing share of trade with the U.S., such as China and other emerging countries, but long quarterly series are not available.

We aggregate the nominal GDPs of the R.W. using PPP nominal exchange rates to convert each national nominal output to current U.S. dollars, and then use the output deflator of the U.S. (base year 2000) to convert the R.W. nominal output to constant U.S. dollars. We obtain aggregate R.W. hours data by simply aggregating the number of employees times hours per employee for each country. We aggregate real capital stocks (base year 2000) using the base year

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9We are thankful to K. Christo¤el and K. Kuester for providing us with the data.
2000 PPP RERs. Then, we construct the (log) TFP processes as follows:

\[ \log A(s^t) = \log \frac{Y(s^t) - (1 - \alpha) \log L(s^t) - \alpha \log K(s^{t-1})}{1 - \alpha} \]

and

\[ \log A^*(s^t) = \log \frac{Y^*(s^t) - (1 - \alpha) \log L^*(s^t) - \alpha \log K^*(s^{t-1})}{1 - \alpha} \]

where \( \alpha \) is the capital share of output and takes a value of 0.36. Backus, Kehoe, and Kydland (1992) and Heathcote and Perri (2002, 2009) use a similar approach when constructing (log) TFP series for the U.S. and a R.W. aggregate but ignoring capital dynamics. Given our focus on long-run properties, we add capital stock into the analysis.

4.2. Integration and Cointegration Properties

In this section, we present evidence supporting our assumption that the (log) TFP processes for the U.S. and the R.W. are cointegrated of order C(1,1). First, we will empirically support the unit root assumption for the univariate processes. Second, we will test for the presence of cointegrating relationships using the Johansen (1991) procedure. Both the trace and the maximum eigenvalue methods support the existence of a cointegrating vector.

<table>
<thead>
<tr>
<th>Table 2: Unit Root Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>log TFP U.S.</strong></td>
</tr>
<tr>
<td>Method</td>
</tr>
<tr>
<td>ADF</td>
</tr>
<tr>
<td>DF-GLS</td>
</tr>
<tr>
<td>P(_T)-GLS</td>
</tr>
<tr>
<td>MZ(_\alpha)</td>
</tr>
<tr>
<td>MZ(_t)</td>
</tr>
<tr>
<td>MSB</td>
</tr>
</tbody>
</table>

Notes: ADF stands for augmented Dickey-Fuller test. DF-GLS stands for Elliott-Rothenberg-Stock detrended residuals test statistic. 
\( P_T \)-GLS stands for Elliott-Rothenberg-Stock Point-Optimal test statistic. 
MZ\(_\alpha\), MZ\(_t\), and MSB stand for the class of modified tests analyzed in Ng-Perron (2001).
For ADF and DF-GLS we present t-Statistics, for \( P_T \)-GLS we present P-Statistics and for the MZ\(_\alpha\), MZ\(_t\), and MSB we present the Ng-Perron test statistics.
\(^+\) denotes null hypothesis of unit root not rejected at 5 percent level.
\(^\circ\) denotes null hypothesis of unit root not rejected at 5 percent level but rejected at 10 percent.
Univariate analysis of the (log) TFP processes for the U.S. and the R.W. strongly indicates that both series can be characterized by unit root processes with drift. Table 2 presents results for the U.S. (log) TFP process using the following commonly applied unit root tests: augmented Dickey-Fuller (Dickey and Fuller, 1979, and Said and Dickey 1984); the DF-GLS and the optimal point statistic ($P_{T\text{GLS}}$), both of Elliott et al. (1996); and the modified $M_{z_1}$, $M_{z_2}$, and MSB of Ng and Perron (2001). The lag length is chosen using the Schwarz information criterion. In each case a constant and a trend are included in the specification. Table 2 also presents the same unit root test results for the R.W. (log) TFP process. None of the test statistics are even close to rejecting the null hypothesis of unit root at the 5 percent critical value. Using the same statistics, unit root tests on the first difference of the (log) TFP processes for the U.S. and the R.W. are stationary. For the U.S. all the tests reject the null hypothesis of unit root at the 5 percent critical value. For the R.W. all the tests reject the null hypothesis of unit root at the 5 percent critical value except the MSB test which rejects it at the 10 percent value.

Table 3: Cointegration Statistics II: Johansen’s test

<table>
<thead>
<tr>
<th>Number of Vectors</th>
<th>Eigenvalue</th>
<th>Trace</th>
<th>p-value</th>
<th>Max-Eigenvalue</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.14</td>
<td>24.93</td>
<td>0.001</td>
<td>21.52</td>
<td>0.003</td>
</tr>
<tr>
<td>1</td>
<td>0.02</td>
<td>3.86</td>
<td>0.07</td>
<td>3.84</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Note: p-values as reported in MacKinnon-Haug-Michelis (1999).

Once we have presented evidence that strongly indicates that the (log) TFP for the U.S. and the R.W. is well characterized by integrated processes of order one, we now focus on presenting evidence supporting our assumption that the processes are cointegrated. If $\log A(s^t)$ and $\log A^*(s^t)$ share one common stochastic trend (balanced growth), an estimated VAR has to have a single eigenvalue equal to one and all other eigenvalues have to be less than one. To check this possibility we have estimated an unrestricted VAR with one lag and a deterministic trend for the two-variables system $[\log A(s^t), \log A^*(s^t)]$ where the number of lags was chosen using the Schwarz information criterion. The results are as expected: the highest eigenvalue equals 0.99, while the second highest is 0.95. But this is not a formal test of cointegration. Table 3 reports results from the unrestricted cointegration rank test using the trace and the maximum eigenvalue methods as defined by Johansen (1991). The cointegration test assumes a linear trend and a constant in the cointegrating vector. Clearly, the data strongly support a single eigenvalue.
4.3. The VECM Model

In the last subsection, we presented evidence that $\log A(s)$ and $\log A^*(s)$ are cointegrated of order C(1,1). In this subsection we provide some additional results. First, we show that the null hypothesis of $\gamma = 1$ cannot be rejected by the data using a likelihood ratio test. This is very important because a cointegrating vector $(1,-1)$ implies that the balanced growth path hypothesis cannot be rejected. Second, we also use the likelihood ratio test to present evidence supporting the null hypothesis that the coefficients related to the speed of adjustment in the cointegrating vector are equal and of opposite sign, i.e., $\kappa = -\kappa^*$. In Table 4, we present the outcome of the two likelihood ratio tests. Note that the tests are incremental.

<table>
<thead>
<tr>
<th>Restriction</th>
<th>Likelihood value</th>
<th>Degrees of freedom</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>992.88</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\gamma = 1$</td>
<td>992.88</td>
<td>1</td>
<td>0.96</td>
</tr>
<tr>
<td>$\kappa = -\kappa^*$</td>
<td>992.3</td>
<td>2</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Next, we estimate the restricted VECM with zero lags (corresponding to one lag in the VAR as the Schwarz information criterion suggests). The estimated restricted model delivers the parameter estimates reported in Table 5. It is worth noting that the coefficient of the speed of adjustment, while significant, is quantitatively small, denoting that (log) TFP processes converge slowly over time. As we will see later, low speed of adjustment parameter ($\kappa$) implies slow spillover of TFP shocks across countries. As we will see later, this finding is key to explaining our results. The constant terms $c$ and $c^*$ are estimated to be different. However, this does not imply that the growth rates of both (log) TFP processes are different. Indeed, because the cointegrating vector is $(1,-1)$, they must grow at the same rate along the balanced growth path. Given our parameter estimates, the implied long-run growth rate of (log) TFP processes is 1.44 percent (in annualized terms).

<table>
<thead>
<tr>
<th>Table 5: VECM Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
</tr>
<tr>
<td>0.001$\dagger$</td>
</tr>
<tr>
<td>(1.76)</td>
</tr>
</tbody>
</table>

$t$-statistics in parenthesis. $^+$ denotes significance at the 5 percent level and $\dagger$ denotes significance at the 10 percent level.
Finally, we also estimate the standard deviation of the innovations $\sigma^e$ to be 0.0105 and $\sigma^{e,*}$ equal to 0.0088. In our simulations, we will also assume that $\varepsilon^a (s^t)$ and $\varepsilon^{a,*} (s^t)$ are uncorrelated, since the null hypothesis could not be rejected in the data.

5. Results

5.1. Parameterization

Our baseline parameterization follows that in Heathcote and Perri (2002) closely. The discount factor $\beta$ is set equal to 0.99, which implies an annual rate of return on capital of 4 percent. We set the consumption share, $\tau$, equal to 0.34 and the coefficient of risk aversion, $\sigma$, equal to 2. Backus, Kehoe, and Kydland (1992) assume the same value for the latter parameter. We assume a cost of bond holdings, $\phi$, of 1 basis point (0.01). Parameters on technology are fairly standard in the literature. Thus, the depreciation rate, $\delta$, is set to a quarterly value of 0.025, the capital share of output is set to $\alpha = 0.36$, and home bias for domestic intermediate goods is set to $\omega = 0.9$, which implies the observed import/output ratio in the steady state.\footnote{In this model, $1-\omega$ is the fraction of imported goods in the steady state (see equation 19). The average import of goods/GDP ratio since 1973 is 9.6 percent. Typically, models that focus on trade in final goods set $\omega = 0.9$, by assuming that all trade is in final goods. In our model, all trade occurs at the intermediate goods level. Note that out of aggregate U.S. imports of goods, about 50 percent are final goods and 50 percent are intermediate goods. Hence, we follow the literature and set $\omega = 0.9$ because the only channel through which trade occurs in the models is in intermediate goods. But below, we also discuss what is the role of assuming $\omega = 0.95$.} We assume two possible values for the elasticity of substitution between intermediate goods, $\theta = 0.85$ and $\theta = 0.62$. The first value is based on Heathcote and Perri (2002); the second value is a bit higher than the lower bound of 0.5 considered by Corsetti, Dedola, and Leduc (2008b). The VECM is calibrated as described in Table 5. In most cases, we will compare our results to the ones one would obtain when using stationary TFP shocks. For the stationary case, we set the parameters of the (log) TFP shocks as in Heathcote and Perri (2002). In particular, Heathcote and Perri (2002) use the following VAR process with one lag:

$$a_t = \rho_0 a_{t-1} + \rho_0^* a_{t-1}^* + \varepsilon^a_t$$

and

$$a_t^* = \rho_0 a_{t-1}^* + \rho_0^* a_{t-1} + \varepsilon^{a,*}_t$$
where $\rho_a = 0.97$, $\rho_a^* = 0.025$, $Var(\varepsilon_a^t) = Var(\varepsilon_a^{0,*}) = 0.0073^2$, and $corr(\varepsilon_a^t, \varepsilon_a^{0,*}) = 0.29$.

### 5.2. Matching RER Volatility

In this subsection we analyze the performance of our model in generating enough RER volatility. Results are shown in Table 6. Since our model is non-stationary, we need to rely on simulations to compute the HP-filtered statistics. Hence, we simulate a series of (log) TFP shocks of length 125 periods, and we feed these shocks to the model. To avoid dependence on initial values, we first simulate the model for 1000 periods and discard those. We HP-filter the relevant series from the model (output, consumption, investment, employment, and the RER) and compute second moments. We repeat this procedure 5000 times and we report the average of the simulations. We solve the model taking a log-linear approximation around the steady state.

#### Table 6: Results

<table>
<thead>
<tr>
<th></th>
<th>$SD(Y)$</th>
<th>$SD(C)$</th>
<th>$SD(X)$</th>
<th>$SD(N)$</th>
<th>$SD(RER)$</th>
<th>$\rho(RER)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Full Sample: 1973-2006</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>1.58</td>
<td>0.76</td>
<td>4.55</td>
<td>0.75</td>
<td>3.06</td>
<td>0.82</td>
</tr>
<tr>
<td>Cointegrated TFP, $\theta = 0.85$</td>
<td>0.93</td>
<td>0.65</td>
<td>2.31</td>
<td>0.29</td>
<td>1.31</td>
<td>0.72</td>
</tr>
<tr>
<td>Cointegrated TFP, $\theta = 0.62$</td>
<td>0.85</td>
<td>0.66</td>
<td>2.46</td>
<td>0.27</td>
<td>3.13</td>
<td>0.70</td>
</tr>
<tr>
<td>Stationary TFP, $\theta = 0.85$</td>
<td>1.19</td>
<td>0.52</td>
<td>2.53</td>
<td>0.32</td>
<td>0.75</td>
<td>0.77</td>
</tr>
<tr>
<td>Stationary TFP, $\theta = 0.62$</td>
<td>1.12</td>
<td>0.54</td>
<td>2.51</td>
<td>0.31</td>
<td>1.41</td>
<td>0.75</td>
</tr>
<tr>
<td><strong>1973-1993</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>1.89</td>
<td>0.78</td>
<td>4.47</td>
<td>0.79</td>
<td>2.72</td>
<td>0.82</td>
</tr>
<tr>
<td>Cointegrated TFP, $\theta = 0.85$</td>
<td>1.12</td>
<td>0.59</td>
<td>2.25</td>
<td>0.27</td>
<td>1.32</td>
<td>0.72</td>
</tr>
<tr>
<td>Cointegrated TFP, $\theta = 0.62$</td>
<td>1.01</td>
<td>0.62</td>
<td>2.17</td>
<td>0.25</td>
<td>2.85</td>
<td>0.71</td>
</tr>
<tr>
<td><strong>1994-2006</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>0.88</td>
<td>0.78</td>
<td>4.82</td>
<td>0.90</td>
<td>5.01</td>
<td>0.82</td>
</tr>
<tr>
<td>Cointegrated TFP, $\theta = 0.85$</td>
<td>0.79</td>
<td>0.55</td>
<td>2.74</td>
<td>0.38</td>
<td>1.45</td>
<td>0.71</td>
</tr>
<tr>
<td>Cointegrated TFP, $\theta = 0.62$</td>
<td>0.73</td>
<td>0.66</td>
<td>2.01</td>
<td>0.42</td>
<td>4.31</td>
<td>0.72</td>
</tr>
</tbody>
</table>

$^\circ$ denotes relative to output.
The first and second rows of Table 6 report the results of the economy with cointegrated (log) TFP and high and low values for the trade elasticity, \( \theta \), respectively. For comparison with a model such as the one in Heathcote and Perri (2002), we also report the results for the economy with stationary (log) TFP shocks in the next two rows. Overall, models with cointegrated shocks generate higher relative volatility of the RER with respect to output than models with stationary (log) TFP shocks. Note that with high trade elasticity and cointegrated (log) TFP shocks, the relative volatility of the RER more than doubles with respect to the model with stationary shocks (1.31 versus 0.75). We go from explaining less than 25 percent of the observed relative volatility of the RER to explaining more than 40 percent.

Table 7: Results

<table>
<thead>
<tr>
<th>Full Sample: 1973-2006</th>
<th>( CORR(Y, N) )</th>
<th>( CORR(Y, C) )</th>
<th>( CORR(Y, X) )</th>
<th>( CORR(RER, C/C^*) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.87</td>
<td>0.84</td>
<td>0.91</td>
<td>-0.04</td>
</tr>
<tr>
<td>Cointegrated TFP, ( \theta = 0.85 )</td>
<td>0.94</td>
<td>0.95</td>
<td>0.97</td>
<td>0.95</td>
</tr>
<tr>
<td>Cointegrated TFP, ( \theta = 0.62 )</td>
<td>0.97</td>
<td>0.98</td>
<td>0.98</td>
<td>0.97</td>
</tr>
<tr>
<td>Stationary TFP, ( \theta = 0.85 )</td>
<td>0.97</td>
<td>0.93</td>
<td>0.97</td>
<td>0.99</td>
</tr>
<tr>
<td>Stationary TFP, ( \theta = 0.62 )</td>
<td>0.97</td>
<td>0.93</td>
<td>0.97</td>
<td>0.99</td>
</tr>
</tbody>
</table>

As expected for lower values of the trade elasticity, the relative volatility of the RER increases under both the stationary and cointegrated models. The striking finding is that the model with cointegrated (log) TFP shocks and elasticity equal to 0.62 is able to closely match the relative volatility of the RER (3.13 in the model versus 3.06 in the data), while the model with stationary shocks and the same elasticity can get only to 1.41 (which represents only about 40 percent of the fluctuation in the data). Interestingly, even though the model with cointegrated (log) TFP shocks improves significantly in matching the RER volatility, it does not affect the fit of other unconditional moments. Both the stationary and the cointegrated (log) TFP shocks models display very similar volatilities of consumption, hours, and investment relative to output. Also, both models (stationary and cointegrated) display similar cross-correlations between consumption, hours, and investment relative to output and autocorrelations of RERs and it is the case that none of the models can explain the Backus and Smith (1993) puzzle (Tables 6 and 7).

A similar pattern is observed on the international side (Table 8). Both specification models display very similar cross-correlations between U.S. and R.W. output, consumption, investment,
and hours. For the case of $\theta = 0.62$, the stationary model shows a cross-correlation between domestic and foreign output of 0.33, while the model with cointegrated shocks reports 0.48 and the observed one is 0.44. For the case of consumption the stationary and cointegrated models report 0.81 and the 0.69, respectively, while the observed correlation is 0.36. For investment, the numbers are $-0.05$, 0.15, and 0.28, respectively. Finally, for hours, we find that the stationary and cointegrated shock models show a correlation of $-0.05$ and 0.13, respectively, while the observed one is 0.40. Unfortunately, the model with cointegrated shocks, like the model with stationary shocks, cannot solve the “quantity puzzle.” In the data, output is more correlated than consumption across countries, while, in the model, consumption is more correlated than output. In any case, the cointegrated model with $\theta = 0.62$ does better than the other (cointegrated and stationary) versions.

<table>
<thead>
<tr>
<th>Table 8: Results</th>
<th>(\text{CORR}(Y, Y^*))</th>
<th>(\text{CORR}(C, C^*))</th>
<th>(\text{CORR}(X, X^*))</th>
<th>(\text{CORR}(N, N^*))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.44</td>
<td>0.36</td>
<td>0.28</td>
<td>0.40</td>
</tr>
<tr>
<td>Cointegrated TFP, $\theta = 0.85$</td>
<td>0.11</td>
<td>0.45</td>
<td>-0.21</td>
<td>-0.29</td>
</tr>
<tr>
<td>Cointegrated TFP, $\theta = 0.62$</td>
<td>0.48</td>
<td>0.69</td>
<td>0.15</td>
<td>0.13</td>
</tr>
<tr>
<td>Stationary TFP, $\theta = 0.85$</td>
<td>0.18</td>
<td>0.70</td>
<td>-0.18</td>
<td>-0.21</td>
</tr>
<tr>
<td>Stationary TFP, $\theta = 0.62$</td>
<td>0.33</td>
<td>0.81</td>
<td>-0.05</td>
<td>-0.05</td>
</tr>
</tbody>
</table>

Although not reported in the tables, we have also simulated the model under two alternative asset market structures: complete markets and financial autarky. In the first case, we assume that agents have access to a full set of state-contingent bonds that pay one unit of the domestic intermediate good in every state of the world. In the second case, we calibrate the cost of holding bonds $\Phi' [D (s^*)]$ to a very large number such that intertemporal trade never occurs. As expected (see Heathcote and Perri, 2002), the version of the model with complete markets generates lower relative volatility of the RER (in particular, it goes from 1.31 to 0.90 when $\theta = 0.85$, and from 3.13 to 1.01 when $\theta = 0.62$), while the version of the model with financial autarky delivers a larger relative volatility of the RER (in particular, it goes from 1.31 to 1.51 when $\theta = 0.85$, and from 3.13 to 4.14 when $\theta = 0.62$). Therefore, the presence of incomplete markets helps the cointegrated (log) TFP shocks to do their job in increasing the relative volatility of RER (at least with respect to the complete markets case).
Finally, it is important to recognize that RERs in the data differ in two important ways from our theory. First, existing empirical evidence shows deviations from LOP, which our model ignores.\footnote{Most papers report deviations of the law of one price across countries due to the lack of full price adjustment to exchange rate fluctuations, as in, for instance Goldberg and Knetter (1997) and Campa and Goldberg (2005). For recent micro-evidence on differences in the response of importers to exchange rate shocks, see Gopinath and Itskhoki (2010).} With respect to this point, Crucini and Shintani (2008) argue that LOP deviations lack persistence at the microeconomic level and therefore we will follow them. In any case, it is important to mention that ignoring the deviations makes it more difficult for our model to match the data: in the data deviations from LOP are one important source of the fluctuations. In any case, we would like to mention that we have simulated a version of our model with Calvo-type sticky prices and domestic currency pricing to find that the results do not change significantly.

Second, our measure of RER uses trade weights applied to CPIs. CPIs include non-traded goods. Thus, there is the issue of whether our measure of RER is the right empirical counterpart to our model-based RER, since we do not have a non-traded goods sector. Without a non-traded goods sector, there is a gap between theory and measurement. To try to fill the gap, we compute two alternative measures of RER. The first measure uses PPIs instead of CPIs. The idea, going back to Engel (1999), is that CPIs have a larger share of non-traded goods in the basket than the PPIs, and hence the PPI is a better measure of tradable goods price index.\footnote{We construct the PPI-based real exchange rate series between 1973 and 2009. We obtain PPI series for Australia, Canada, Japan, the United Kingdom and the United States using national sources data. Data for the Euro area since 1981 come from Eurostat. Since many countries in the Euro area do not produce these series before 1973 and 1981, we use the West German PPI in that period.} Our second measure uses export deflators as the relevant tradable goods price indices, since it measures the prices of goods that are actually shipped internationally.\footnote{We construct the export deflator-based real exchange rate series between 1973 and 2009. We obtain export deflators series for Australia, Canada, Japan, the United Kingdom and the U.S. using national data sources, while data for the Euro area come from the AWM.}

We find that the volatility of the PPI- and CPI-based RERs is similar which reinforces Engel’s (1999) result. In particular, for the period 1973:1 to 2006:4 the standard deviation of the PPI-based RER is 2.96 larger than the standard deviation of output while using the CPI-based RER, the standard deviation of RER is 3.06 larger than the standard deviation of output (as shown in Table 6). For the same period, the standard deviation of the export deflator-based RER is 2.34 larger than the standard deviation of output. Hence, if we use any of these alternative measures, our main message does not change, although our model does even better when compared with
the data.

5.3. Intuition

In this subsection we try to explain the intuition behind our results. In our model, four key parameters drive the behavior of the volatility of the RER with respect to output: (i) the elasticity of substitution between home and foreign goods, (ii) the fraction of intermediate goods in the production of the final good (or “home bias”), (iii) the persistence of the (log) TFP process, and (iv) the persistence in the differential of (log) TFP processes across countries. In this subsection, we study how these four elements shape up our results. The first two components are well-known in the IRBC literature, so we briefly elaborate on them here. The second two components are new to this paper, so we spend more time talking about them.

5.3.1. The Role of the Elasticity of Substitution and Home Bias

It is important to note that since we have only tradable intermediate goods in the model, the RER relates to input prices as follows:

\[
\text{RER}(s^t) = \frac{\omega P_F(s^t)^{1-\theta} + (1 - \omega) P_H(s^t)^{1-\theta}}{\omega P_H(s^t)^{1-\theta} + (1 - \omega) P_F(s^t)^{1-\theta}}^{\frac{1}{1-\theta}}.
\]

Thus, for a given volatility of the relative prices of intermediate goods, the model needs low elasticity of substitution (low \(\theta\)) and/or high home bias (high \(\omega\)) in order to match the observed RER volatility. However, this is largely an artifact of ignoring non-traded intermediate goods. An alternative would be introducing a non-traded intermediate good as a substitute to the asymmetric aggregate among traded goods. However, there are challenges associated with measuring (log) TFP in the tradable and non-traded sectors in each of the two countries at quarterly frequency. Since one of the key ingredients of our paper is, indeed, to properly measure (log) TFP, it is not possible for us to follow this route.

In our model, it is difficult to obtain one expression linking the volatility of the RER to output. However, by combining the two demand equations for home and foreign intermediate goods (18)
and (19), normalizing by the TFP in each country, and log-linearizing, we obtain that:

\[ \hat{y}_{H,t} - \hat{y}_{F,t} = \frac{\theta}{2\omega - 1} \text{rer}_t - (a_{t-1} - a_{t-1}^*) \]

therefore, for a given volatility of relative quantities and relative (log) TFP, lowering θ or increasing \( \omega \) would imply higher RER fluctuations (see also Backus, Kehoe and Kydland, 1994).

Of course, this is not a full argument, since changes in \( \theta \) and/or \( \omega \) will affect all variables in a general equilibrium model. The full general equilibrium argument is directly shown in Figure 2. In Figure 2 we compute the volatility of (HP-filtered) output and the RER and the ratio between the two for different combinations of \( \theta \) and \( \omega \). As expected, increasing home bias \( (\omega) \) or decreasing the elasticity of substitution \( (\theta) \) leads to higher relative volatility of the RER with respect to output. Also, as shown in Figure 2, the behavior of the volatility of the RER mostly drives the behavior of the relative volatility of the RER with respect to output. Finally, we would like to note that if he had calibrated the model with \( \omega = 0.95 \), which implies a ratio of intermediate goods imports to output of 5 percent, then we would have obtained a relative volatility of RER of 3.5 when \( \theta = 0.62 \) and 1.5 when \( \theta = 0.85 \), increasing somewhat the numbers reported in Table 6.

5.3.2. The Role of the Coefficients of TFP

The key facet of productivity in Baxter and Crucini (1995), at least in terms of the relevance of asset market restrictions they explored, is not the persistence of (log) TFP processes but the persistence of the productivity gap. The reason is clear from their Hicksian decompositions. Home and foreign wealth effects, due to incomplete markets, arise when productivity diverges across countries in a persistent fashion. Hence, in this subsection we explore the role of both persistence in productivity and in productivity differentials across countries for RER volatility.

We can write the joint process of (log) TFP shocks across countries as follows:

\[
\begin{align*}
    a_t &= \rho a_{t-1} + \nu a_{t-1}^* + \varepsilon_t^a \\
    a_t^* &= \rho a_{t-1}^* + \nu a_{t-1} + \varepsilon_t^{a,*}
\end{align*}
\]

\[\text{We are thankful to associate editor Mario Crucini for suggesting this presentation.}\]
as in Backus, Kehoe and Kydland (1994) and Heathcote and Perri (2002). The level of (log) TFP in a country is related to its own lag with coefficient \( \rho \), and to the lag of the other country’s (log) TFP with coefficient \( \nu \). The coefficient \( \nu \) is also known as the TFP spillover from one country to the other. Note that when \( \rho = (1 + \kappa) \) and \( \nu = -\kappa \), this VAR equals the VECM with cointegrating vector (1,-1) and symmetric convergence speed to the cointegrating relationship that we have considered throughout the paper.

Given that the two countries are of the same size, we can define world (log) TFP as \( a^w_t = a_t + a^*_t \), and (log) TFP differential as \( a^r_t = a_t - a^*_t \). Then, these new processes evolve as:

\[
\begin{align*}
a^w_t &= (\rho + \nu)a^w_{t-1} + \varepsilon^w_t \\
a^r_t &= (\rho - \nu)a^r_{t-1} + \varepsilon^r_t
\end{align*}
\]

where \( \varepsilon^w_t = \varepsilon^a_t + \varepsilon^{a,*}_t \), and \( \varepsilon^r_t = \varepsilon^a_t - \varepsilon^{a,*}_t \). Therefore, the key parameters in the model become \( (\rho + \nu) \), the persistence of the world (log) TFP shock, and \( (\rho - \nu) \), the persistence of the (log) TFP differential. In the VECM that we have been considering so far, \( (\rho + \nu) = 1 \) and \( (\rho - \nu) = 1 - 2\kappa \). Therefore, given a common unit root in the system, a slower convergence to the cointegrating relationship (smaller \( \kappa \)) implies a higher persistence of the (log) TFP differential (higher \( \rho - \nu \)).

In Figure 3 we plot the standard deviation of (HP-filtered) the RER and output, and the ratio of the two, as a function of \( (\rho + \nu) \) and \( (\rho - \nu) \). As we can see, increasing the persistence of the world (log) TFP shock increases the volatility of output but not to a large extent. However, for a given level of persistence in the world (log) TFP shock, an increase in the persistence of (log) TFP differential reduces the standard deviation of output. Why is this the case? With a higher persistence of (log) TFP shocks, households experience a larger positive wealth effect and therefore supply less labor and capital. In the limit, when (log) TFP shocks follow a random walk, output jumps almost immediately to its new steady-state level with little persistece in its dynamics and HP-filtered output volatility declines. Similarly, when the persistence of the (log) TFP differential decreases, home-country households know that productivity will increase sooner in their country. Hence, because of a wealth effect, they supply less labor and capital and output volatility increases. A higher persistence in (log) TFP differentials leads to lower output volatility by delaying the wealth effect.\(^{15}\)

\(^{15}\)Impulse responses of all variables when both \( \rho \) and \( \nu \) change are available from the authors upon request.
What happens with the volatility of the RER? The intuition is clear when we comment on changes in $\rho$ and $\nu$ in isolation. When $\rho$ increases but $\nu$ remains fixed (this implies moving along a 45-degree line in the top right panel of Figure 3), the wealth effect leads home households to demand more consumption goods. In order to produce more final goods, the home country demands more intermediate goods from the foreign country, which, provided that the elasticity of substitution between home and foreign intermediate goods is low enough, leads to larger RER depreciations. Hence, the volatility of the (HP-filtered) RER increases. When $\rho$ is fixed but $\nu$ increases, the exact opposite happens (this would be a movement with slope $-1$ in the top right panel of Figure 3). In this case the wealth effect hits the foreign country households. They know that productivity will increase sooner in their country and they demand more consumption goods than they would if spillovers were slower. Thus, the demand for home intermediate goods increases because foreign final goods producers substitute away from domestic intermediate goods. As a consequence, foreign intermediate goods become less scarce and the RER depreciates less than in a model with slower (or no) spillovers.

Therefore, an increase in world (log) TFP productivity ($\rho + \nu$) holding constant the persistence of the (log) TFP differential ($\rho - \nu$) means that both $\rho$ and $\nu$ increase. As we have discussed, these two coefficients have opposite effects on the volatility of the RER and, on net, increasing ($\rho + \nu$) leads to a decline in the volatility of the RER. On the other hand, an increase in the persistence of the (log) TFP differential ($\rho - \nu$) holding constant the persistence of world (log) TFP shock ($\rho + \nu$) means that $\rho$ increases by as much as $\nu$ decreases, which unequivocally increases RER volatility.

The case we examine throughout the paper, that is, a VECM with cointegrating vector (1, -1) and symmetric speed of convergence $\kappa$, would be horizontal movements along the line when $\rho + \nu = 1$. As we can see in Figure 3, decreasing $\kappa$ (i.e. increasing $\rho - \nu$ while keeping $\rho + \nu = 1$) leads to a decline in the volatility of output, an increase in the volatility of the RER, and hence an increase in the relative volatility of the RER with respect to output that matches the one observed in the data.\footnote{Note that, theoretically, it would be possible to calibrate a stationary model with low persistence in world productivity and high persistence in the relative productivity that implies high RER volatility. However, this would require that $\rho - \nu > \rho + \nu$ and hence that $\nu$ is negative. None of the available estimates suggest that spillovers are negative, indicating that this theoretical result is not supported by empirical evidence.}
5.4. Matching the Increase in RER Volatility

As described in Section 2, the volatility of the RER with respect to the volatility of output has increased in the last decade for U.S. economy. As shown by the Quandt-Andrews test, the increase seems to be dated in 1993:4. As Table 6 shows, the volatility of the RER has gone from less than three times the volatility of output during the period 1973:1 to 1993:4 to more than five times during the period 1994:1 to 2006:4. Using U.S. and R.W. data, in this section we present evidence that relates a decrease in the speed of convergence to the cointegrating relationship, i.e., lower $\kappa$, with the increase in the relative volatility of the RER.

To present our evidence, we estimate our VECM for two non-overlapping sub-samples.\textsuperscript{17} The first sample goes from 1973:1 to 1993:4, while the second sub-sample goes from 1994:1 to 2006:4. We split the sample to match the results of the Quandt-Andrews test. For the first subsample we observe that the estimated value of the speed of adjustment term is larger in absolute value than the value estimated with the entire sample. In particular, $\kappa$ moves from $-0.007$ to $-0.008$. Also, the standard deviation of the stochastic process for the U.S., $\sigma$, is estimated to be $0.012$, while the standard deviation for the R.W., $\sigma^*$, is estimated to be $0.011$. Both values are larger than the ones obtained when the whole sample is used.

In the second sub-sample, 1994:1 to 2006:4, the estimated speed of adjustment coefficient dramatically decreases with respect to both the full sample and the first sub-sample: the point estimate is $-0.002$. This means that the catching up process is much slower in the second part of the sample. This result indicates that the co-movement between TFPs in the post-1994 period is characterized by a very slow return to the long-run level. Finally, the standard deviations $\sigma$ and $\sigma^*$ are estimated to be $0.009$ and $0.007$, respectively. Our sub-sample estimates of $\sigma$ and $\sigma^*$ reflect both our sample period and the countries that we include in the R.W. The big drop in $\sigma$ and $\sigma^*$ across sub-samples reveals the reduction in output volatility that the U.S. and other countries experienced after the 1980s (see Kim and Nelson, 1999, and McConnell and Perez-Quirós, 2000).

We now simulate the model under the estimates of the VECM for each of the sub-samples. Table 6 reports the results. At this point it is important to point out that the change in VECM parameters is entirely unexpected and then fully understood by the agents in the model. Our results indicate that the change in the estimates of the VECM across samples is an important

\textsuperscript{17}We assume that the cointegrating relationship is the same across samples.
force behind the increase in the relative volatility of the RER. While in the data the relative volatility of the RER increases by 80 percent across samples, our simulations show that the model generates increases in relative volatility of around 50 percent for both low and high values of $\theta$; i.e., our model can explain more than 60 percent of the increase in RER volatility.

6. Concluding Remarks

In this paper, we document two empirical facts. First, that (log) TFP processes of the U.S. and the R.W. are cointegrated with cointegrating vector $(1, -1)$ and, second, that the relative volatility of the RER with respect to output has increased in the U.S., the United Kingdom, Canada, and Australia during the last 20 years.

Then, we have shown that introducing cointegrated (log) TFP processes in an otherwise standard IRBC model increases the model’s ability to explain RER volatility, without affecting the fit to other second moments of the data. We have also documented that if we allow the speed of convergence to the cointegrating vector to change, as it does in the data, the model can also explain the observed increase in the relative volatility of the RER.

For future research, it would be interesting to introduce cointegrated (log) TFP processes in medium-scale open economy macroeconomic models, which typically include more frictions, and try to match a larger set of domestic and international variables (see Adolfson et al., 2007). Also, instead of analyzing the volatility of the RER between the U.S. and a synthetically constructed R.W., one could compute bilateral RERs and compare them to bilateral output. This exercise is beyond the scope of this paper, but it could be an interesting line of research given that the equilibrium trade literature is consistently finding that RERs and gross bilateral trade flows are closely related.

References


Standard deviation of HP-filtered output and RER and the ratio of standard deviations

Standard deviation of HP-filtered output and RER when $\theta$ and $\omega$ change
Standard deviation of HP-filtered output and RER when $\rho$ and $\nu$ change