A Model of Patent Trolls

Jay Pil Choi
Michigan State University and Hitotsubashi University
e-mail: choijay@msu.edu

Heiko Gerlach
University of Queensland
e-mail: h.gerlach@uq.edu.au

March 2017

Abstract

This paper develops a model of patent trolls to understand various litigation strategies employed by non-practicing entities (NPE). We show that when an NPE faces multiple potential infringers who use related technologies, it can gain a credible threat to litigate even when it has no such credibility vis-à-vis any single potential infringer in isolation. This is due to an information externality generated by an early litigation outcome for subsequent litigation. Successful litigation creates an option value against future potential infringers through Bayesian updating. This renders a credible litigation threat against the initial defendant and allows the NPE to extract more rents. We discuss policy implications including the adoption of the British system of “loser-pays” fee shifting and the use of injunctive relief.

Keywords: patent portfolios, patent litigation, non-practicing entities, patent troll

JEL: D43, L13, O3

We would like to thank the Editor Harold Cole, two anonymous referees as well as Scott Baker, Erik Hovenkamp, Louis Kaplow, Scott Duke Kominers, Jorge Lemus, Alvaro Parra, Kathy Spier, Emanuele Tarantino, and participants in various conferences and seminars for valuable discussions and comments.
1 Introduction

The patent system is designed to protect and promote innovation by granting innovators exclusive rights to commercially exploit their inventions for a limited period of time. However, patent law does not require that only the inventor enforce the patent. Patents can be transferred to other parties and be enforced by whoever owns them (Lemley and Melamed 2013). Recently, the emergence of non-practicing entities (NPEs) as a major driver of patent litigation has spawned a heated debate on their role in the overall patent system and their impacts on innovation. NPEs, also derisively called “patent trolls,” are a new organizational form whose sole purpose is to use patents primarily to obtain license fees rather than to support the development of technology. They amass patents not for the purpose of commercializing a new product, but to litigate and demand licensing fees.

The proponents of NPEs emphasize potential positive roles of NPEs. They argue that NPEs help small independent inventors to monetize their intellectual property (IP) rights against potential misappropriation by established companies, thereby inducing more innovation by small inventors. In contrast, the opponents are concerned that NPEs simply raise the costs of innovation and can drag the innovation process. Due to their business models, they seek patents to pursue “freedom to litigate” rather than “freedom to operate.” The value of a patent thus can be based on the “exclusion value” rather than the “intrinsic value” when it is held by NPEs (Chien 2010). More importantly, the recent surge in the number of lawsuits initiated by patent trolls became a cause for concern for businesses and policy-makers alike. One recent statistic shows that patent trolls are responsible for 67 percent of all patent lawsuits (Morton and Shapiro 2014). Bessen, Ford, and Meurer (2011) estimate that trolls cost the economy $500 billion over the last twenty years, mostly in the IT industry.

This paper develops a model of patent trolls to understand various litigation strategies employed by non-practicing entities. We show that when an NPE faces multiple potential infringers who use related technologies, it can gain a credible threat to litigate even when it has no such credibility vis-a-vis any single potential infringer in isolation. This is due to an information externality generated by an early court outcome for subsequent litigation. Successful litigation creates an option value against future potential infringers through Bayesian updating. This renders a credible litigation threat against the initial defendant and allows the NPE to extract more rents.

1 NPEs are also called patent asserting entities (PAEs).
Lemley and Melamed (2013) point out that patent trolls do not employ a unitary business model and there are at least three different troll business models. A “lottery-ticket” troll is an owner of a patent that reads on a significant technology area. They target big established firms with an uncertain shot at a big payout. It is particularly important to these trolls that the perceived probability of infringement is high when they face their big practicing entity (PE) target. By contrast, “bottom-feeder” trolls are not particularly concerned as to whether the patent is infringed or not. They rely on the high cost of patent litigation, and aim to settle for small amounts of money. Finally, “patent aggregator” trolls acquire huge patent portfolios to convince target company to pay royalties for the portfolio license.

Our paper formalizes how the exclusion value is created by the credible threat to litigate and explores its implications for NPEs’ litigation strategies. For instance, consider bottom-feeder trolls who search for “quick, low-value settlements for a variety of patents.” The logic is that the defendants prefer to settle for small amounts of money rather than pay the high cost of patent litigation that could easily run into millions. However, high litigation costs cut both ways and the logic begs the question of why defendants would consider the litigation threat by bottom feeders seriously. Given the considerable litigation costs relative to a meager expected payout, why don’t they ignore the threat? We do not rely on a reputational mechanism, but rather provide a theory of litigation credibility based on information externalities. We show that an NPE may have a patent portfolio that is not strong enough to make its litigation threat credible in isolation, but in the presence of multiple defendants, the litigation threat becomes credible due to its option value for other future defendants. As pointed out by Lemley and Melamed (2013), “the universe of technology users against which a troll might assert patents is ... potentially much larger than the group of competitors against which a practicing entity is likely to assert its patents.”

To understand the role of information externalities in patent litigation, consider the following simple numerical example. First, consider a situation in which an NPE faces only one PE that uses its patented technology. Let the PE’s profit be 20 and if the NPE is successful against the PE in its litigation, it can extract half of the PE’s profit via Nash bargaining with the threat of injunction. The probability that the PE’s patent is valid and infringed by the PE is given by $1/4$, and the legal costs for each party is 4. Then the expected payoff from litigation for the NPE is given by $(1/4) * 10 - 4 = -1.5 < 0$, and the NPE’s litigation threat is not credible. Now suppose that there are two PEs with the same profit level of 20. For simplicity, these two PEs are not competing each other, but assumed to use the same technology. This implies that the infringement by the two PEs is perfectly correlated. In this case, litigation against one PE reveals perfect information about the infringement by the other PE. Thus, if the NPE wins against

\[^3\text{See Lemley and Melamed (2013, p.2126).}\]

\[^4\text{According to the legal principle of “res judicata,” a matter that has been judged on the merits may not, generally, be relitigated.}\]
one PE, it has a credible threat to litigate against the remaining PE and can extract 10 for sure. This implies that the NPE’s threat against the first PE is credible because \((1/4) \times (10 + 10) - 4 = 1 > 0\). The NPE and the targeted PE will settle out of court to save litigation costs. With Nash bargaining, the NPE will be able to receive a licensing fee of 5 from the targeted PE due to the presence of another PE that offers an additional option value for the initial litigation. We thus show how the presence of other potential infringers enhances the credibility of the patent holder’s litigation threats and enables him to “double dip.” However, note that the NPE no longer has any credible threat against the remaining PE once it extracts the licensing fee from the first PE. This simple example also suggests that the NPE may have higher incentives to acquire patent portfolios for the purpose of litigation vis-a-vis PEs. Suppose that the target firm is randomly selected because the two firms are symmetric. Then, each firm’s expected licensing cost is 2.5. As a result, each PE will have incentives to bid up to 2.5 if the patent is up for sale whereas the NPE has incentives to bid up to 5. Acquiring the patent in this example is like providing a public good between the PEs because if one PE acquires the patent, the other PE benefits as much because the acquiring PE will not have any credible threat against the other PE. This type of provision of public good problem can also explain the emergence of defensive aggregators.

Hovenkamp (2013) is related to our paper in that he considers the NPE’s incentives to litigate and the credibility of litigation threat. However, the mechanism by which the NPE gains credibility with weak patents is very different. He develops a dynamic model of predatory litigation that relies on the NPE’s litigious reputation and behavioral type of “impressionable” PEs which are easily intimidated by the NPE’s predatory litigation behavior. In contrast, we do not assume any asymmetric information about firms’ types and our main results are driven by information externalities across litigation suits. Lemus and Temnyalov (2014) analyze the role of patent asserting entities (PAEs) on litigation and innovation incentives. To address this issue, they consider a model in which PAE is allowed to acquire patents from practicing entities, and compare the equilibrium in such a set-up to a situation in the absence of the PAE. They identify two effects created by the PAE that are immune to counter-litigation: enhanced patent monetization effect and loss of the value of defensive patent portfolios. They show that when the former effect dominates the latter, PAEs can enhance innovation incentives and social welfare. The main focus of their paper, however, is different from ours and can be complementary to ours in understanding the tactics and roles of NPEs/PAEs in the overall patent system. They are concerned with the price of patent acquisition by the PAE and how this in turn changes the returns to R&D. We are more interested in the litigation strategies of NPEs and focus on litigation externalities, which are absent in their model.

Choi (1998) considers the implications of information externality in patent litigation, but in a different context. He considers a setting in which a patent holder is the incumbent facing multiple potential
entrants. Launching a patent suit in face entry can be a risky proposition for the incumbent because of potentially harmful information that would invite further entry if its patent is invalidated. He explores the implications of such information revelation on entry dynamics and show that the nature of the entry game can be one of either waiting or preemption depending on the strength of the patent. However, the nature of information revelation in Choi (1998) is different from ours because the patent holder is a practicing entity and the issue is entry dynamics rather than extraction of rents by NPEs.

There is also a small set of papers in the law and economics literature considering strategic implications of information from trial outcomes for subsequent litigation and settlement. Che and Yi (1993) consider a situation in which a single uninformed defendant faces multiple informed plaintiffs and once a precedent is set, it can have a lasting effect on successive trial outcomes. Daughety and Reinganum (1999) consider an incomplete information model in which an initially uninformed plaintiff makes a menu of settlement demands of the informed defendant who faces other potential plaintiffs. They show that the possibility that there are other plaintiffs the defendant might face improves the current plaintiff’s bargaining position as the outcome of the current case may invite further follow-on suits. As a result, the defendant may be willing to pay “hush money” to keep the negotiation outcome confidential. By contrast, we use a different model framework with symmetric information and focus on issues arising from intellectual property litigation incentives of non-practicing entities.

The remainder of the paper is organized in the following way. In Section 2, we set up a simple model of patent litigation with information externalities. To illustrate the main idea, we consider one NPE that can assert its patent portfolio against multiple PEs in a pre-determined order, and analyze the implications for the credibility of the litigation threat. In section 3, we endogenize the NPE’s target choice and derive the optimal sequence of litigation targets. Section 4 extends our analysis to an environment in which PEs compete with each other and we explore the role of injunctive relief. Section 5 considers the implications of the British cost shifting rule under which the loser pays all legal expenses. Section 6 analyze the NPE’s incentives to acquire patent portfolio vis-a-vis PEs’. Section 7 extends the basic model in several directions and checks the robustness of the main results. In particular, we consider the possibility of confidential licensing agreements and the implications of NPE patent enforcement for innovation and entry incentives of PEs as well as endogenous litigation effort and more than two PEs. Section 8 closes the paper with concluding remarks. Longer proofs for lemmas and propositions are relegated to the Appendix.
2 Benchmark Model

We consider a situation in which one NPE, or patent troll, intends to assert its patent portfolio against multiple PEs. The NPE has a patent portfolio of size $S$, which translates into an infringement probability of $\theta \in [0,1]$ for any PE. This infringement parameter can be interpreted as the strength of the NPE’s patent portfolio.\(^5\) For simplicity, let us assume that there are two PEs, firm 1 (PE\(_1\)) and firm 2 (PE\(_2\)), and the NPE is negotiating sequentially with each of them. To illustrate the nature of the information externality across litigation cases, we first assume that the sequence is pre-determined in the benchmark model. This would be the case if PEs are entering the market sequentially over time.\(^6\) By contrast, in the next section, we consider a scenario, in which both PEs are already in the market and the NPE can endogenously choose whether to approach the PEs simultaneously or sequentially, and if sequentially, which firm to target first when the PEs are asymmetric. The PEs are not competing with each other, but they use related technologies. This means that the litigation outcome for one firm does not affect the other firm’s profitability through competitive effects.\(^7\)

Nonetheless, the litigation outcome for one PE may have implications for the likelihood of the other PE’s infringement on the NPE’s patent portfolio when they use related technologies. For instance, many industries have evolved by integrating technologies from a variety of different scientific disciplines. The interdisciplinary approach and convergence of technologies have made it commonplace for the same type of related technologies to be adopted in previously separate industries, blurring the boundaries of traditional industries and creating new ones. Consider the convergence of broadcasting and telephone industries. Traditionally, they represented very different forms of communications in many dimensions, including the mode of transmission and the nature of communication. As a result, they were considered separate industries. Digital convergence now enables both person-to-person communication services and broadcast content with similar technologies. We represent the technological overlap between the two PEs with a parameter $\rho \in [0,1]$.

More specifically, there are four possible litigation outcomes if there are patent suits against both PEs: $(I,I), (I,NI), (NI,I)$, and $(NI,NI)$, where $I$ and $NI$ respectively denote infringement and no

\(^5\)Suppose that the probability that the PE’s product infringes a particular patent is $q$ and this probability is the same and independent of each other across patents. Then, the probability that the PE’s product will infringe at least one patent is given by $\theta = 1 - (1 - q)^S$, where $S$ is the number of patents held by the NPE. More generally, the overall probability of infringing on a patent portfolio depends on the size as well as the infringement chances with respect to individual patents.

\(^6\)The order would be inconsequential if the two PEs are symmetric.

\(^7\)We consider the case of product market competition among the PEs in Section 4.
infringement. The probabilities of each event are given by:

\[
\begin{align*}
\Pr(I, I) &= \theta^2 + \rho\theta(1 - \theta), \quad \Pr(I, NI) = \Pr(NI, I) = (1 - \rho)\theta(1 - \theta) \\
\Pr(NI, NI) &= (1 - \theta)^2 + \rho\theta(1 - \theta).
\end{align*}
\]

We can interpret \( \rho \) as a correlation coefficient in litigation outcomes across the PEs. If \( \rho = 1 \), there is perfect correlation between the litigation outcomes. At the other extreme, if \( \rho = 0 \), the litigation outcomes are independent. As a result, the litigation outcome for one party does not reveal any information about the likelihood of litigation outcomes for the other party. More generally, the updated beliefs about one PE’s infringement probability given the litigation outcome for the other one is given by

\[
\begin{align*}
\Pr(I | I \text{ for the other firm}) &= \frac{\Pr(I, I)}{\Pr(I)} = \theta + \rho(1 - \theta) \equiv \bar{\theta}, \\
\Pr(I | NI \text{ for the other firm}) &= \frac{\Pr(I, NI)}{\Pr(NI)} = (1 - \rho)\theta \equiv \bar{\theta}.
\end{align*}
\]

**Figure 1** below illustrates how the infringement probability is updated depending on the litigation outcome for the other party. The gap between the two lines, \( \bar{\theta} - \theta \), is given by \( \rho \). As expected, a higher \( \rho \) leads to more information revelation from litigation on the infringing probability of other firms.

![Figure 1: Updating of infringement probability](image)

Now let us analyze the NPE’s incentives to litigate against the PEs. Let \( D_i \) denote the expected damage payment or prospective licensing revenue the NPE expects to receive from firm \( i \) if the NPE litigates and firm \( i \) is found to infringe on the NPE’s patent portfolio. We can imagine various scenarios in which \( D_i \) is determined. Until *eBay v. MercExchange*, an injunction order was issued more or less automatically in the absence of exceptional circumstances if a patent was found valid and infringed.
Suppose, for instance, that injunctive relief is granted to the NPE when it wins in the litigation case. Then, the NPE can threaten to shut down the business of the PE and extract licensing revenues. If we assume Nash bargaining between the NPE and the infringing firm, the expected payment from PE\(_i\) would be \(D_i = \frac{\pi_i}{2}\), where \(\pi_i, i = 1, 2\), denotes firm \(i\)’s operating profit without litigation.

However, in the landmark case of eBay, the Supreme Court unanimously ruled that the decision to grant an injunction should be based on traditional principles of equity. In particular, it can be denied if legal damages are “sufficient to compensate for the infringement and an injunction may not serve the public interest.”\(^8\) Justice Kennedy’s concurring opinion, in which NPEs were characterized as firms using patents “not as a basis for producing and selling goods but, instead, primarily for obtaining licensing fees,” was often interpreted as advocating denial of injunctive relief to NPEs.\(^9\) In case an injunction is not available, the legal statute stipulates that the claimant be awarded lost profits “adequate to compensate for the infringement, but in no event less than a reasonable royalty.”\(^10\) For an NPE, the appropriate damage is a reasonable royalty rate because the NPE does not produce any products or services. *Georgia-Pacific* established 15 factors that can be considered in determining the reasonable royalty rate, with the essence being considered as a “hypothetical license” approach that defines the reasonable royalty rate as “[t]he amount that a licensor (such as the patentee) and a licensee (such as the infringer) would have agreed upon (at the time of the infringement began) if both had been reasonably and voluntarily trying to reach an agreement.”\(^11\) For instance, the availability of substitute technologies can be a factor that influences such an agreement and thus the reasonable royalty rate.

To accommodate various scenarios in which the patent holder is compensated, we adopt a general approach with the assumption that \(D_i = D(\pi_i)\) with \(0 < D_i < \pi_i\) and \(\frac{\partial D_i}{\partial \pi_i} \geq 0\). If injunctive relief is available to the NPE, \(D_i = \frac{\pi_i}{2}\). In the reasonable royalty rate case, we simply assumes that the damage payment is increasing in firm profits. Litigation incurs a cost of \(L > 0\) for each involved party.

To highlight the importance of information externalities, we first analyze the NPE’s incentives to litigate against the PEs when they use unrelated technologies \((\rho = 0)\) and, consequently, the litigation outcome against one PE has no implications for the other PE. In this case, the NPE has a credible


\(^9\)See, however, Denicolo et al. (2008) who interpret the *eBay* ruling as more of a call for a balancing test from a nearly automatic granting of injunctive relief in the past.

\(^10\)35 U.S.C. 284 (2006). The claimant can also be compensated for treble damages if a wilful infringement can be demonstrated.

incentive to litigate against firm $i$ if and only if

$$\theta D_i \geq L. \quad (1-i)$$

The NPE wins the infringement case against firm $i$ with probability $\theta$. In this case, the NPE receives a damage award payment $D_i$ from firm $i$.\textsuperscript{12} If the PE wins, it retains its entire market profit as it is not infringing on the NPE’s patent portfolio.

Now assume that the technologies of the PEs are related. The NPE first negotiates with firm 1 and then with firm 2. The NPE’s interaction with firm 1 can end up in three scenarios: They litigate and the NPE wins in court, they litigate and the PE wins, and finally, they do not litigate and settle out of court. Hence, the posterior belief that firm 2 infringes based on the outcome of the NPE’s interaction with firm 1 is given by $\hat{\theta} \in \{\bar{\theta}, \bar{\theta}, \bar{\theta}\}$. The NPE will have a credible threat to litigate against firm 2 if and only if

$$\hat{\theta} D_2 \geq L \quad (1)$$

and the value of the NPE’s patent portfolio with respect to firm 2 is given by

$$V_2(\hat{\theta}) = \begin{cases} \hat{\theta} D_2 & \text{if } L \leq \hat{\theta} D_2, \\
0 & \text{otherwise.} \end{cases}$$

In equilibrium, the NPE and firm 2 never litigate and the extent to which the NPE can extract rents from the PE in Nash bargaining depends on the threat of litigation, that is, the belief that firm 2 infringes.

Now consider the litigation incentives of the NPE with respect to PE$1$. To do so, let $\Psi_2$ define the information externality of litigating the first firm in terms of expected profits with the second firm. If the NPE successfully litigates against firm 1, the probability of infringement of the second firm is revised upwards. On the other hand, upon losing litigation, the probability of the second firm infringing decreases relative to the case where the firms settle. Hence, we have

$$\Psi_2 = \theta V_2(\bar{\theta}) + (1 - \theta)V_2(\bar{\theta}) - V_2(\theta).$$

It follows that the NPE has a credible incentive to litigate against the first firm if the sum of litigation

\textsuperscript{12}An earlier version of this paper also consider PEs with asymmetric infringement probabilities and derives qualitatively similar results. See Choi and Gerlach (2015) for more details.
profits with PE\(_1\) and the information externality with firm 2 is positive,

\[
\theta D_1 - L + \Psi_2 \geq 0.
\]

The next lemma determines the sign of the information externality.

**Lemma 1** If \(\theta D_2 < L < \theta D_2\), the information externality \(\Psi_2\) is negative. If \(\theta D_2 < L < \theta D_2\), then the information externality \(\Psi_2\) is positive. Otherwise, \(\Psi_2 = 0\).

**PROOF:**

\[
\Psi_2 = \begin{cases} 
\theta D_2 & \text{if } L \leq \theta D_2, \\
0 & \text{otherwise}
\end{cases} + \begin{cases} 
\theta D_2 & \text{if } L \leq \theta D_2, \\
0 & \text{otherwise}
\end{cases} - \begin{cases} 
\theta D_2 & \text{if } L \leq \theta D_2, \\
0 & \text{otherwise}
\end{cases} = \begin{cases} 
0 & \text{if } L \leq \theta D_2, \\
-\theta(1 - \theta) D_2 & \text{if } \theta D_2 < L \leq \theta D_2, \\
\theta \theta D_2 & \text{if } \theta D_2 < L \leq \theta D_2, \\
0 & \text{otherwise}.
\end{cases}
\]

The information externality can be positive or negative or positive as a function of the relationship between expected damage payments and cost of litigation. If the expected damage payments from the second firm are high relative to the cost of litigation, the externality can be negative. If the NPE settles with the first firm, no information is revealed to the second firm and the NPE still has a credible incentive to litigate and extract rents from that firm. By contrast, if the NPE litigates and loses, the expected probability of infringement of the second firm decreases and eliminates the threat of litigation and future licensing revenues with firm 2. Hence, the presence of a second PE exerts a negative information externality on the NPE.

If the expected payments from the second firm are small relative to the litigation cost, the externality can be positive. In the absence of litigation against the first firm, the NPE would not have a credible threat to sue the second firm. However, a positive litigation outcome could increase the perceived probability of infringement such that litigating the second firm would become credible. In this case, the presence of firm 2 has a positive externality on the NPE as a successful litigation outcome can raise licensing revenues with the other firm.

Firm 1 and the NPE settle rather than taking the case to court if their joint profits of settlement
exceed their joint profits from litigation, that is, if

\[ \pi_1 + V_2(\theta) \geq \pi_1 - 2L + \theta V_2(\theta) + (1 - \theta)V_2(\theta) \quad \text{or} \quad \Psi_2 \leq 2L. \]

The NPE settles with firm 1 if the information externality from litigation does not exceed the total cost of litigation. This holds, a priori, when the externality is negative or zero. In the presence of a positive externality, the most the NPE can extract, that is, the maximum value of \( \Psi_2 = \theta \bar{\theta} D_2 \) is the expected profits with the second firm, \( \theta D_2 \). Litigation would be optimal if this gain would exceed both parties’ litigation cost. However, a positive information externality requires that the expected profit with firm 2 is more than the litigation cost of the NPE. Hence, litigation never takes place in the benchmark model.\textsuperscript{13}

It follows that when condition (2) is satisfied, firms Nash bargain. The NPE receives a total expected profit of

\[ \Pi^{NPE} = \begin{cases} \theta D_1 + \Psi_2/2 & \text{if } L \leq \theta D_1 + \Psi_2, \\ 0 & \text{otherwise.} \end{cases} \]

We can thus characterize the outcome of the benchmark model as follows.

**Proposition 1** Consider the equilibrium of the benchmark model with exogenously ordered sequential litigation.

(i) There is no litigation in equilibrium.

(ii) When the information externality is negative and \( D_1 \leq (2 - \theta)D_2 \), there exist parameter values such that the NPE has no credible litigation threat with respect to the first firm although litigation would be credible if it would deal with this firm in isolation.

(iii) When the information externality is positive, there exist parameter values such that the NPE has a credible threat to litigate against firm 1 although it would not be credible to sue that firm in isolation.

(iv) Compared to the case with unrelated technologies, the NPE may be able to extract higher (lower) total licensing fees when the information externality is positive (negative).

In equilibrium, the NPE never sues the first firm for infringement and no information is revealed in the process. The NPE then interacts with the second firm as in isolation. The information externality affects the NPE through its effect on the credibility of litigation incentives and on the Nash bargaining settlement with firm 1 via the negotiation threat points.

\textsuperscript{13}The model should thus be viewed as informing the terms of settlement in licensing contracts rather than offering predictions about the conditions under which litigation takes place. However, see the analysis in section 4, where litigation can take place in equilibrium.
The presence of information externalities can explain different types of troll business models. For instance, consider a case where the NPE would have an incentive to litigate against firm 1 in isolation, that is, \( L \leq \theta D_1 \). However, due to the fact that the NPE might lose licensing revenues with firm 2 if it loses litigation with firm 1, the NPE will not enforce its property rights with the first firm and wait for the other, more lucrative target. This equilibrium outcome can explain the behavior of “lottery ticket” patent trolls that aim at and wait for opportunities for a big payout rather than pursuing every licensing opportunity in the presence of a negative information externality. In contrast, the “bottom feeder” business model of patent trolls can be explained by the presence of a positive information externality. Consider a case where \( L > \theta D_i, \ i = 1, 2 \). In this case, both PEs’ profits are too low relative to the litigation cost to make litigation profitable for the NPE when dealing with them in isolation. However, the possibility of a positive information externality from a successful litigation outcome increases the threat of litigation against firm 1 and allows the NPE to extract additional rents in negotiations. Despite the relatively high cost of litigation, information externalities allow the NPE to create a litigation threat and make profit.

Figure 2 below shows a diagram in the \((L, \rho)\) space that illustrates the credibility of the litigation threat against firm 1 for the symmetric case of \( \pi_1 = \pi_2 = \pi \) and \( D_1 = D_2 = D(\pi) = D \). The grey shaded area depicts all parameter values for which litigation credibility against PE_1 is affected due to the presence of an information externality. Area A in the graph refers to point (ii) of the Proposition. As \( L \leq \theta D \), the NPE would have an incentive to litigate against firm 1 in isolation. Nonetheless, it will not enforce its property rights with the first firm due to the possibility of a negative information externality in dealing with the other PE. Area B refers to point (iii) of the Proposition. As \( L > \theta D \), both PEs’ profits are too low to make litigation profitable for the NPE when dealing with them in isolation. However, the presence of a positive information externality allows the NPE to retain litigation credibility and extract licensing revenues from firm 1.

How does the information externality affect total licensing revenues of the NPE relative to a situation with unrelated technologies? In regions A and C the NPE is worse off, whereas in region B profits are higher. In region A, due to the negative information externality, the NPE is only able to extract rents from firm 2 whereas it would be able to extract rents from both firms in isolation. In region C, the NPE is able to sell a license to firm 1 but negotiated license fees are lower due to the lower threat point of litigation. Finally, in region B, the NPE would not receive any license income with uncorrelated technologies. However, due to the positive information externality, litigation becomes credible and the NPE can extract rents from firm 1.
3 Strategic Litigation Target Choice

In the previous section we have assumed that the PEs arrive in a predetermined order. Suppose now that the PEs are both operating in their respective market. The NPE can thus choose whether to approach the PEs simultaneously or sequentially, and which firm to target first. Suppose, without loss of generality, that PE$_2$ is the more profitable target, that is $\pi_1 < \pi_2$ and $D_1 \leq D_2$. In this section we investigate the optimal negotiation strategy and target choice for the NPE in the presence of information externalities.

In analyzing strategic litigation target choice, we assume that the NPE negotiates with the PEs only once, either in sequence or simultaneously. For instance, when the NPE approaches one PE and the first target refuses to pay, the NPE has two choices: either litigate against the first target or just move on to the other target; it cannot come back to the first target again later after it strikes a deal or engages in litigation with the remaining PE. We justify this assumption on two grounds. First, the assumption is made for analytical simplicity. Even if we allow the NPE to come back later to the first target who refuses to pay, we can derive qualitatively the same results. Second, when the first target refuses to pay and the NPE decides not to act on its threat, the NPE’s inaction may be interpreted as a tacit withdrawal of patent claims and the NPE may be barred to bring an infringement suit against the first target based on equitable estoppel.\footnote{See, for example, \textit{Aspex Eyewear Inc. v. Clariti Eyewear, Inc.}, Nos. 09-1147, -1162 (Fed. Cir. May 24, 2010).}

Similar to the benchmark model in the previous section, define $V_1$ and $\Psi_1$ as the continuation value.
and information externality when the NPE approaches firm 2 first. Furthermore, let us write the litigation credibility constraint of the NPE when he approaches the PEs in the order of firm $i$ first and firm $j$ second, where $j \neq i$, as

$$\theta D_i - L + \Psi_j \geq 0. \quad (2-i)$$

Moreover, let $\Pi_{ij}^{NPE}$ denote the NPE's payoff from sequentially approaching the PEs in the order of firm $i$ first and firm $j$ second. Then it follows from our analysis above that

$$\Pi_{ij}^{NPE} = \begin{cases} 
\theta D_i + \Psi_j/2 & \text{if (2-i) holds,} \\
0 & \text{otherwise.} 
\end{cases} + V_j(\theta). \quad (3)$$

Now consider the situation where the NPE approaches both firms simultaneously. In this case, there are no information externalities and the NPE settles with both firms at the same terms as if the technologies were unrelated. Hence, with simultaneous negotiations, the NPE makes a profit of $V_1(\theta) + V_2(\theta)$.

The NPE chooses the negotiation strategy, sequential or simultaneous, that maximizes his expected profit. When solving for the optimal litigation strategy, we adopt the following tie-breaking rule. If the NPE achieves the same expected profits with simultaneous negotiations and with the best sequential negotiation strategy, he chooses the former. The NPE’s optimal target choice can then be characterized as follows.

**Proposition 2** Consider the NPE’s optimal target choice with two PEs and $D_2 \geq D_1$.

(i) If $\Psi_1 \leq 0$ and $\Psi_2 \leq 0$, simultaneous negotiations are at least as profitable as sequential negotiations.

(ii) If $\Psi_1 > 0$ and (2-2) is satisfied, the NPE chooses sequential negotiations with the more profitable firm 2 as first target.

(iii) If $\Psi_2 > \Psi_1 = 0$ and (2-1) is satisfied, the NPE chooses sequential negotiations with the less profitable firm 1 as first target.

The timing of negotiations is irrelevant when there are no information externalities. In this case, it follows from Eq. (3) that the NPE’s profits with sequential negotiations are the same as with simultaneous negotiations and equal to $V_1(\theta) + V_2(\theta)$. **Figure 3** below illustrates in the $(L, \rho)$ space under which conditions this irrelevance result holds. If at least one of the externalities is negative while the other

---

16We implicitly assume that there are no economies of scale when the NPE litigates both PEs with the same patent portfolio. Economies of scale would make simultaneous litigation more attractive without changing the qualitative result of this section.
is not strictly positive, the NPE is at least as well off with simultaneous negotiations as with the best sequential strategy. When the NPE has a credible threat against either PE in isolation, both information externalities can be strictly negative. In this case, simultaneous negotiations are strictly preferred by the NPE. In particular, note that if litigation against the smaller firm is credible in isolation, then sequential negotiations never arise in equilibrium as no firm can create a positive externality.

What is the optimal first target for the NPE in sequential negotiations? Consider the litigation credibility constraints (2-i). If the NPE has a credible threat against firm \( i \) in isolation, then this firm can only create a negative externality as second target. Vice versa, if there is no credible threat in isolation against a firm, then this firm can only generate a positive externality as second target. This suggests that the NPE should target first the firm against which he has a credible threat in isolation and then approach the firm against which he has no threat. Consider Figure 3 below for values \( \theta D_1 \leq L \leq D_2 \) such that \( \Psi_1 \geq 0 \) and \( \Psi_2 \leq 0 \). It is easy to check that the sequence where the NPE approaches PE \( 1 \) and then PE \( 2 \) is not sustainable against the first target. Since the NPE has no threat against PE \( 1 \) in isolation and the information externality of the other PE is non-positive, condition (2-1) cannot be satisfied. By contrast, approaching PE \( 2 \) first is always sustainable since the NPE has a credible threat in isolation and the information externality of PE \( 1 \) is non-negative. Sequential negotiations with firm 2 as first target dominates simultaneous negotiations if and only if a positive information externality can be generated. This is part of the statement in point (ii) of the proposition. In the figure, the dark shaded area depicts the parameter values such that targeting the bigger firm first is optimal.

Now consider values such that the NPE has no credible threat against either firm in isolation. It is clear that if \( \Psi_1 = 0 \), then condition (2-2) cannot be satisfied and the 21-sequence is not sustainable. At the same time there exist values such that (2-1) holds and \( \Psi_2 > 0 \). For these values, which are light shaded in Figure 3, approaching the small firm first is optimal. This yields point (iii) in the proposition. Note that, if \( \theta D_2 < D_1 \), then there exist values such that the NPE has no credible threat against either firm in isolation and both generate a positive information externality. Here the only source of revenue is with the first target. Even though the magnitude of the information externality is larger when the less profitable firm is the first target, the direct effect of extracting licensing income from the more profitable firm outweighs the indirect effect of the positive information externality from targeting the less profitable firm. For the same reason, the credibility constraint is weakly easier to satisfy with firm 2 as the first target. Hence, whenever targeting 2 is credible for \( \Psi_2 \geq \Psi_1 > 0 \), it arises as the optimal strategy for the NPE. This completes point (ii) of the proposition above.

To sum up, we would expect to observe NPEs targeting small firms first in situations where both firms’ profits are small relative to the cost of litigation. In this case the NPE would have no incentive to
litigate against either firm in isolation and the only way to extract money is to approach the small firm and use the positive externality from the large firm to extract rents. This strategy is thus an extreme form of the “bottom feeder” business model. By contrast, if the NPE faces one firm that is sufficiently profitable such that there is a credible threat in isolation and the technologies are closely related, then the optimal first target is the bigger firm.

Figure 3: Strategic litigation targets for NPE for $\theta \pi_2 \leq \pi_2$.

Our results on strategic sequencing of litigation targets are related to the literature on optimal negotiation sequence. Krasteva and Yildirim (2012), for instance, consider the sequencing choice of a buyer who negotiates with the sellers of two complementary objects with uncertain payoffs. They show that the buyer’s optimal sequencing is to negotiate with the weak seller if the sellers have diverse bargaining powers. If all buyer’s valuations were common knowledge, the buyer would be indifferent to the sequence. Our model, however, analyzes very different issues and assumes symmetric information between the NPE and PEs. Nonetheless, sequencing matters for the NPE. In addition, we allow simultaneous bargaining whereas most papers in the literature do not allow such a possibility.\textsuperscript{17}

\textsuperscript{17}Krasteva and Yildirim (2014) extend their analysis to allow for endogenous information acquisition. They show that for moderate complements, the value of information is negative and the buyer would optimally commit to be uninformed even with costless information. In our model, the NPE’s choice of simultaneous bargaining can be interpreted as a way of limiting information externalities.
4 Downstream Competition and Injunctive Relief

We have analyzed the NPE’s litigation strategies in the framework that can encompass both enforcement by injunction and enforcement by liability for damages. One policy question in relation to NPEs has been the availability of injunctive relief. More specifically, people expressed concerns that injunctive relief confers NPEs the ability to “hold up” PEs with the threat to shut down their businesses once they have made sunk investments, which can lead to licensing royalties far in excess of the true value of the patents involved (Lemley and Shapiro, 2007; Shapiro 2010). This hold-up concern led to the landmark case of *eBay* in which the US Supreme Court established four equitable factors that should be considered in determining whether an injunction should issue. In particular, Justice Kennedy recognized NPE business models in which “firms use patents not as a basis for producing and selling goods but, instead, primarily for obtaining licensing fees.” This concurring opinion in the case has been interpreted as denial of injunctive relief to NPEs, as injunctions can be used “as a bargaining tool to charge exorbitant fees.”

When an NPE litigates against competing PEs, we point out that the availability of injunctive relief raises a new set of issues as the NPE acquires the ability to monopolize the downstream market to increase its profits. Consider our benchmark and suppose that the two PEs are product market competitors. Negotiations are again sequential; the NPE first bargains with firm 1 and then with firm 2. Let $\pi^d$ denote the duopoly profits each PE is making when they both either own a license for the NPE’s technology or are not found infringing or are not challenged by the NPE. If exactly one firm gets a license or exactly one firm is not infringing, then this firm earns monopoly profits $\pi^m(\geq 2\pi^d)$.

Consider the bargaining with the second firm when firm 1 is active in the product market. This could be either due to the fact that firm 1 settled (out of court or after an infringement verdict) or the court found firm 1’s technology not to be infringing on the NPE’s patent. Given the product market presence of firm 1, the NPE and the second firm always prefer to settle to avoid the cost of litigation. With the availability of injunction as a remedy, the NPE’s profits with the second firm are thus

$$V^d(\hat{\theta}) = \begin{cases} \hat{\theta} \pi^d / 2 & \text{if } L \leq \hat{\theta} \pi^d / 2, \\ 0 & \text{otherwise.} \end{cases}$$

---

18 According to legal theory, injunction is a property rule of entitling the claimant in that patent owners have a right to completely prevent all uses of the patented technology. In contrast, under a liability rule, someone might use the patent with adequate compensation to the patent holder. See Calabresi and Melamed (1972) for a distinction between property rules and liability rules.

19 For injunctive relief, a plaintiff need “to demonstrate: (1) that it has suffered an irreparable injury; (2) that remedies available at law are inadequate to compensate for that injury; (3) that considering the balance of hardships between the plaintiff and defendant, a remedy in equity is warranted; and (4) that the public interest would not be disserved by a permanent injunction.” See *eBay Inc. v. MercExchange, L.L.C.*, 547 U.S. 388 (2006).
Now consider the second case in which the first firm was found infringing and the NPE did not sell a license to firm 1. With the updated belief of \( \hat{\theta} = \bar{\theta} \), the NPE again settles with the second firm and receives

\[
V^m(\bar{\theta}) = \begin{cases} 
\bar{\theta}\pi^m/2 & \text{if } L \leq \bar{\theta}\pi^m/2, \\
0 & \text{otherwise.}
\end{cases}
\]

Let us turn to the NPE’s negotiations with the first firm. Suppose the NPE decides to litigate and wins the court case. The NPE now has the choice to either license the technology or use the injunction to exclude PE\(_1\) from the market. Exclusion occurs when the expected profits of the NPE when selling a single license to PE\(_2\) exceed the joint profits of the NPE with PE\(_1\), that is,

\[
V^m(\bar{\theta}) \geq \pi^d + V^d(\bar{\theta}).
\]

The next lemma characterizes under which condition this equation is satisfied.

**Lemma 2** Suppose the first PE has been found infringing on the NPE’s patent. If product market competition is sufficiently intense, the NPE uses the injunction to exclude the infringing PE.

An injunction allows the NPE to exclude the first PE from the market and reach a license agreement with the second PE as the monopolistic supplier in the market. By contrast, if the NPE sells a license to the first PE, the available rent he can extract from the second PE depends on the degree of product market competition. The lower the duopoly profits \( \pi^d \), the lower the joint profits of PE\(_1\) and NPE, and the more profitable is exclusion. Note that exclusion can be optimal in situations where the NPE has a credible threat against PE\(_2\) and, a fortiori, when there is no such threat.

If exclusion is not optimal, PE\(_1\) and NPE Nash bargain and share their joint surplus. Let \( V^I \) and \( J^I \) denote the NPE profits and the joint profits of PE\(_1\) and NPE, respectively, when the court finds that PE\(_1\) infringes on the patent and injunctive relief is available. In its initial negotiations with the first PE, the NPE has a credible threat to litigate if

\[
[\theta V^I(\bar{\theta}) + (1 - \theta)V^d(\bar{\theta})] - L \geq V^d(\theta).
\]

Litigation arises in equilibrium if the joint profits from licensing for NPE and firm 1 are less than the profits from litigation, that is

\[
\theta J^I(\bar{\theta}) + (1 - \theta)[\pi^d + V^d(\bar{\theta})] - 2L \geq \pi^d + V^d(\theta).
\]
The following proposition characterizes the equilibrium with product market competition and explores the role of injunctions on litigation incentives.

**Proposition 3** Suppose the PEs are competitors and injunctive relief is available.

(i) If product market competition is sufficiently intense and the cost of litigation relatively low, the NPE litigates against the first PE and, if successful, uses the injunction to exclude the firm while selling a license to the second PE.

(ii) When the technologies are perfectly related, the NPE has a credible litigation threat with the first PE, which is at least as strong as if the NPE would face a monopolist in the product market.

In the presence of injunctive relief, the NPE is able to exclude a PE, which has been found infringing on its patent. Such exclusionary licensing is profitable if downstream competition reduces the amount of rents that the NPE could extract in negotiations with competing PEs. Hence, if product market competition is intense and litigation cost is relatively low, the NPE is suing the first PE for infringement in an attempt to increase downstream profits.\(^{20}\) Even when the NPE prefers to settle with the first PE, the NPE is still able to use injunctive relief as the threat point of his bargaining with PE\(_1\). This is particularly effective when the technologies of the firms are closely related. Consider the case where the NPE has no litigation incentive against PE\(_2\) once he sells a license to PE\(_1\). If \(\rho\) approaches 1, condition (4) holds and the NPE always has an incentive to exclude PE\(_1\) should he prevail in court. Hence, the threat point of the NPE with the first NPE is \(\theta \pi^m/2\), which is the same as if the NPE faces a monopolist in the product market. Overall, injunctive relief gives NPEs a stronger litigation threat and it may lead to costly litigation and inefficient exclusion of competitors.

### 5 Cost Shifting and Litigation Incentives of NPEs

In the US, the default rule for patent litigation is that each party bears its own attorneys’ fees. In patent cases, under 35 U.S.C. §285, attorneys’ fees are only shifted in exceptional cases, which have been very rare. The Congress is currently considering different pieces of legislation that all aim to reduce NPE patent litigation by adopting “loser-pays” fee shifting, also called the British rule of legal fee allocation. The idea is that if NPEs face the possibility of paying the target firm’s attorneys’ fees, they would not initiate litigation unless the case has sufficient merit. In this section, we analyze whether the British rule of legal fee allocation reduces the NPE’s incentives to litigate and the profitability of their business model relative to the American rule.

\(^{20}\)Note that Figure 4 in the Appendix depicts the relevant constraints in a \(L - \pi^d\) diagram to illustrate our results.
Consider the set-up of our benchmark model under the British rule with symmetric firms such that $\pi_1 = \pi_2$ and $D_1 = D_2$. Suppose the NPE is facing the second PE. Under the British rule, the NPE only pays attorneys’ fees when losing the court case but then also has to cover the defendant’s legal fees. Hence, for a given belief $\hat{\theta}$, the NPE has an incentive to litigate the second PE if and only if

$$\hat{\theta}D_2 - (1 - \hat{\theta})2L \geq 0 \text{ or } L \leq \frac{\hat{\theta} D_2}{1 - \hat{\theta}} \equiv L(\hat{\theta}).$$

(7)

As a benchmark, consider $\hat{\theta} = \theta$. In this case, the condition coincides with the incentive constraint for litigation when the NPE faces each PE in isolation. Under the British rule the expected cost of litigation is the total legal fees $2L$ times the probability of losing. Under the American rule the cost of litigation is $L$ independent of the outcome. Hence, the British rule gives more litigation credibility than the American rule if and only if the perceived infringement is greater than $1/2$.

When the NPE has a credible threat against the second PE for a given belief $\hat{\theta}$, Nash bargaining ensues and the value of the NPE’s patent portfolio with respect to firm 2 is given by

$$V^B(\hat{\theta}) = \begin{cases} \hat{\theta}D_2 + 2L(\hat{\theta} - 1/2) & \text{if } L \leq L(\hat{\theta}), \\ 0 & \text{otherwise}. \end{cases}$$

With the British cost allocation, the NPE receives the expected damage payment plus or minus an amount that reflects who is more likely to pay all legal fees should they proceed to a trial. The party, that is more likely to win litigation, can leverage the expected overall legal fees to increase its profit share in Nash bargaining. In particular, if $\bar{\theta} > 1/2$ ($\bar{\theta} > 1/2$), then following a win (loss) against PE1, the NPE earns more than the expected damage with the second PE.

We can now analyze the decision to litigate against the first firm. The NPE has a credible threat of litigation if

$$\theta D_1 - (1 - \theta)2L + \Psi^B \geq 0$$

(8)

where the information externality under the British rule is defined as

$$\Psi^B = \theta V^B(\bar{\theta}) + (1 - \theta)V^B(\theta) - V^B(\theta).$$

Let $L^B$ be the value of the litigation cost at which the information externality is equal to zero for $L(\bar{\theta}) \leq L \leq L(\theta)$, that is,

$$L^B = \frac{\theta D_2}{1/2 - \theta}.$$
and consider the sign of the information externality under the loser-pays rule.

**Lemma 3** If $L(\theta) \leq L < \min\{L(\theta), L^B\}$, then the information externality is negative. If $\min\{L(\theta), L^B\} \leq L \leq L(\bar{\theta})$, then the information externality is positive. Otherwise, it is zero.

The British cost allocation can generate a positive information externality in situations where litigating PE$_2$ is credible in isolation but not after a loss against PE$_1$. This is due to the fact that winning against the first PE allows the NPE a stronger leverage of the legal fees in Nash bargaining with PE$_2$ and a higher settlement payment. This effect is stronger when the technologies are closer related and the actual cost of litigation is high. By contrast, if $\rho < 1/2$, then the information externality is always negative. When the NPE only has a credible threat against PE$_2$ after a win against the first PE, the information externality is always positive.\footnote{These results are illustrated in Figure 5 in the appendix to Lemma 3.}

The next proposition characterizes the incentives to litigate against the first PE under the British cost allocation rule.

**Proposition 4** Consider litigation incentives under the British fee allocation rule.

(i) When the information externality is negative and $\rho < 1/2$, there exist parameter values such that the NPE has no incentive to litigate against firm 1 whereas it would have a credible threat against each PE in isolation.

(ii) When the information externality is positive, the NPE’s incentives to litigate are at least as strong as when it faces each PE in isolation.

(iii) With perfectly related technologies and $\theta > 2/3$, the NPE always has a credible threat to litigate against the first PE.

We thus get qualitatively similar results to the benchmark model with the American rule. In particular, a change to the British rule does not affect the existence of the two NPE business models. Point (iii) gives conditions under which the bottom feeder model works particularly well under the British rule. When facing independent PEs, a NPE is unable to have a credible threat for any level of litigation costs unless $\theta = 1$ (see condition (7)). With multiple PEs and perfectly correlated technologies, a win against the first PE enables the NPE to extract all legal fees in the settlement with the second PEs. Hence, if the initial merit is sufficiently high, $\theta > 2/3$, the NPE always has a credible threat against the first PE. Finally,
note that the equilibrium profits of the NPE under the British rule are given by

$$\Pi_{B}^{NPE} = \theta D + 2L(\theta - 1/2) + \Psi^B/2 + V^B(\theta)$$

Let us now compare litigation incentives and NPE profitability under the American and the British rule. First, consider equations (2) and (8) and note that the latter condition is easier to satisfy if

$$2L(\theta - \frac{1}{2}) \geq \Psi - \Psi^B.$$  \hspace{1cm} (9)

The LHS gives the savings in legal fees when litigating against the first PE. The RHS is difference in the informational externality that the American and British rule generate. Without information externalities, we get the standard result that the British rule makes litigation more credible if and only if the case is more likely to be won by the NPE. Information externalities can tilt this balance one way or the other. If the information externality under the American rule is stronger than there exist values $\theta > 1/2$ such that litigation under the American rule is more credible. Vice versa, if the information externality under the British rule is stronger, then there exist values $\theta < 1/2$ where litigation is more credible when the loser pays all legal fees.

Now consider the NPE’s profits with the two cost allocation rules. The NPE makes higher profits under the British rule if

$$2L(\theta - \frac{1}{2}) \geq (\Psi - \Psi^B)/2 + V(\theta) - V^B(\theta).$$  \hspace{1cm} (10)

The RHS is now the difference in profits with the American and British rule composed of the gains in negotiation with the first PE and the actual profits with the second PE after settling with PE$_1$. This leads us to the main result of this section.

**Proposition 5** Compare litigation credibility and NPE profits under the American and British rule.

(i) Consider parameter values such that the bottom feeder model can arise under both rules. There exists a $\theta' < 1/2$ such that if $\theta > \theta'$, the NPE has more credibility and higher profits under the British rule.

(ii) Consider parameter values such that the lottery ticket model can arise under both rules. There exists a $\theta'' < 1/2$ such that if $\theta > \theta''$, the NPE has more credibility and higher profits under the British rule.

The proposition implies that both NPE business models can gain more credibility and yield higher profits with the British rule of legal cost allocation. Since the threshold values for $\theta$ are less than 1/2, this result holds in situation where, in the absence of any information externality, the American rule would provide more credibility and higher profits. The first point refers to situations in which litigation against PE$_2$
is only credible if the NPE wins against firm 1 under both the American and British rule. In this case, the British rule provides a stronger information externality if \( \bar{\theta} > 1/2 \) as the NPE can extract legal fees from the second PE. This stronger information externality can outweigh the advantage the American rule has when the merit of the case is less than 1/2. The second point applies to parameter values such that, under both rules, litigation against PE\_2 is credible after settlement with the first PE but not after a loss in court. Here the British rule yields a stronger information externality if and only if \( \bar{\theta} < 1/2 \). This condition is satisfied for any \( \theta \leq 1/2 \) and, again, the stronger information externality can make up for the credibility advantage the American rule has when it is more likely that the first PE wins.

For other parameter values than the ones given in Proposition 5, the results are more ambiguous. In particular, if \( \theta > 1/2 \), then there exist values such that under the British rule, the information externality is negative and litigation not credible while under the American rule, it is positive and the NPE has a credible threat. Vice versa, if \( \theta < 1/2 \), then there are situations where the American rule is generating a negative externality and there is no credibility while under the British rule, the externality is positive and litigation is credible.\(^{22}\) Hence, overall, the relative credibility and profitability of the legal fee allocation rules depends on specific parameter values. This precludes any general policy conclusions as discussed in the introduction to this section.

6 Patent Portfolio Acquisition

Our analysis so far has assumed that a non-practicing entity has a patent portfolio of certain strength. We now analyze the NPE’s incentives to acquire patent portfolio vis-a-vis PEs’. Suppose that a patent portfolio of strength \( \theta > 0 \) has been put up for sale. We ask which type of entities is more likely to acquire the patent portfolio. To illustrate the implications of litigation externalities for patent portfolio acquisition incentives, we consider the simplest setting of one NPE and two PEs bidding for the available patent packet. To simplify the analysis, we analyze a setting in which all parties have no existing patent portfolios.\(^{23}\) As a benchmark case, we first establish that all firms have the same willingness to pay for the patent portfolio in the absence of any litigation externalities.

**Lemma 4** If there are no litigation externalities (i.e., \( \rho = 0 \)), all firms bidding for the patent portfolio have the same willingness to pay.

The intuition for this result is simple. The acquisition incentives for the NPE are determined by the

---

\(^{22}\) We show this point more formally in the appendix under the proof of the previous proposition.

\(^{23}\) We can easily extend the analysis of the game to a setting in which firms have existing patent portfolios. See Choi and Gerlach (2017) for more details of such an analysis.
amount of licensing revenues it can extract from the two PEs. Let \( R_1 \) and \( R_2 \) be the amount of revenues the NPE can extract from each firm with the acquisition of the patent portfolio. Then, the NPE’s maximum willingness to pay is \((R_1 + R_2)\). For PE_1, its payoff from the acquisition of the patent portfolio is \( R_2 \), which is the licensing income it can generate from PE_2.\(^{24}\) If PE_1 does not acquire the patent portfolio, its payoff will be \(-R_1\). As a result, PE_1’s maximum willingness to pay for the patent portfolio is the same as the NPE’s and given by \((R_1 + R_2)\). The same logic applies to PE_2.

However, if we allow for the possibility of litigation externalities, we can show that the NPE has higher incentives to acquire patent portfolios than any PEs due to a free rider problem between PEs. To see this, consider the case of two symmetric PEs with positive information externalities, that is, \( D_1 = D_2 = D \) and \( \theta D < L < \bar{\theta} D \) with \( \Psi = \theta \bar{\theta} D \). In this case, the NPE can only extract rents from the first PE it approaches while PEs have no credible threat against each other if they acquire the patent. If the NPE gets the patent, it randomly chooses one of the PEs and extracts \( \theta D + \Psi / 2 \). From the PEs’ perspectives, they will be chosen as the first target with probability \( 1/2 \). Acquiring the patent only serves a defensive purpose as each PE is willing to bid up to the expected losses from the NPE’s purchase of the patent packet, that is up to \([\theta D + \Psi / 2]/2\). Hence, the NPE has a higher willingness to pay.

For this type of result to hold, some uncertainty about the identity of the NPE’s first litigation target is required at the time of patent portfolio acquisition. This could be due to the fact that the NPE considers the PEs as equally valuable targets like above. This argument also applies to situations where the expected damage payment of each PE is unknown to the NPE and only revealed in litigation. If the ex ante expected damage payment from each PE is the same, the NPE will be indifferent about which firm to target first. A second possibility is that, at the moment of acquisition, the PEs are equally valuable but at the moment of choosing the first litigation target, the NPE strictly prefers one PE.

Let us briefly sketch this argument. If we consider positive information externalities with \( D_1 < D_2 \), the NPE will choose PE_2 as the first target and once the NPE settles with PE_2, it does not have any litigation credibility against PE_1. In this case of complete information, the NPE and PE_2 will have the same willingness to acquire the patent portfolio. Now suppose that acquisition takes place before the market matures and the values of the products are uncertain until they are actually introduced. We thus introduce uncertainty about \( D_i \) at the time of patent portfolio acquisition. Consider the following timing. When a patent portfolio is up for sale, the potential damage payments \( D_i \) are not known but they are distributed according to \( F(.) \). Once the patent portfolio acquisition takes place, the values of

\(^{24}\)Here the assumption that PE_2 does not have any existing patent portfolio it can use against PE_1 as a countermeasure is important. If there is any existing patent portfolio for PE_2, the licensing income PE_1 can generate will depend on the relative patent portfolio strength and can be different from what the NPE can extract from PE_2.
Lemma 5 Assume that \( D_1 \) and \( D_2 \) are distributed according to a joint distribution \( F(.,.) \) on \( R_+^2 \) and revealed after acquisition but before any litigation decision. Then, the NPE has a strictly higher willingness to pay for the patent portfolio for sale.

Suppose the distributions were restricted to values of \( D_i \) such that there are only positive information externalities for both firms. In this case, the firm with the higher realized \( D_i \) is the first target. As the ranking is uncertain at the time of acquisition, our result follows. For other realizations of \( D_i \), the NPE has at least as high a willingness to pay for the patent portfolio as the maximum of the PEs' willingness to pay. This is due to the fact that the NPE can avoid negative information externalities by litigating simultaneously. Thus, the NPE will have a strictly higher willingness to pay and we can summarize the findings as follows.

Proposition 6 In the presence of a positive information externality and if there is some uncertainty about the identity of the first litigation target at the time of acquisition, the NPE has a higher willingness to pay for the patent portfolio than the PEs.

7 Extensions

In this section, we extend the model in several directions and checks the robustness of the main results.

7.1 Unobservable Settlements

In this extension we show that our results are robust to the possibility that the NPE signs confidential licensing agreements. When the fact whether a licensing agreement has been signed is not observable, an individual PE does not know whether the other PE has signed and the amount the NPE can extract depends on the beliefs. Consider the following simple set-up. First, the NPE decides whether to approach PE\(_1\) or PE\(_2\). Let \( p_1 \) be the probability with which he first approaches PE\(_1\). This choice is not observed by the PEs who believe that PE\(_1\) is approached first with probability \( b_1 \). When the NPE initiates a patent dispute with a PE, the PE makes a settlement proposal \( s_i \). The NPE can accept or litigate and then

\(^{25}\)Given the two PEs are active and known at the time of acquisition, we assume that the NPE can choose to litigate simultaneously or sequentially. If the NPE does not have the option to litigate both PEs simultaneously, the PEs might have a higher willingness to pay due to negative information externalities.

\(^{26}\)The assumption that the uninformed party makes the settlement proposal leads to a “screening” game and is used, for example, in P’ng (1983), Bebchuk (1984) and Daughety and Reinganum (1999).
approaches the remaining PE. Suppose that an acceptance is not observed whereas litigation becomes public. After observing the litigation outcome with \( \text{PE}_j \), \( \text{PE}_i \) proposes a settlement \( s_i^j \). Again the NPE can either accept or litigate. For simplicity, we focus on symmetric PEs, \( \pi_1 = \pi_2 = \pi \) and \( D = D(\pi) \). We also restrict attention to parameter values such that the NPE would have no incentives to litigate against one PE when he has settled with the other one, that is, \( \theta D < L \leq \theta D \). We look for Perfect Bayesian Equilibria of this game.

Suppose there is litigation with \( \text{PE}_i \) and the NPE wins. Under our parameter assumption, the NPE now has a credible threat to litigate \( \text{PE}_j \). When the NPE approaches, \( \text{PE}_j \) optimally offers \( s_j^i = \theta D - L \) to avoid litigation. Now suppose there is no litigation with the first PE. The NPE always accepts the settlement proposal of the second PE that he approaches. Moreover, he accepts the offer from the first PE he approaches if his total license revenue exceeds the expected profits from going to trial with the first PE,

\[
s_1 + s_2 \geq \theta D - L + \theta(\theta D - L). \tag{11}
\]

Now consider the PE’s incentives. When the NPE approaches a PE, the PE believes \( b_1 = p_1 \). \( \text{PE}_1 \) prefers to make a settlement offer \( s_1 \) that satisfies (11) to an offer that does not satisfy this condition (and induce litigation) if and only if

\[
p_1(\theta D + L) \geq s_1. \tag{12}
\]

Settlement at \( s_1 \) is optimal for \( \text{PE}_1 \) if the belief that he is the first PE to be approached is sufficiently high. Similarly, \( \text{PE}_2 \) prefers to induce a settlement at \( s_2 \) rather than risking litigation if

\[
(1 - p_1)(\theta D + L) \geq s_2, \tag{13}
\]

that is, when the probability that \( \text{PE}_1 \) has been approached first is small. It is immediate that conditions (11), (12) and (13) can be satisfied jointly if and only if

\[
L \geq \frac{\theta \theta}{2 + \theta} D
\]

which always holds under our assumption for the parameter values. Since this condition is not a function of \( p_1 \) we get the following result.

**Proposition 7** Consider unobservable settlements and \( \theta D < L \leq \theta D \). There exists a Perfect Bayesian Equilibrium in which the NPE first approaches \( \text{PE}_1 \) with probability \( p_1 \in [0, 1] \) and extracts positive payments from both PEs.
Hence, an equilibrium where the NPE randomly picks the first PE exists alongside an equilibrium where the PEs know with certainty which firm the NPE is approaching first. The maximum amount the NPE can extract in an equilibrium is given by $\theta D + L$ and the PE that is more likely to be approached first is paying a higher share in settlement. In Daughety and Reinganum (1999), the informed party (defendant) benefits from confidential settlement to avoid publicity and follow-on suits from potential future plaintiffs and is willing to pay hush money. In this simple context, the NPE does not strictly benefit from a confidential settlement. In the next extension, we explore how this conclusion changes when we endogenize entry of potentially unaware PEs.

7.2 Secrecy, Awareness and Endogenous Entry

A natural extension of our framework is to explore the implications of NPE patent enforcement for innovation and entry incentives of PEs. In what follows we analyze the impact of endogenous entry on the incentives of the NPE to settle or litigate the first PE and whether to keep settlements open or confidential.

To fix ideas, suppose PE$_2$ is a potential entrant who has to sink an investment of $k < \pi_2$ dollars in order to enter the industry. Initially, PE$_2$ is not aware of the patents held by the NPE. However, if PE$_1$ and NPE initiate a patent dispute and settle or go to trial, PE$_2$ might become aware of the NPE’s intellectual property. In particular, let $p_S$ and $p_T$ be the probability that PE$_2$ becomes aware after a settlement or a trial, respectively. We assume that $0 < p_S \leq p_T < 1$ to reflect that a trial is more likely to create publicity than a settlement. Furthermore, we assume that the NPE and PE$_1$ have the option to keep the settlement open or confidential. If the settlement is kept open, the probability of PE$_2$ becoming aware is $p_S = \overline{p}_S$ whereas if the settlement is confidential it is $p_S = p_S < \overline{p}_S$.

We start solving the game from the back. If the PE$_2$ remains unaware, he will enter the industry. If he becomes aware of a trial (and its outcome) or a settlement, he enters if (i) litigation is not credible or if (ii) litigation is credible but the expected profit after settlement with the NPE exceeds the cost of entry, that is,

$$\pi_2 - \hat{\theta} D_2 \geq k.$$ 

Let $\theta^E$ denote the belief for which this condition holds with equality. If $\hat{\theta}$ is smaller than $\theta^E$, PE$_2$ enters the market. Assume that $\theta^E < \theta$. This implies that, given litigation is credible, PE$_2$ does not enter the market after a settlement or a trial in which the NPE won. If the NPE loses the trial (and litigation is credible), then PE$_2$ enters if and only if $\theta \leq \theta^E$ which holds for $\rho \geq \rho^E = \{ \rho | \theta^E(\rho) = \theta^E \}$. Hence, if the technologies of PE$_1$ and PE$_2$ are sufficiently close and the NPE loses in court against PE$_1$, then
the updated belief that PE₂ infringes decreases sharply and makes entry profitable. By contrast, when technologies are rather unrelated, a loss in court reduces the belief only slightly and PE₂ does not enter when he becomes aware of the trial outcome.

Now consider the NPE’s incentive to initiate litigation against the first PE. The NPE has an incentive to litigate rather than to settle with \( p_S \in \{ p_S, \overline{p}_S \} \) if

\[
\theta D_1 - L + \Psi^E_2 \geq 0
\]

where the information externality is given by

\[
\Psi^E_2 = \theta \begin{cases} 
(1 - p_T)\overline{\theta}D_2 & \text{if } L \leq \overline{\theta}D_2, \\
0 & \text{otherwise}
\end{cases} + (1 - \theta) \begin{cases} 
(1 - p_T)\overline{\theta}D_2 & \text{if } L \leq \overline{\theta}D_2 \land \rho < \rho^E, \\
\overline{\theta}D_2 & \text{if } L \leq \overline{\theta}D_2 \land \rho \geq \rho^E, \\
0 & \text{otherwise}
\end{cases}. 
\]

We immediately obtain the following results.

**Lemma 6** Consider the information externality \( \Psi^E_2 \) with endogenous entry.

(i) If \( L \leq \overline{\theta}D_2 \), the externality can be positive or negative. Otherwise, the information externality has the same sign as in the benchmark model.

(ii) The information externality (weakly) increases in \( p_S \).

When litigation costs are small relative to damages, litigation against the second PE₂ is always credible, independent of the outcome with PE₁. In our benchmark model, the fact that there is always credibility implied that the information externality is equal to zero. With endogenous entry, there are two additional effects. A trial makes it more likely that the potential entrant is aware of the NPE’s patents and refrains
from entering the industry. This introduces a negative information externality. At the same time, litigation can also promote entry in situations where the technologies of the PEs are closely related. If the NPE loses in court, the infringement belief of the potential entrant is reduced which lowers the expected payment for PE₂ and entry becomes profitable while there would have been no entry after settlement. This second effect can outweigh the first effect if \( \rho \geq \rho^E \) and

\[
\rho \leq \frac{p_S - \theta p_T}{(1 - \theta)p_T} \equiv \rho'.
\]

Hence, when \( \rho^E < \rho' \), there exist parameter values such that the information externality is positive. This is more likely the case if settlements create sufficient publicity and the initial infringement belief is low.

For higher litigation costs, the qualitative result of the benchmark model carry over to the set-up with endogenous entry of a potentially aware PE₂. For \( \theta D_2 < L \leq \theta D_2 \), the information externality is negative like in the benchmark. It is smaller (that is, the negative externality is stronger) with endogenous entry if and only if \( \bar{\theta} p_T > p_S \). Finally, for \( \theta D_2 < L \leq \bar{\theta} D_2 \), the information externality is positive but smaller relative to the benchmark model.

The second point in the lemma implies that more secretive settlements reduce the litigation credibility of the NPE with respect to the first PE. More secretive settlements make the entry of PE₂ more likely. This, however, increases the NPE’s incentive to settle rather than to litigate. As a consequence, open settlements are easier to enforce for the NPE.

This leads us to the question as to whether the NPE and PE₁ choose open or confidential settlements. When they settle, their joint profit is the profit of PE₁ and the NPE’s continuation profit with PE₂. A confidential settlement increases the continuation profit as it reduces the awareness of potential entrants. Hence, confidential settlement are more profitable. However, since confidentiality reduces the NPE’s litigation credibility, open settlements arise when secret settlements are not enforceable.

Finally, let us consider the joint incentives to litigate. The NPE and PE₁ prefer litigation to settlement with \( p_S \in \{p_S, \bar{p}_S\} \) if \( \Psi^E_2 \geq 2L \). As shown above, the information externality can be positive when the NPE always has a credible litigation threat. It is then clear that if the litigation cost is sufficiently small relative to the damages, there exist parameter values such that litigation is optimal. In those instances, litigation is used to encourage entry by the second PE who would not enter in case of a settlement. If the NPE loses in court, the entrant believes that he is less likely to infringe on the NPE’s patents and enters the industry. We can thus summarize as follows.

**Proposition 8** Consider litigation and disclosure incentives with endogenous entry. If litigation costs
are small relative to damages, the NPE and PE choose secret settlements. Otherwise, settlements are open. When technologies are of intermediate correlation, the NPE might litigate against the first PE in equilibrium.

7.3 Endogenous Litigation Effort

We have assumed that the probability that the patent holder prevails in litigation is exogenously given by the relationship between patent claims and the technologies of the PEs. The probability of winning in the court may also depend on the litigation efforts by both parties. In this extension we show that when litigation efforts are endogenous, two additional effects arise, which might increase or decrease the credibility of the NPE.

Consider the set-up of the benchmark model. In each period one PE enters. The market profit of both PEs is given by $\pi$ and the associated expected damage payment is $D$. To focus on the effects from endogenous litigation effort, suppose the fixed cost of litigation is zero, that is, $L = 0$. Instead there are discretionary legal expenses that each party can spend to influence the court outcome. Let $e_N$ and $e_P$ be the variable legal expenses incurred by the NPE and a PE, respectively. We consider a Tullock type contest to model the strategic litigation effort of the parties. In this contest, the effectiveness of legal expenses depends on the perceived strength of the infringement case. This could, for example, reflect the fact that producing convincing evidence is harder, the worse the case is stuck against a party. Hence, assume that the expected probability of the NPE winning the court case is given by

$$p(e_N, e_P) = \frac{\theta e_N}{\theta e_N + (1 - \theta)e_P}.$$ 

At equal levels of expenditure, the winning probability is equal to the expected merit of the case $\theta$.

We solve backwards and first look at the NPE facing the second PE when the expected probability of infringement is $\hat{\theta}$. Suppose the NPE and PE take their case to court. The NPE maximizes his expected profit net of his legal expenses,

$$\max_{e_N} p(e_N, e_P)D - e_N.$$ 

The PE chooses the litigation effort that minimizes his expected loss and legal expenses or

$$\max_{e_P} \pi - p(e_N, e_P)D - e_P.$$ 

It is straightforward to show that there exists a Nash equilibrium with positive effort from both parties.
The following lemma summarizes this equilibrium.\footnote{A more detailed analysis of this set-up including all proofs can be found in Choi and Gerlach (2015).}

**Lemma 7** The NPE and the second PE exert the same amount of effort in court. The equilibrium winning probability of the NPE is \( \hat{\theta} \). Both parties make positive profits in equilibrium and the NPE’s litigation threat is always credible.

In the absence of fixed cost, the NPE’s threat of litigation is always credible. However, both parties are better off avoiding costly litigation in court. In the Nash bargaining settlement, the NPE gets

\[
V_2(\hat{\theta}) = D + \frac{1}{2}[p^* D - e_N^*] - \frac{1}{2}[\pi - p^* D - e_P^*] = \hat{\theta} D.
\]

In other words, the NPE can expect the same value from the second PE as in the benchmark model while always maintaining a credible litigation threat.

Let us turn to the NPE’s interaction with the first PE. Suppose the NPE initiates litigation. The first PE’s litigation effort solves the same maximization problem as the second PE. In contrast, the NPE takes into account the effect of the litigation outcome on the infringement belief and the profits with the second PE. This means the NPE choose his effort following

\[
\max_{e_N} \Pi_N = p(e_N, e_P)[D + V_2(\theta)] + (1 - p(e_N, e_P))V_2(\theta) - e_N.
\]

In the unique Nash equilibrium with positive effort levels, we get

\[
e_P^* = p^*(1 - p^*)D, e_N^* = (1 + \rho) e_P^* \text{ and } p^* = \theta + \frac{\rho \theta (1 - \theta)}{1 + \rho \theta} \geq \theta.
\]

Hence, with correlated technologies, the NPE invests more than the first PE and the probability of winning is higher than the initial merit of the case.

Now consider the individual litigation incentive constraint for the NPE. Litigation is credible if the current and future expected gains from litigation outweigh the future profits from settlement when there is no additional information revealed via litigation, that is, if

\[
p^* D - e_N^* + \Omega L \geq 0
\]

(14)
where the externality term is defined as

$$\Omega^L = p^*V_2(\theta) + (1 - p^*)V_2(\theta) - V_2(\theta) = (p^* - \theta)\rho D \geq 0.$$ 

Unlike in the benchmark model, the information externality is not driven by the litigation incentive against the second PE as the NPE’s threat is always credible with endogenous litigation costs. Litigating against the first PE has the benefit of endogenously raising the success probability above the initial merit of the case. This makes it more likely to face the second PE with a strong case for infringement and raises the expected future settlement payoff. Hence, like in the benchmark model, there is a positive externality of litigation.

However, there might also be a negative effect. In the presence of a second PE, PE1 and NPE invest more in legal expenses relative to what they would with isolated PEs. This means that the contest dissipates more rents and litigation is more costly for the NPE. In particular, it can be shown that if $\theta$ is sufficiently small, the incentive constraint (14) is not satisfied and the NPE has no incentive to litigate against the first PE. To see this, consider a marginal increase in $\theta$ at $\theta = 0$. This change has a first-order effect on future profits in the incentive constraint but a negligible effect on the endogenous litigation efforts with the first PE.

**Proposition 9** Consider the model with endogenous litigation effort. If the initial merit of the case $\theta$ is sufficiently low, the NPE has no credible threat of litigation against the first PE and is worse off relative to facing the PEs in isolation. Otherwise, if the threat against the first PE is credible, the NPE always extracts more rents relative to facing two isolated PEs. Litigation never occurs in equilibrium.

With endogenous litigation efforts, two cases arises. If the NPE has a credible threat to litigate against the first PE, then the positive externality allows the NPE to extract more rents relative to facing unrelated, isolated PEs. However, if litigating the first PE is too costly and condition (14) fails to hold, the NPE is worse off in the presence of a second PE.

### 7.4 More Than Two Practicing Entities

Consider an extension of our model with $N$ symmetric PEs ($D_1=\ldots=D_N=D$), which the NPE approaches in an exogenous order starting from PE1. To reduce the number of cases to consider, we focus on the “bottom-feeder” NPE business model and use a simple information structure, where the litigation outcomes for PE$_n$, where $n \in \{2, 3, \ldots, N\}$, are either perfectly correlated or uncorrelated with each other, but the same correlation assumed earlier holds for PE1 and others.
7.4.1 The Case of Perfectly Uncorrelated Technologies among \( n \in \{2,3,\ldots,N\} \)

In this case, litigation outcomes for any \( n \in \{2,3,\ldots,N\} \) have no further consequences in terms of information revelation for other PEs. Thus, the NPE’s litigation incentives against firms in this set can be independently analyzed, that is, the NPE will have a credible incentive to litigate against firm \( n \in \{2,3,\ldots,N\} \) if and only if \( \hat{\theta}D \geq L \). It is clear that if this condition is not satisfied, the NPE will have no credible litigation incentive against any firm. However, if it holds, the information externality of litigating the first firm in terms of expected profits with the rest of the firms can be written as \( \Psi(N) = \theta(N-1)\hat{\theta}D \).

The NPE has a credible threat against PE\(_1\) if

\[
\theta D - L + \Psi(N) = \theta D - L + \theta(N-1)\hat{\theta}D \geq 0
\]

Thus, there exists a critical level of \( N^* \) such that for all \( N \geq N^* \) the NPE has a credible threat against PE\(_1\). Now let us analyze whether litigation actually takes place in equilibrium. If the NPE does not litigate against PE\(_1\) and settles out of court, the NPE will not have any credible threat against any other PEs. A settlement maximizes the joint profits of NPE and PE\(_1\) if

\[
\Psi(N) = \theta(N-1)\hat{\theta}D \leq 2L
\]

This implies that there is another critical number of \( N^{**} \) such that for all \( N \leq N^{**} \), the NPE and PE\(_1\) settle out of court. By contrast, if \( N > N^{**} \), the magnitude of the information externality from litigating PE\(_1\) is too large. PE\(_1\)’s legal cost saving is not large enough to compensate the positive information externality that would enable the NPE to collect further licensing fees from other PEs in case it wins against the first PE.

**Proposition 10** In the case of uncorrelated technologies \( n \in \{2,3,\ldots,N\} \) we have two critical number of firms \( N^* \) and \( N^{**} \) such that (i) if \( N < N^* \), the NPE has no credible threat to litigate against any PEs, (ii) if \( N^* < N < N^{**} \), the NPE has a credible threat against PE\(_1\) and collects a license fee, but has no credible threat against the remaining PEs, and (iii) if \( N > N^{**} \), the NPE litigates against PE\(_1\) and if the NPE wins it will collect further license fees from the remaining firms.

7.4.2 The Case of Perfect Correlation among \( n \in \{2,3,\ldots,N\} \)

In this case, litigation outcomes for any \( n \in \{2,3,\ldots,N\} \) reveal full information for the remaining PEs. Similar to the case of uncorrelated technologies, we find that the NPE can have credible threat against
early PEs, but not from late PEs because litigation does not arise on the equilibrium path. In the case of uncorrelated technologies among \( n \in \{2, 3, ..., N\} \), the only way to establish litigation credibility against \( \text{PE}_n, n \in \{2, 3, ..., N\} \), is actually to win against \( \text{PE}_1 \), and once the NPE wins it can collect license fees from all other firms. However, with perfect correlation the NPE can still maintain credibility against early PEs without litigating \( \text{PE}_1 \). We summarize our findings in the following proposition with the details in the Appendix.

**Proposition 11** In the case of perfect correlation among \( n \in \{2, 3, ..., N\} \), the NPE can have credible litigation threats against early PEs in the sequence but there is no litigation on the equilibrium path.

8 Concluding Remarks

We have developed a model of patent trolls to understand NPEs’ business models and litigation tactics used to maximize their licensing revenues. We considered a setting in which the technologies covered by NPEs’ patent portfolios can be potentially infringed by multiple PEs who use related technologies. The main driver of our analysis is information that can be revealed in litigation for future licensing negotiations with other potential infringers. If the outcome of current litigation affects the credibility of future litigation threat, information externalities arise. The presence of a positive information externality can explain the “bottom feeder” business model of patent trolls. The existence of other PEs from which the NPE can extract rents in case of a win in court against the current PE increases the credibility of the NPE. This allows the NPE to extract rents even in situations where the settlement payments are low relative to the expected cost of litigation. A negative information externality is consistent with the business model of a “lottery ticket” patent troll who foregoes smaller licensing targets in order not to risk the chance of a big payout with a large target in the future.

Moreover, our simple framework yields several interesting predictions and policy implications. Within the “bottom feeder” business model, early entrants in an industry are more likely to be approached and involved in patent disputes than later entrants. The opposite is true for “lottery ticket” patent trolls where patent disputes arise for later entrants. With two active PEs in the marketplace, we expect the NPE to target the less profitable firm first when both firms’ profits are small relative to the expected cost of litigation. When one firm is sufficiently more profitable and technologies are strongly related, then the NPE targets the more profitable firm first. When NPEs have access to injunctive relief in order to stop infringement, patent litigation arises when PEs are close competitors in the product market and firms’ profits are high relative to litigation cost. In the context of patent acquisition, we show that the NPE is
willing to bid more than any of the PEs individually and is thus more likely to obtain patent portfolios that are put up for sale. With respect to policy recommendations, our analysis shows that a change in litigation fee allocation to the British rule ("loser pays") has ambiguous effects on the credibility and profitability of NPEs. Finally, giving NPEs access to injunctive relief against infringing PEs can have negative welfare effects as it may lead to excessive litigation and exclusion of competitors in the marketplace.

While our analysis does touch on the issues of PE entry and innovation, more research is needed to explore the implications of NPEs’ litigation incentives for dynamic industry efficiency. For instance, Bessen et al. (2011) estimate that NPE lawsuits are associated with half a trillion dollars of lost wealth to defendants from 1990 to 2010 and reduced innovation incentives. In a different vein, Cohen et al. (2015) empirically document that increased litigation risk by NPEs has driven innovators to shield themselves by shifting innovation away from public and private firms into universities. To mitigate negative impacts of NPEs, they suggest “cost-shifting” in legal fees to limit the power of NPEs. However, our analysis indicates that the effect of cost-shifting on litigation incentives can be subtle in the presence of multiple PEs, reversing the standard result that the British rule is less favorable to the NPE when the merit of the case is low. Our analysis of litigation externalities also suggests that there could be unmeasured costs of reduced entry of PEs when additional entry restores litigation credibility for NPEs.

Appendix

PROOF of Proposition 1: (ii) Assume $\theta D_2 < L \leq \theta D_2$. Condition (2) holds if

\[ L \leq \theta(D_1 - (1 - \theta)D_2). \]

The RHS is increasing in $\rho$ and takes value $\theta D_1$ at $\rho = 1$. There always exist parameter values to satisfy this condition if the RHS at $\rho = 0$ is less than $\theta D_2$ or

\[ \theta(D_1 - (1 - \theta)D_2) \leq \theta D_2. \]

This gives the condition in the text. For point (iii), if $\theta D_2 < L \leq \bar{\theta} D_2$, then (2) holds if

\[ L \leq \theta(D_1 + \bar{\theta}D_2). \]

The RHS is increasing in $\rho$ and takes value $\theta(D_1 + D_2) > \theta D_1$ at $\rho = 1$. For point (iv), suppose $\Psi_2 < 0$. In this case, the NPE makes profits of $\theta D_1 + \theta D_2$ when the PEs technologies are unrelated. Consider
correlated technologies. If litigation against PE is credible, the NPE earns \( \theta D_1 + \Psi_2/2 + \theta D_2 \) while if litigation is not credible it gets \( \theta D_2 \). Suppose \( \Psi_2 > 0 \). If the NPE faces unrelated PEs he obtains \( V_1(\theta) \). With correlated technologies, the NPE \( \theta D_1 + \Psi_2/2 \) if litigation against PE is credible. If litigation is not credible, the NPE makes zero profits with and without correlated technologies. The proposition follows.

**PROOF of Proposition 2:** For notational convenience, let \( L_{ij} \) be the threshold value for which condition (2-i) holds with equality when firm \( i \) is the first target. With the assumption of \( D_1 \leq D_2 \), it can easily be shown that parameters such that \( \Psi_1 < 0 \) and \( \Psi_2 > 0 \) do not exist. For point (ii), there are two distinct cases to consider with \( \Psi_1 > 0 \). First consider \( \Psi_2 \leq 0 < \Psi_1 \), which corresponds to parameter values such that \( \theta D_1 < L \leq \min\{\theta D_1, \theta D_2\} \). Here the 21-sequence allows credibility against the first target as (2-2) is satisfied since \( L < \theta D_2 + \Psi_1 \). However, there is no credibility against the second target after settlement with the first one. By contrast, since \( L > \theta D_1 + \Psi_2 \), the 12-sequence is not credible against the first target. Finally, the 21-sequence strictly dominates simultaneous negotiations since

\[
\theta D_2 + \frac{1}{2} \Psi_1 > \theta D_2.
\]

The second case is where \( \Psi_2 > \Psi_1 > 0 \) which holds if \( \theta D_2 < L \leq \theta D_1 \). Such values only exist if \( D_1 > \theta D_2 \).

In this case, litigation against the second target is not credible for either sequence. If litigation with the first target is credible, profits are weakly higher in the 21-sequence since

\[
\theta D_2 + \frac{1}{2} \Psi_1 = \theta (1 - \theta) D_2 \geq \theta D_1 + \frac{1}{2} \Psi_2 = \theta (1 - \theta) D_1.
\]

Moreover, we have

\[
L_{21} = \theta D_2 + \theta \theta D_1 \geq L_{12} = \theta D_1 + \theta \theta D_2.
\]

The 21-sequence is always credible while the 12-sequence is credible and there exist values where the 21-sequence is the only sustainable strategy. Finally, since simultaneous negotiations yield no returns, the NPE strictly prefers the 21-sequence as long as (2-2) is satisfied. For point (iii): The case \( \Psi_2 > \Psi_1 = 0 \) corresponds to parameter values such that \( L > \theta D_1 \) and \( \theta D_2 < L \leq \theta D_2 \). Litigation is not credible in the 21-sequence and with simultaneous negotiations. However, if \( L \leq L_{21} \), the 12-sequence has a credible threat with the first target. As \( L_{21}(\rho = 0) = \theta D_2 + \theta^2 D_1 > \theta D_2 \) such parameter values always exist. For point (iv), three more cases are possible. The case \( \Psi_1 < \Psi_2 = 0 \) corresponds to parameter values such that \( \theta D_1 < L \leq \theta D_1 \) and \( L < \theta D_2 \). Litigation is always credible against the second target in both sequences. Litigation is also always credible with the first target in the 12-sequence. If \( L \leq L_{21} \), then litigation is
credible against the first target in the 21-sequence. However, the 12-sequence is more profitable since
\[ \theta D_1 + \frac{1}{2} \Psi_2 + \theta D_2 > \theta D_2 + \frac{1}{2} \Psi_1 + \theta D_1 \]
for \( \Psi_1 < \Psi_2 = 0 \). However, the best sequential strategy cannot improve on simultaneous litigation which yields \( \theta(D_1 + D_2) \). The second case to consider is \( \Psi_2 < 0 = \Psi_1 \) corresponding to parameter values such that \( L > \bar{\theta}D_1 \) and \( \theta D_2 < L \leq \theta D_2 \). The 12-sequence has no credibility. The 21-sequence has a credible threat against the first firm but as \( \Psi_1 = 0 \), the NPE makes the same profit, \( \theta D_2 \), as with simultaneous litigation. The last case is \( \Psi_2 < \Psi_1 < 0 \) which holds if \( \bar{\theta} \pi_2/2 < L \leq \bar{\theta} \pi_1/2 \). If sustainable the equilibrium profit for the NPE with firm \( i \) as first target is always less than what it can earn with simultaneous negotiations,
\[ \theta D_i + \frac{1}{2} \Psi_j + \theta D_j < \theta D_i + \theta D_j \]
since \( \Psi_j < 0 \). The proposition follows.

**PROOF of Lemma 2 and Proposition 3:** Consider \( L \leq \bar{\theta} \pi^d/2 \). Condition (4) holds if and only if
\[ \pi^d \leq \frac{\bar{\theta}}{2 + \bar{\theta} \pi^m}. \]
Let \( \Gamma_1 \) be the set of all parameter values \( (L, \pi^d) \) such that for \( L \leq \bar{\theta} \pi^d/2 \), the NPE prefers licensing. Then consider \( \bar{\theta} \pi^d/2 < L \leq \bar{\theta} \pi^m/2 \). Condition (4) holds if and only if
\[ \pi^d \leq \frac{\bar{\theta} \pi^m}{2 \bar{\theta} \pi^m}. \]
Let \( \Gamma_2 \) be the set of all parameter values \( (L, \pi^d) \) such that \( \bar{\theta} \pi^d/2 < L \leq \bar{\theta} \pi^m/2 \) and the NPE prefers licensing. The statement in the lemma follows. The NPE’s profits from a successful litigation outcome with PE\(_1\) are
\[ V^I(\bar{\theta}) = \frac{\pi^d}{2} + \begin{cases} \bar{\theta}(\pi^d + \pi^m)/4 & \text{if } (L, \pi^d) \in \Gamma_1, \\ \bar{\theta}(\pi^m)/4 & \text{if } (L, \pi^d) \in \Gamma_2, \\ (\bar{\theta} \pi^m - \pi^d)/2 & \text{otherwise,} \end{cases} \]
while the joint profits are

\[ J^I(\bar{\theta}) = \pi^d + \begin{cases} \bar{\theta}\pi^d / 2 & \text{if } (L, \pi^d) \in \Gamma_1, \\ 0 & \text{if } (L, \pi^d) \in \Gamma_2, \\ \bar{\theta}\pi^m / 2 - \pi^d & \text{otherwise.} \end{cases} \]

For point (i) of the proposition, consider condition (6). Upon inspection, if \((L, \pi^d) \in \Gamma_1\) or if \((L, \pi^d) \in \Gamma_2\), the condition cannot be satisfied. For the remaining parameter values, successful litigation leads to the exclusion of PE_1. Three cases need to be distinguished. First, if additionally \(L \leq \theta\pi^d / 2\), the litigation condition holds if

\[ L \leq \frac{1}{4} \bar{\theta}\pi^m - |3\theta - (1 - \theta)\bar{\theta}| \frac{1}{4} \pi^d. \]

The RHS is decreasing in \(\pi^d\) and positive if and only if \(\pi^d \leq \bar{\theta}\pi^m / (2 + \bar{\theta})\). Hence, the condition is satisfied if \(L\) and \(\pi^d\) sufficiently small. Second, if \(\theta\pi^d / 2 < L \leq \theta\pi^d / 2\), condition (6) is satisfied if

\[ L \leq \frac{1}{4} \bar{\theta}\pi^m - \frac{3\theta}{4} \pi^d, \]

which always holds for \(L\) and \(\pi^d\) sufficiently small. Finally, if \(L > \theta\pi^d / 2\), the condition is

\[ L \leq \frac{1}{4} \bar{\theta}\pi^m - \theta\pi^d \]

and point (i) of the proposition follows. For point (ii), note that the set \(\Gamma_2\) is empty at \(\bar{\theta} = 1\). Consider \((L, \pi^d) \in \Gamma_1\). If \(0 < L \leq \theta\pi^d / 2\), condition (5) is satisfied for \(\rho = 1\) if

\[ L \leq \frac{1}{4} \theta(\pi^d + \pi^m). \]

The RHS is always larger than \(\theta\pi^d / 2\). Hence, the condition is satisfied. If \(\theta\pi^d / 2 < L < \pi^d / 2\), the condition is

\[ L \leq \frac{1}{4} \theta(\pi^m + 3\pi^d). \]

The RHS is larger than \(\theta\pi^m / 2\) if \(\pi^d \geq \pi^m / 3\), that is for any admissible value of this parameter region. Finally consider \((L, \pi^d)\) such that the NPE would exclude the first PE after winning the court case. If \(0 < L \leq \theta\pi^d / 2\), condition (5) is satisfied for \(\rho = 1\) if

\[ L \leq \frac{1}{2} \theta(\pi^m - \pi^d). \]
The RHS is always larger than $\theta \pi^d/2$ and the condition holds. If $L > \theta \pi^d/2$, the condition is $L \leq \theta \pi^m/2$ and the point follows.

![Figure 4: Litigation Incentives with Product Market Competition and Injunctive Relief.](image)

**PROOF of Lemma 3:**

$$
\Psi^B = \begin{cases}
\theta D_2 + 2L(\bar{\theta} - 1/2) & \text{if } L \leq L(\bar{\theta}), \\
+ (1 - \theta) & \text{if } L \leq L(\bar{\theta}), \\
0 & \text{otherwise}.
\end{cases}
$$

Note that $\Psi^B$ is linear and continuous in $L$ for $L(\theta) < L \leq L(\bar{\theta})$. At $L = L(\theta)$ we have $\Psi^B = \theta(\rho - 1/2)D_2$ which can be positive or negative. Next, as $L$ approaches $L(\theta)$ from above, $\Psi^B$ goes to $\theta D_2[\theta + (1 - \theta)2\rho]/[2(1 - \theta)] > 0$. $\Psi^B$ is linear in $L$, takes value $\theta D_2[\theta + \rho]/[2(1 - \theta)(1 - \rho)] > 0$ at $L(\bar{\theta})$ and, hence, the externality is strictly positive for $L(\theta) < L \leq L(\bar{\theta})$.

**PROOF of Proposition 4:** For point (i), note that for $L(\theta) < L \leq L(\theta)$ and $D = D_1 = D_2$ condition
(8) is

\[ L \leq \frac{\theta \bar{\theta}}{1 + \theta - 2\theta \bar{\theta}} D. \]

The RHS is increasing in \( \rho \) and is less or equal than \( L(\theta) \) if and only if

\[ \frac{\theta D(1/2 - \rho)}{(1 - \theta)(1 + \theta - 2\theta \bar{\theta})} \geq 0 \]

or \( \rho \leq 1/2 \). For \( \rho \geq 1/2 \), there is always an incentive to litigate for \( L(\theta) < L \leq L(\bar{\theta}) \). Point (i) follows directly from Lemma 3. For point (iii) note that \( L(\theta) \) goes to \( \infty \) as \( \rho \) goes to 1. For \( L(\theta) \leq L \leq L(\bar{\theta}) \), condition (8) holds if

\[ L \leq \frac{\theta(1 + \bar{\theta})}{1 - \theta(1/2 + \bar{\theta})} D. \]

The RHS is larger than \( L(\bar{\theta}) \) if and only if

\[ \rho \leq \frac{\theta^2}{\theta^2 + 2 - 3\theta}. \]

This holds for \( \rho = 1 \) if and only if \( \theta \geq 2/3 \). The proposition follows.

**Figure 5:** Litigation incentives with the British rule of fee allocation.

**PROOF of Proposition 5:** For point (i) assume \( \theta D \leq L \leq \bar{\theta} D \) and \( L(\theta) \leq L \leq L(\bar{\theta}) \). Since \( V(\theta) = V^B(\theta) = 0 \) condition (10) holds when (9) is satisfied. Condition (9) is then satisfied if and only if

\[ 2L(\theta - 1/2) \geq 2\theta L(1/2 - \bar{\theta}) \quad \text{or} \quad \theta \bar{\theta} + \theta/2 - 1/2 \geq 0. \]

Since the LHS is increasing in \( \theta \) and \( \rho \), we have \( d\theta'/d\rho < 0 \). From \( \theta'(\rho = 0) = 1/2 \) then follows the first
point. For point (ii) assume \( \theta D \leq L \leq \theta D \) and \( L(\theta) \leq L \leq L(\theta) \). Condition (9) is satisfied if and only if

\[
2L(\theta - 1/2) \geq 2(1 - \theta)L(\theta - 1/2) \quad \text{or} \quad \bar{\theta} \geq 1/2.
\]

The LHS increases in \( \theta \) and \( \rho \) and at \( \rho = 0 \) it has to hold that \( \theta \geq 1/2 \). Finally, condition (10) holds if and only if

\[
2L(\theta - 1/2) \geq (1 - \theta)L(\theta - 1/2) - 2L(\theta - 1/2) \quad \text{or} \quad (3 + \theta)(2\theta - 1) + 2\rho\theta(1 - \theta) \geq 0.
\]

Again the LHS increases \( \theta \) and \( \rho \) and at \( \rho = 0 \) it has to hold that \( \theta \geq 1/2 \). It follows that \( \theta' < 1/2 \) for all \( \rho > 0 \). This completes point (ii). Finally, to prove the claim in the text after the proposition, assume

\[
\max\{\theta\bar{\theta}D, \theta D\} \leq L \leq \theta D \quad \text{and} \quad L(\theta) \leq L \leq L(\bar{\theta}).
\]

This requires \( \theta < 1/2 \) and ensures that litigation is not credible with the American rule while there is a positive information externality with the British rule. Litigation is credible under the British rule if

\[
L \leq \min\{\frac{\theta(1 + \bar{\theta})}{2(1 - \theta/2 - \theta\bar{\theta})}D, L(\bar{\theta})\}.
\]

It is easy to check that for \( \rho \) sufficiently large, the binding constraint is the first term. It suffices to prove that

\[
z(\theta) = \frac{\theta(1 + \bar{\theta})}{2(1 - \theta/2 - \theta\bar{\theta})}D - \theta\bar{\theta}D > 0
\]

for \( \theta \) sufficiently small. This follows directly from \( z(0) = 0 \) and \( z'(0) = (1 - \rho)D/2 > 0 \).

**PROOF of Proposition 9:** Condition (14) can be rewritten as

\[
\varphi = p^*[D + V_2(\bar{\theta}) - V_2(\theta)] - e^{*}_N - [V_2(\theta) - V_2(\theta)] \geq 0
\]

\[
= (1 + \rho)(p^*)^2D - \rho\theta D = [(1 + \rho)^3\theta^2/(1 + \rho\theta)^2 - \rho\theta]D \geq 0
\]

Check that

\[
\frac{\partial \varphi}{\partial \theta} = \frac{2(1 + \rho)^2\theta}{(1 + \rho\theta)^3} - \rho \quad \text{and} \quad \frac{\partial^2 \varphi}{(\partial \theta)^2} = \frac{2(1 + \rho)^2(1 - 2\rho\theta)}{(1 + \rho\theta)^4}.
\]

The second derivative is positive if \( \theta < 1/(2\rho) \) and negative otherwise. Hence, \( \varphi \) takes value 0 at \( \theta = 0 \), has a negative slope there and is convex for low values of \( \theta \) and concave for higher values. Moreover, since \( \varphi(\theta = 1) = 1 \), there exists a unique \( \theta' \) such that if \( \theta \geq \theta' \), the condition is satisfied. Vice versa, if \( \theta < \theta' \),
the condition does not hold. Litigation does not maximize the joint profits of the first PE and NPE since

\[ e^*_N + e^*_P - \Omega = \frac{\theta(1 - \theta)[2 + 3\rho - \theta\rho^3]}{(1 + \rho\theta)^2} \pi \frac{\pi}{2} > 0 \]

Now consider the difference between the NPE profits from bargaining with the first PE and the profits when facing two unrelated PEs, that is,

\[ p^*D + V_2(\theta) + \frac{1}{2}[\Omega - (e^*_N - e^*_P)] - 2V_2(\theta) = \frac{\theta(1 - \theta)[1 + \theta(2 + \rho\rho)]}{2(1 + \rho\theta)^2} D \geq 0. \]

**PROOF of Lemma 5:** Assume that \( D_1 \) and \( D_2 \) are distributed according to a joint distribution \( F(.,.) \) on an interval \([\bar{D}, \overline{D}]^2 \subset [L/\theta, L/\theta]^2\), that is, the sign of the information externality is always positive for all realizations of \( D_1 \) and \( D_2 \). Let \( D^{(1)} \) and \( D^{(2)} \) denote the first and second order statistic, respectively, i.e., \( D^{(1)} = \min\{D_1, D_2\} \) and \( D^{(2)} = \max\{D_1, D_2\} \). Then, given the parameter restrictions, the NPE’s willingness to pay for the patent portfolio is given by

\[ E_F[\theta D^{(2)} + \Psi^{(1)}]/2], \text{ where } \Psi^{(1)} = \theta D^{(1)}. \]

In contrast, each PE’s willingness to pay is \( E_F[\theta D_i + \Psi_j/2|D_i > D_j]\Pr[D_i > D_j], i = 1, 2 \) and \( j \neq i \). Since \( E_F[\theta D_i + \Psi_j/2|D_i > D_j]\Pr[D_i > D_j] > 0 \) and \( \sum_{i=1}^2 E_F[\theta D_i + \Psi_j/2|D_i > D_j]\Pr[D_i > D_j] = E_F[\theta D^{(2)} + \Psi^{(1)}]/2], \) the NPE has a higher willingness to pay for the patent portfolio than PEs. This result extends to distributions on \( R^2_+ \) as argued in the main text.

**PROOF of Proposition 11:** Let us consider the NPE’s incentives to litigate against firm \( n \) given that there was no litigation up to that point. The NPE will have a credible incentive to litigate against firm \( n \in \{2, 3, ..., N\} \) if and only if

\[ \theta D + \theta(N - n)/D \geq L \]

We consider three cases depending on the number of PEs \( N \). First, assume \( \theta(N - 1)/D \geq L \) such that the NPE retains incentives to litigate against at least some firms in the set of \( \{2, 3, ..., N\} \). More precisely, there exists an \( n^* (< N) \) such that the NPE will have a credible threat to litigate if and only if \( n \leq n^* \). Settlement with this firm is jointly optimal for the NPE and firm \( n^* \) if and only if \( \theta(N - n^*)D \leq 2L \). Note that this condition is always satisfied because the definition of \( n^* \) implies that \( \theta(N - n^*)D < L \). Following Nash bargaining, the licensing fee the NPE receives from firm \( n^* \) is given by \( s_{n^*} = \theta D + \theta(N - n^*)D/2 \). Now we use backward induction to analyze the NPE’s incentives against firm \( n^* - 1 \). By applying a
similar logic, there will be a settlement between the NPE and firm \(n^* - 1\) if

\[ \theta(N - n^*)D \leq 2L + \frac{\theta(N - n^*)D}{2}, \]

which is always satisfied. Nash bargaining leads to a licensing fee of \(s_{n^*-1} = \theta D + \theta(N - n^*)D/4\). In general, all firms \(n \in \{2, \ldots, n^*\}\) settle with the NPE who receives a licensing fee given by

\[ s_n = \theta D + \frac{\theta(N - n^*)D}{2n^*-n+1}. \]

We denote by \(S\) the sum of all settlement offers that can be collected from the set of \(\{2, 3, \ldots, N\}\) when the strength of the patent is given by \(\theta\). We can define the critical firm \(n^*\), settlement offers \(s_n\) and their sum \(S\) corresponding to \(\theta\), and similarly \(\pi^*\), \(s_n\) and \(S\), respectively, for \(\bar{\theta}\). Now consider incentives to litigate against firm 1. There will be a settlement if and only if \(\Psi \leq 2L\) where \(\Psi = \theta S + (1 - \theta)S - S\). By treating \(n\) as a continuous variable, we can show that this condition is always satisfied. To see this, note that

\[ S = \sum_{n=2}^{n^*} s_n = (\pi^* - 1)\bar{\theta}D + \sum_{n=2}^{n^*} \frac{\theta(N - \pi^*)D}{2n^*-n+1} < (\pi^* - 1)\bar{\theta}D + L \]

because \(\theta(N - \pi^*)D < L\) by the definition of \(\pi^*\). In addition, we have \((\pi^* - 1)\bar{\theta}D = \bar{\theta}N - L\) since \(\bar{\theta}(N - \pi^* + 1)D = L\) by the definition of \(\pi^*\). Thus, \(S < \bar{\theta}ND\). Similarly we have \(S < \theta ND\) and \(\theta ND - L < S < \theta ND\). Hence, \(\Psi < [\theta \bar{\theta} + (1 - \theta)\bar{\theta}]ND - (\theta ND - L) = L\) because \(\theta \bar{\theta} + (1 - \theta)\bar{\theta} = \theta\).

Under our maintained assumption of \(\theta D < L\), we can also show that \(\Psi \leq 2L\) when we explicitly consider the integer constraint. The second case arises when \(\theta(N - 1)D < L < \bar{\theta}(N - 1)D\) and the NPE does not have a credible threat against any firm \(n \in \{2, 3, \ldots, N\}\) if there is a settlement with firm 1. Against PE1, the NPE has a credible threat if \(\theta(D + S)) \geq L\) and a settlement occurs in equilibrium if \(\theta S \leq 2L\).

This condition always holds because \(S \leq (N - 1)D\) and thus \(\theta S \leq \theta(N - 1)D < L\) under the parameter restrictions we consider. Finally, in the last case \(\bar{\theta}(N - 1)D < L\) and the NPE does not have a credible threat against any firm \(n \in \{2, 3, \ldots, N\}\) regardless of litigation outcomes against PE1. Thus, the NPE simply has no threat against any firm including firm 1.
References


