Too Correlated To Fail: Bailouts and Systemically Important Institutions

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Introduction

- Financial crises are often preceded by herd behavior in investment
- These periods also feature growth of systemically important (SI) financial institutions
- We show how SI institutions emerge endogenously in a setting where government bailouts create a collective moral hazard problem
    - SI institutions emerge as an equilibrium outcome of *rent-seeking behavior* on the part of investors when there are incentives to correlate risk-taking

Basic Intuition

- Government bailouts generate a collective moral hazard problem in which investors have incentive to fail in the same states of the world
- Firms have incentive not only to undertake the same risky projects, but also to expose themselves to the same firm-level (idiosyncratic) shocks
- They way they coordinate this exposure is by lending, directly or indirectly, to the same firm, resulting in the endogenous emergence of SI institutions

Basic Intuition

- These firms are SI in that their failure is sufficient to trigger a system-wide misallocation of resources, and therefore a government bailout
- Futhermore, those firms which become SI are precisely the *riskiest* firms
- SI institutions thus emerge in equilibrium as a vehicle to maximize systemic risk and facilitate this collective rent-seeking behavior

Related Literature

- *Ex post* efficient bailouts and *ex ante* inefficiency: Farhi and Tirole (2012), Bianchi (2016)
- PE models of government bailouts and herding: Acharya and Yorulmazer (2007, 2008)
- Pecuniary externalities and over-borrowing and investment: Lorenzoni (2008), Davila and Korinek (2017)
- Financial networks and systemic risk: Erol (2017), Leitner (2005), Elliot, Golub, Jackson (2014)

Model Environment

- 3 periods: 0, 1, 2
  - All uncertainty resolved at date 1
- 2 types of goods: a consumption good and capital
- 2 types of agents: a risk-averse household, which owns 2 types of firms:
  - N investment firms, who make productive use of capital
  - 'Traditional' firms, who operate a less productive technology
- Consumption good can be converted freely into capital 1-1 at any date
- Capital can be converted into the consumption good only by firms, who have access to 2 projects

Investment Opportunities

- Firms have access to 2 investment projects, both with constant returns-to-scale
  - A 'prudent' project with risk-free date 1 return R_C
  - A risky project with date 1 return $R^i_A(s)$

- Any firm's risky project offers the same expected return as the prudent project, but with higher risk: $R_C = E[R_A]$
  - By construction, the first-best allocation of capital features investment in **only the prudent project**

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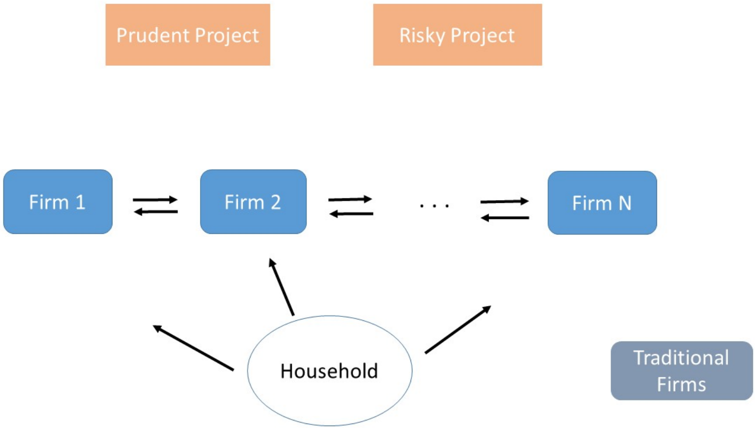
Firm Heterogeneity in 'Riskiness'

$$R_A^i(s) \;=\; \rho^i \underbrace{R_A(s)}_{\textit{common component}} + \underbrace{\sigma^i \varepsilon^i}_{\textit{idiosyncratic risk}}$$

- Firms are heterogeneous in their 'riskiness' along two separate dimentions:
  - Exposure to aggregate risk via ρ^i, and in their idiosyncratic risk via σ^i
- For now, take special case in which no idiosyncratic risk, so that $\sigma^i = 0$ for all i
  - Analytically more tractable; delivers key insights
- To finance investment, a firm can raise funds from two sources: the household or other firms.
  - Structure of a firm's liabilities will be governed in equilibrium by optimal contracts between the two types of agents

Environment



Contracting problem between household and firms

Key friction which drives inefficiency results

- The household and firms have a limited ability to commit to future payments
  - If a firm defaults, the household can seize the value of the firm's capital, net of liabilities to other creditors
  - No-default conditions: the liquidation value of a firm acts as collateral for its loan

$$0 \leq \underbrace{d_1^i(s) + d_2^i(s)}_{repayments} \leq \underbrace{L\left(s, k_0^i\right)}_{liquidation\ value\ of\ i}$$

- The optimal state-contingent contract limits firms' ability to efficiently allocate funds across states of the world
  - This opens the door for a misallocation of capital in the bad state

Household problem

- Household has access to financial contracts offered by each firm at date 0, and a riskless storage technology at date 1
- Chooses how much to consume, save, and whether to accept contracts, subject to budget constraints in each period

$$\max_{\{\{c_t(s)\}_s\}_t, \{f_0^i\}_i, \{B_1(s)\}_s} E\left[u(c_0) + u(c_1(s)) + u(c_2(s))\right]$$

$$c_0 + \Sigma_i f_0^i d_0^i \leq e_0 - T_0 \tag{1}$$

$$c_1(s) + B_1(s) \leq e_1 + \sum_i f_0^i d_1^i(s) - T_1 \tag{2}$$

$$c_2(s) \leq e_2 + (1+r)B_1(s) + \sum_i f_0^i d_2^i(s) + \Pi_2(s) - T_2 \tag{3}$$

Contracting environment between firms

- A financial contract between i and j $\{\ell^{ij}, \{r^{ij}(s)\}_s\}$
  - A date 0 transfer from i to j, and state-contingent repayments at date 1
- For simplicity, assume no incentive problems between any two firms
  - At date 1, firm j decides whether to make contractual payments to i
  - If j defaults, creditor i can seize at least the full value of the liability
- Defaulting on an inter-firm loan is never optimal. Therefore, a firm can credibly commit to honoring its contract with another firm
  - No default constraints that the contract needs to satisfy

Participation constraint

- However, an optimal contract needs to satisfy two other sets of constraints:
  - Feasibility constraints in each state at date 1: a firm's funds in state s should be sufficient to meet its contractual oblgations
  - A participaiton constraint at date 0, to incentivize another firm to accept the contract
- Firm j faces a participation constraint: value of firm i under contract must exceed its reservation value

$$\underbrace{u^{ij}\left(\ell^{ij},\left\{r^{ij}(s)\right\}_s\right)}_{\text{value of } i \text{ under contract with } j} \geq \underbrace{\bar{u}^i}_{\text{reservation value of } i}$$

- The reservation value of a firm depends on its opportunity cost of funds
- Thus, firms' investment decisions will determine the contracts offered in GE

Inter-firm lending

- The structure of inter-firm lending will be determined endogenously by the optimal contracts which prevail in equilibrium
- But fricitonless environment will mean that the structure the inter-firm credit network is largely irrelevant for allocations
  - So we don't need to keep track of the full structure of the network
- Inter-firm lending will matter only to the extent that some firms become 'systemically important'

Firms' choices at date 0

- At date 0, each firm first chooses how to allocate funds across loans to other firms, vs capital holdings
- Then chooses how to allocate capital holdings across projects, given each project's state-dependent returns

Firm's Problem

Each firm solves a joint contracting and portfolio allocation problem to maximize the value of the firm

$$\max \ E_0\left[m_2(s)\left(A-b_2^i(s)\right)k_1^i(s)\right] \tag{4}$$

subject to budget constraints at date 0 and in each state at date 1

$$k_0^i+\sum_j \ell^{ij} \leq n+d_0^i+\sum_j \ell^{ji} \tag{5}$$

$$q(s)k_1^i(s) \leq \left(q(s)+\vec{\omega}^{i\prime}\vec{R}^i(s)-\gamma-b_1^i(s)+g^i(s,\omega_C^i)\right)k_0^i-b_1^i(s)\left(\sum_j \ell^{ij}-\sum_h \ell^{hi}\right)+b_2^i(s)k_1^i(s)-\sum_h r^{hi}(s)\ell^{hi}+\sum_j r^{ij}(s)\ell^{ij} \tag{6}$$

Firm's Problem

no-default constraints for the household contract

$$0 \leq d_1^i(s) + d_2^i(s) \leq L\left(s, k_0^i\right)$$

the household participation constraint

$$u'(c_0)d_0^i \geq E\left[u'(c_1(s))\, b_1^i(s)\left(k_0^i - \sum_h \ell^{hi} + \sum_j \ell^{ij}\right) + \left(u'(c_2(s)) - u'(c_1(s))\right) b_2^i(s) k_1^i(s)\right] \tag{7}$$

and the other firms' participation constraints for each j

$$u^{ji}\left(\ell^{ji}, \left\{r^{ji}(s)\right\}_s\right) \geq \bar{u}^j \tag{8}$$

Date 0 investment decisions of firms

- At date 0, each firm first chooses how to allocate funds across the prudent project, its risky project, and loans to other firms
- In general, firms face a risk-return tradeoff for each asset type
  - Firm i cares about the covariance between the return on an asset with the shadow value of its funds

$$E\left[z_1^i(s)\right] R_C \leq E\left[z_1^i(s) R_A^i(s)\right]$$

- Participation constraint is therefore

$$\underbrace{E\left[z_1^j(s) r^{ji}(s)\right]}_{\text{value of contract}} \geq \underbrace{E\left[z_1^j(s) \bar{\sigma}^j(s)\right]}_{\text{opportunity cost of funds}}$$

FOC for inter-firm contract

- Lagrange multiplier η^{ji} on pariticipation constraint determined in GE by all firms' investment decisions, which i takes as given

- Firm i's optimality conditions for choosing contract $\left\{r^{ji}(s)\right\}_s$ to offer to j

$$-\eta^{ji} \underbrace{z_1^j(s)}_{j's\ rate\ of\ return\ of\ funds} \leq \underbrace{z_1^i(s)}_{i's\ rate\ of\ return\ of\ funds} \tag{9}$$

- So the optimal contract equates the rates of return of i and j, to the extent that the participation constraint will allow
  - The optimal inter-firm contract is efficient (in a partial equilibrium sense)

- Can show that $\left\{\hat{r}^{ji}(s)\right\}_s = \left\{r^{ji}(s) \,|\, r^{ji}(s) = \bar{o}^j(s)\right\}_s$ is an optimal contract
  - Contract pays lenders their opportunity cost of funds in each state

Optimality conditions

- Recall that firms face a risk-return tradeoff for each asset type
- But it turns out that in equilibrium, each firm is always at a corner solution in its investment decision
  - CRS technology means returns of all assets are linear
  - And no idiosyncratic risk implies no diversification motive
- So in equilibrium, each firm falls into one of two categories: investors and intermediaries
- How do firms sort themselves into these groups in equilibrium?
  - Through competition via inter-firm financial contracts

Competition through inter-firm contracts

- Optimal inter-firm contracts determine how firms share risks in equilibrium

- Firms choose among contracts $\rightarrow$ firms compete for funds

- Can model competition as a static game between firms at date 0
  - Firm i offers a financial contract $\left\{r^{ji}(s)\right\}_s$ to each j
  - Firms then evaluate all contracts based on their risk-return profiles

- Best response functions are characterized by optimality conditions for contract $r^{ji}(s)$
  - Firm i's offer to j depends on the offers made by other firms to j through the participation constraint

$$z_1^i(s) = -\eta^{ji} z_1^j(s) \tag{10}$$

Characterizing optimal inter-firm contracts in PE

- There is a unique Nash equilibrium in which each firm j is offered the same contract by all other i, in which each j's participation constraint holds exactly
- Hence, the Nash equilibrium pins down N different contracts (one for each firm), as a function of each firm's $z_1^i(s)$
- In GE, portfolio of firms will pin down rates of return on wealth and contracts
  - In particular, intermediaries' date 1 returns depend on portfolio of investing firms
- Frictionless contracting environment and competition $\Rightarrow$ optimal contracts channel funds to investments which maximize surplus

Date 1 demand for capital

- At date 1, firms decide how much additional capital to acquire for the new project

- Demand by productive firms
  - CRS and no uncertainty after date 1 $\rightarrow$ net investment proportional to net return at date 1

$$\underbrace{k_1^i(s) - k_0^i}_{\text{net investment}} = \underbrace{\left(\frac{1}{q(s) - b_2^i(s)}\right)}_{\text{leverage factor}} \underbrace{\left(\vec{\omega}^{i\prime}\vec{R}(s) - \gamma - b_1^i(s) - b_2^i(s)\right)}_{\text{net return}} k_0^i$$

- Net return 0 at R_C

- Demand by traditional firms

$$k_1^T(s) = F'^{-1}(q(s))$$

Date 1 market for capital

- Date 1 price of capital in state s must clear the market

$$\underbrace{\sum_i k_0^i}_{\textit{Total Supply}} \leq \underbrace{\sum_i k_1^i(s,\omega^i) + k_1^T(s)}_{\textit{Total Demand}}$$

- In any equilibrium, we have two possible cases:
  - *Normal times*: $q(s) = 1$, $k_1^T(s) = 0$, and the productive sector makes positive investment in capital
  - *Systemic crisis*: $q(s) < 1$, and $k_1^T(s) > 0$ as the productive sector sells capital to the traditional sector

- Thus, capital misallocated to traditional firms iff net demand by productive firms is less than net supply

Capital misallocation at date 1

- In the high aggregate state, net investment in capital is positive
- In the bad state, firms which invested heavily in the risky project are facing losses
  - Borrowing constraints $\Rightarrow$ firms must sell capital to finance the losses
  - This happens when its return is less than R_C

Capital misallocation at date 1

2 possible cases, depending on size of aggregate losses:

- a) If aggregate losses are low, then productive firms collectively have enough funds to absorb supply of capital
- b) But when aggregate losses exceed some threshold - i.e. a 'systemic crisis':
  - Price of capital must fall to induce less productive firms to buy capital, so $k_1^T(s) > 0$

Thus, capital is misallocated at date 1 iff aggregate losses exceed some threshold

General equilibrium in absence of bailouts

- In the unique equilibrium, all firms invest only in the prudent project
  - No risk-return tradeoff: $E\left[u'\left(c_1(s)\right)\right] R_C \; > \; E\left[u'(c_1(s)) R^i_A(s)\right]$
- This pins down reservation value of firm and inter-firm contracts:
  - The best inter-firm contract any firm can offer is to pay the return on capital from investing in the prudent project

$$\hat{r}(s) = R_C$$

- Then each firm is indifferent between lending to another firm vs investing in prudent project on own behalf

Unique equilibrium features prudent investment

- Thus, in the unique equilibrium
  - No inter-firm lending
  - All firms invest in the prudent project only

- In the absence of government bailouts, the competitive equilibrium achieves the constrained-efficient allocation
  - Capital is never misallocated in equilibrium

Now introduce a benevolent government

- We now introduce a government which seeks to maximize household welfare subject to budget constraints
  - Takes agents' optimizing behavior as given
- Government only has tools available *ex post*
  - Taxes and transfers at dates 1 and 2, after uncertainty resolved
  - For now we rule out macro-prudential instruments
- Bailouts can be only imperfectly targeted: the government cannot directly observe date 0 investments, but only observe date 1 returns

Government's problem

- The government's objective is to maximize household welfare, using only tools available *ex post* (dates 1 and 2)

- Faces standard government budget constraints in each state

$$\sum_i g_1^i(s, \omega_C^i) = T_1 + B_1(s) \qquad \sum_i g_2^i(s, \omega_C^i) + B_1(s)(1+r) = T_2$$

- A necessary and sufficient condition for optimality is that $k_1^T(s) = 0$ all states
  - First show that this is true from an *ex post* standpoint
  - Later show that this is true even when the government internalizes how its policy affects agents' date 0 decisions, due to a time-consistency problem

Optimal government bailout policy

- From an *ex post* standpoint, an inefficiency occurs in s iff $k_1^T(s) > 0$ (a systemic crisis)
  - In all other situations, the optimal behavior of agents ensures output is maximized in state s
- In a systemic crisis, some firms are facing losses that need to be financed
  - Recall that a firm sells capital to finance its losses iff its return $R_A^i(L)$ is less than R_C
- The bailout enables these investors to finance their losses without selling capital

$$g^i(L) = R_C - R_A^i(L)$$

Kink in optimal bailout policy

- The optimal bailout policy is *ex post* efficient: capital is never misallocated across firms *ex post*
- The policy features kink in aggregate losses:
  - A bailout occurs in state s iff there is a systemic crisis
  - So the occurrence of a bailout depends on the collective investment decisions of firms
- This kink will introduce a strategic complementarity in date 0 investment
  - This will lead to *ex ante* inefficiency

Portfolio allocation decision with bailouts

- Agents rationally anticipate the government's optimal bailout policy
  - Bailout is effectively an implicit insurance policy on the risky investment, as long as enough firms undertake it
  - Bailout puts a floor on return of risky project equal to R_C
- The kink in the policy creates a strategic complementarity in investment
  - If other firms are heavily exposed to risky project then firm i has incentive to be heavily exposed as well
- 2 ways in which a firm can create exposure to risky project:
  - Invest directly in own risky project
  - Lend funds to another firm to invest in its risky project

Best response functions for date 0 investment decisions

- Firm optimality conditions implicitly depend on actions of others

- Consider a firm's choice between prudent vs risky project
  - If the conditions for a government bailout do not hold, then the prudent project dominates

$$E\left[z_1^i(s)\right] R_C > E\left[z_1^i(s) R_A^i(s)\right]. \tag{11}$$

  - If they do hold, then the risky project dominates

$$E\left[z_1^i(s)\right] R_C < E\left[z_1^i(s)\left(R_A^i(s) + g^i(s,1)\right)\right]$$

- In both cases, the firm's choice between lending to another firm vs investing on its own behalf is determined by the participation constraint

$$E\left[z_1^i(s)\hat{r}(s)\right] \geq E\left[z_1^i(s)\left(q(s) + \omega^i R^i(s) - \gamma\right)\right]$$

Inter-firm contract in GE

- If the conditions for a bailout do not hold:
  - All investing firms undertake only the prudent project
  - The inter-firm contract is the same as in the economy without bailouts: $\hat{r}(s) = R_C$
- If the conditions for a bailout hold, then all investing firms undertake the risky project
  - This pins down firms' opportunity cost of funds, and therefore inter-firm contracts
  - Namely, firm i's opportunity cost of funds is given either by $R_A^i(s)$ or the return provided by the best available contract $\hat{r}(s)$
- In equilibrium, all firms have the same opportunity cost of funds and so the same inter-firm contract is offered to all firms

Emergence of systemically important firm

- Let 1 denote the riskiest firm: the firm with greatest exposure to aggregate risk
- If firm 1's risky investment is insured by the government, then it can out-compete other firms in offering an inter-firm contract
  - In the low (bailout) state, firm 1 can match any firm's offer, since all get R_C anyway
  - But in the high state, firm 1 can offer a higher return, since its returns are higher than any other firm's ($R_A^1(H) > R_A^j(H)$)

Emergence of systemically important firm

- Then other firms find it optimal to forgo their own investment opportunities in favor of lending to 1
  - As a result, firm 1 becomes systemically important: its capital holdings are large enough to trigger a systemic crisis
- This generates collective exposure to the riskiest project
- The fact that the riskiest firm becomes systemically important means *systemic risk is maximized*

General equilibrium with bailouts

- Two equilibria:
  - 'Prudent' equilibrium: all investors undertake only the prudent project
  - 'Risky' equilibrium: all investors undertake only the risky project
- Risky equilibrium:
  - All firms lend funds to riskiest firm (directly or indirectly)
  - Riskiest firm invests only in the riskiest project
- Thus, collective rent-seeking behavior causes emergence of SI firm
- The emergence of a SI firm maximizes systemic risk

Welfare and inefficiency in the risky equilibrium

- The risky equilibrium features excessive consumption volatility
- The (*ex post* efficient) government bailout policy prevents capital from being misallocated
- However, rent-seeking results in excessive risking taking *ex ante*
  - In the bad state, there are losses to be absorbed
  - These are absorbed by the government via the bailout
  - And passed on to the household via lump-sum taxes
- Hence, household consumption is excessively volatile due to excessive risk-taking

Next steps: macro-prudential policy

- Interconnectedness and too-big-to-fail problems are closely linked
  - Rents accrue to all firms, not just SI firms
  - So optimal regulatory policy will target all firms, not just SI firms
- A fundamental tension between achieving efficient allocation of resources across firms *ex post* vs an efficient allocation of resources across risks *ex ante*
  - Bailouts correct an *ex post* inefficiency, but introduce one *ex ante*
  - In the absence of macro-pru, a time-consistency problem prevents the government from striking the socially-optimal balance between these two
- In future work, we will characterize optimal macro-prudential policy in light of this tradeoff

Conclusion

- Showed that emergence of systemically important institutions can be explained by a collective moral hazard problem in the presence of government bailouts
- Interconnectedness and TBTF may be closely linked
- Collective nature of moral hazard problem: to solve TBTF, focusing on SI firms alone is insufficient
- Poses a challenge for optimal policy in balancing *ex post* efficiency with *ex ante* incentives

Supplemental Slides

Borrowing constraints on household credit

$$0 \le b_1^i(s) \le \frac{(q(s)-\gamma)\,k_0^i + \sum_j \left(r^{ij}(s)\ell^{ij} - r^{ji}(s)\ell^{ji}\right)}{k_0^i - \sum_h \ell^{hi} + \sum_j \ell^{ij}} \tag{12}$$

$$0 \le b_2^i(s) \le \Gamma A \tag{13}$$

Household optimality conditions

$$u'(c_0)d_0^i \geq E\left[u'(c_1(s))\,d_1^i(s) + u'(c_2(s))\,d_2^i(s)\right] \tag{14}$$

$$u'(c_1(s)) = u'(c_2(s)) \tag{15}$$

Firm optimality conditions

$$\frac{\partial L}{\partial k_0^i} \leq 0 \iff z_0^i \left(\frac{1}{u'(c_0)} E\left[u'(c_1(s))\, b_1^i(s) \right] - 1 \right) + E\left[z_1^i(s) \left(q(s) + \vec{\omega}^{i\,\prime} \vec{R}^i(s) - \gamma - b_1^i(s) + g^i(s, \omega_C^i) Pr\left[g^i > 0 | bailout \right] \right) \right] \leq 0 \tag{16}$$

$$\frac{\partial L}{\partial k_1^i(s)} \leq 0 \iff m_2(s) \left(A - b_2^i(s) \right) \leq z_1^i(s) \left(q(s) - b_2^i(s) \right) \tag{17}$$

$$\frac{\partial L}{\partial b_1^i(s)} \leq 0 \iff \left[\frac{u'(c_1(s))}{u'(c_0)} z_0^i - z_1^i(s) \right] \left(k_0^i - \sum_h \ell^{hi} + \sum_j \ell^{ij} \right) \leq \lambda_1^i(s) - \lambda_0^i(s) \tag{18}$$

$$\frac{\partial L}{\partial b_2^i(s)} \leq 0 \iff \left[z_1^i(s) - m_2(s) \right] k_1^i(s) \leq \mu_1^i(s) - \mu_0^i(s) \tag{19}$$

$$\frac{\partial L}{\partial \omega_C^i} \leq 0 \iff E\left[z_1^i(s) k_0^i \left(R_C - R_A^i(s) + g'(s, \omega_C^i) Pr\left[g^i > 0 | bailout \right] \right) \right] \leq 0 \tag{20}$$

$$\frac{\partial L}{\partial \ell^{ij}} \leq 0 \iff E\left[z_1^i(s) \left(r^{ij}(s) - b_1^i(s) \right) \right] \leq z_0^i \left(1 - E\left[\frac{u'(c_1(s))}{u'(c_0)} b_1^i(s) \right] \right) \tag{21}$$

$$\frac{\partial L}{\partial r^{ji}(s)} \leq 0 \iff -\nu^{ji} \frac{\partial u^{ji}\left(\ell^{ji}, \left\{ r^{ji}(s) \right\}_s \right)}{\partial r^{ji}(s)} \leq \pi(s) z_1^i(s) \ell^{ji} \tag{22}$$

Key assumptions

- Limited commitment - opens door for pecuniary externality in certain states
- Inter-firm contracts allow firms to share risks, expose to others losses
- Limited ex ante intervention
- Imperfectly targeted bailouts
- Multiplicity due to kink in bailout function (nonlinearity of household welfare in aggregate losses), which creates strategic complementarity in firm investment decisions (systemic events)

Household contracting problem

- Borrower decides whteher to honor contractual obligations and makes a take-it-or-leave-it offer
- If the household refuses, the firm is liquidated. The household can seize
  - The firm's date 1 capital holdings, but none of the firm's date 1 returns
  - A fraction Γ of the firm's date 2 profits
- Any profits not seized by Hh remains with firm
- The household cannot reposess firm j's claim $\sum_j \left(r^{ij}(s)\ell^{ij} - r^{ji}(s)\ell^{ji} \right)$ on i's assets.
- Similarly, the household can always walk away from the contract without consequence
- No-default conditions:

$$0 \leq d_1^i(s) + d_2^i(s) \leq \underbrace{(q(s) - \gamma)\, k_0^i + \sum_j \left(r^{ij}(s)\ell^{ij} - r^{ji}(s)\ell^{ji} \right)}_{\text{date 1 liquidation value of firm } i}$$

$$0 \leq d_2^i(s) \leq \underbrace{\Gamma A k_1^i(s)}_{\text{date 2 liquidation value of firm } i}$$

Inter-firm contracts

- Maximizes firm value subject to 2 sets of constraints:
  - Feasibility constraint in each state (ex post): has to have sufficient funds to make contractual payments
  - A pariticipation constraint (ex ante): value of lender if accepts contract at least as large as reservation value of firm
    - Reservation value of firm depends on firm's opportunity cost of funds, TBD in GE
- In equilibrium, investment decisions of firms will be symmetric
- So opportunity cost of funds that the firm needs (across states) to meet equals the return to capital from its own investment opportunity.
- Then only way to simultaneously meet participation constraint ex ante and feasibility constraints in each state ex post is to pay return on capital in each state.Only constraint is participation const
- Thus, conctractual repayments equal borrowers return to capital in each state

Inter-firm contracts in prudent equilibrium

- Optimal contracts solve participation constraint and optimality condition
- Take the contract in which borrower pays lender's opportunity cost of funds in each state
- In the prudent equilibrium, each firm's opportunity cost of funds is given by R_C in all states
- Therefore, the optimal ontract is given by $\hat{r}(s) = R_C$
- Firms are indifferent between accepting a contract vs investing in prudent project themselves
- So inter-firm lending is indeterminate and inconsequential

Inter-firm contracts in risky equilibrium

- In the risky equilibrium, each firm's opportunity cost of funds is given by $R_A^i(s) + g^i$, or taking best contract
- The riskiest firm 1 can offer the best contract by offering up to $\hat{r}(s) = R_A^1(s) + g^1$
- With this contract, firm 1 can out-compete all others in inter-firm lending

$$E\left[z(s)\hat{r}(s)\right] = E\left[z(s)R_A^1(s)\right] > E\left[z(s)R_A^j(s)\right] \quad \forall j$$

- All firms prefer to forgo their own investment opportunities in favor of lending to 1