Monetary and Macropudential Policies against Currency Crises

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Abstract

This paper studies optimal monetary and macroprudential policies in a small open economy model which borrows from abroad in foreign currency subject to sudden stops of capital inflows. Socially excessive foreign borrowing by private agents causes inefficiently large currency depreciation during crises, which increases the domestic-currency value of foreign debt and exacerbates crises. Contractionary monetary policy can mitigate depreciation during crises, but anticipation of this policy intervention induces larger borrowing ex ante and destabilizes the economy. Contractionary monetary policy during crises improves social welfare over inflation targeting monetary policy only if it is combined with an ex ante macroprudential tax on foreign borrowing to prevent over-borrowing. Macroprudential taxes on foreign borrowing prevent sharp currency depreciation and substantially stabilize the economy, thereby improving welfare, regardless of whether monetary policy is inflation targeting or discretionary.

Keywords: Sudden Stops, Pecuniary Externality, Monetary Policy, Macropudential Policy, Currency Crisis, Liability Dollarization

JEL code: F31, F32, F38, F41

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1 Introduction

Emerging economies have been accelerating their international bond issuance in the last decade. Acharya, Cecchetti, De Gregorio, Kalemli-Özcan, and Panizza (2015) report that international bond issuance by financial corporations in 15 emerging economies has risen from less than $400 billion per year in 2010 to nearly $1 trillion per year in 2014. Nonfinancial corporate issuance has also doubled in the same period to $400 billion. One of the key risk factors in this accelerating bond issuance is that these bonds are dominantly issued in foreign currencies. According to the same paper, about 80% of the nearly $4 trillion outstanding bonds in 2014 is denominated in foreign currencies. It is well understood that foreign-currency debt may deteriorate financial stability of borrower countries when exchange rate abruptly depreciates and the domestic-currency value of foreign debt increases. Currency crises of this type have occurred repeatedly in emerging economies since the 1990s, and the crises in Argentina and Turkey in 2018 are the recent examples. To date, however, there has been no consensus on how emerging economies should prepare for and respond to this type of crises. Should monetary policy be contractionary to avoid sharp currency depreciation, or expansionary to stimulate the economy? Can macroprudential policies prevent currency crises? Should monetary and macroprudential policies cooperate to tackle currency crises?

This paper addresses these questions by developing a small open economy model which borrows from abroad in foreign currency subject to the risk of sudden stops of capital inflows. There is a growing literature that studies optimal macroprudential policies for emerging economies subject to sudden stops, represented by Bianchi (2011), Bianchi and Mendoza (2018), and other papers. The main contribution of this paper is to develop a novel mechanism in which sudden stops modeled as an occasionally binding constraint on foreign borrowing cause sharp currency depreciation, which in turn increases the domestic-currency value of foreign debt and exacerbates crises. Using the model, this paper provides policy implications for the optimal combination of monetary and macroprudential policies against currency crises under foreign-currency denominated foreign debt.1

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1There is a literature that studies the implication of foreign-currency debt on monetary and exchange rate policies in financial accelerator models, but this literature does not consider macroprudential policies. More recently, Aoki, Benigno, and Kiyotaki (2018) model currency mismatches in the balance sheet of banks and study monetary and macroprudential policies. See the literature review for details.
The model is a small open economy similar to Bianchi and Mendoza (2018), in which foreign borrowing is subject to an occasionally binding borrowing constraint. The borrowing limit is determined by the collateral asset price, and an endogenous drop in the asset price when the constraint binds causes a pecuniary externality and over-borrowing ex ante. I introduce three innovations into the model. First, foreign borrowing is explicitly denominated in foreign currency, which is the key feature of foreign borrowing in emerging economies. Second, exports of tradable goods produced in this economy (home tradable goods) face downward-sloping demand from abroad. A larger amount of exports is associated with real exchange rate depreciation. The third innovation is New-Keynesian sticky price, which enables me to study the role of monetary policy.\(^2\)

The key mechanism of the model is an amplification loop of real depreciation which arises through the interaction between the balance-of-payments adjustment and the borrowing constraint. It works as follows: when the borrowing constraint binds, the country needs to repay outstanding foreign debt with limited new borrowing, which causes large net capital outflows. Large net capital outflows imply large net exports and cause real depreciation through the balance-of-payments adjustment, which endogenously determines the real exchange rate so that net capital outflows are equal to net exports. Real depreciation in turn increases the domestic-currency value of outstanding foreign debt, but the value of new foreign borrowing is constrained by the borrowing limit. In this case, real depreciation implies further net capital outflows, which trigger second-round real depreciation. The loop continues and leads to large real depreciation during crises.

Because private agents take the real exchange rate as given, this amplification loop of real depreciation causes an externality and distorts private agents’ decisions, both before and during crises. When the borrowing constraint is not binding today but may bind next period, reducing foreign borrowing would reduce the debt repayment and mitigate real depreciation when the constraint actually binds next period. Private agents do not internalize this effect, and take socially excessive foreign borrowing in normal times. I call this externality an ‘ex

\(^2\)Devereux, Young, and Yu (2018) and Coulibaly (2018) also introduce New-Keynesian sticky price into the models with pecuniary externalities, and study optimal monetary and macroprudential policies. See the literature review for the similarities and differences between this paper and their papers.
ante’ real exchange rate externality.\(^3\) When the borrowing constraint is binding, private agents do not internalize that reducing imported inputs for production would improve the trade balance and mitigate real depreciation. This externality induces private agents to use socially too much imported inputs during crises. I call this externality an ‘ex post’ real exchange rate externality. Both of these externalities result in inefficiently large real depreciation during crises through the amplification loop. This implies that an inefficiently large fraction of output is exported, and domestic consumption becomes inefficiently low.

Given this model economy, I study discretionary policies without commitment to future policies. I first consider the case where monetary policy is the only policy tool, and analytically characterize the optimal discretionary monetary policy. When the borrowing constraint is not binding today but may bind next period, the optimal discretionary monetary policy is to lower inflation, i.e. contractionary. This is because contractionary monetary policy would discourage use of imported inputs for production, thereby improving the trade balance and causing real appreciation. Real appreciation today would imply expected real depreciation from today to tomorrow, which increases the effective interest rate and discourages foreign borrowing, partially correcting over-borrowing. When the constraint is binding, the optimal discretionary monetary policy is again contractionary. Contractionary monetary policy would discourage use of imported inputs for production, thereby causing real appreciation and partially correcting the ex post real exchange rate externality.

Next, I characterize the optimal combination of discretionary monetary policy and a time-consistent macroprudential tax on foreign borrowing. I show that monetary policy focuses only on minimizing the inflation cost in normal times, because over-borrowing is handled by a macroprudential tax. However, during crises, monetary policy should still intervene by lowering inflation. This is because a macroprudential tax cannot correct the ex post real exchange rate externality that induces too much imported inputs, thus monetary policy should deal with it during crises.

In the quantitative analysis, I calibrate the model parameters using the standard values in the literature and targeting the average of middle income countries. I solve the model.

\(^3\)This implies that both the pecuniary externality and the ex ante real exchange rate externality induce over-borrowing by private agents.
numerically using a global method to deal with an occasionally binding constraint. Non-linear crisis dynamics in the model are consistent with the empirical regularities of emerging economies’ crises, characterized by drops in output, consumption, asset price, a sharp reversal of capital flows, and sharp real depreciation. I compare the crisis dynamics under four different policy regimes: strict inflation targeting policy, optimal discretionary monetary policy, and each monetary policy with the optimal time-consistent macroprudential tax on foreign borrowing. Model simulations show that discretionary monetary policy without taxes induces larger borrowing than inflation targeting in normal times and destabilizes the economy. This is because anticipation of monetary policy intervention during crises reduces the effective interest rate ex ante and induces larger borrowing. Simulations also show that the macroprudential taxes on foreign borrowing substantially stabilize the economy under both monetary policy regimes. Drops in output and consumption during crises are more than 5% and 10% respectively without taxes, but only 1 – 2% with the taxes. The real exchange rate depreciates by 10% without taxes, but barely depreciates with the taxes.

Finally, I compare the welfare implications by different policy regimes. Without macroprudential taxes, discretionary monetary policy gives lower expected welfare than inflation targeting by 0.03% in terms of permanent consumption. Macroprudential taxes improve expected welfare. Compared to inflation targeting without taxes, the optimal macroprudential taxes improve welfare by 0.070% under inflation targeting and by 0.075% under discretionary monetary policy. This implies that combined with the optimal macroprudential tax, discretionary monetary policy gives higher expected welfare than inflation targeting, although the welfare gain is as small as 0.005%. To measure the effectiveness of monetary policy intervention during crises, I consider the case where the optimal taxes on both foreign borrowing in normal times and imported inputs during crises are available, and monetary policy is inflation targeting. The expected welfare under this policy is 0.022% higher than inflation targeting with the tax only on foreign borrowing, in contrast to a 0.005% welfare gain by discretionary monetary policy. The reason for this limited gain by discretionary monetary policy is that contractionary monetary policy during crises discourages entire production including labor inputs. I also show that the fixed-rate macroprudential tax gives about 75% of welfare gains by the optimal time-varying taxes.
The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 lays out the model. Section 4 examines the flexible price version of the model to characterize the externalities in the model. Section 5 analytically characterizes the optimal discretionary policies. Section 6 conducts quantitative analyses. Section 7 concludes.

2 Related Literature

This paper belongs to the growing literature that studies optimal policies to manage sudden stops in capital inflows. Mendoza (2010) develops the key mechanism of a pecuniary externality triggered by an occasionally binding collateral constraint. Following his pioneering work, many papers such as Bianchi (2011), Jeanne and Korinek (2013), Benigno, Chen, Otrok, Rebucci, and Young (2013), Benigno, Chen, Otrok, Rebucci, and Young (2016), and Bianchi and Mendoza (2018) study the optimal macroprudential policies to correct pecuniary externalities. These papers are real models and do not consider monetary or exchange rate policies. Fornaro (2015) and Ottonello (2015) introduce nominal wage rigidities and study the optimal exchange rate policy. These papers highlight the benefit of depreciation during crises and discuss that exchange rate depreciation helps surviving sudden stops by boosting exports or reducing unemployment. Mendoza and Rojas (2017) and Mendoza and Rojas (2018) introduce into the model of Bianchi (2011) a simple financial intermediary that transforms foreign-currency foreign debt into domestic-currency domestic loan, and study the policy implications. In contrast to my work, real depreciation in their model reduces the burden of outstanding debt during crises, because real depreciation is a decline in the non-tradable goods price relative to the tradable goods price, and real depreciation lowers the real interest rate denominated in consumption composites.

Most closely related to my work in this literature are Devereux, Young, and Yu (2018) and Coulibaly (2018), both of which introduce New-Keynesian sticky price into the models with pecuniary externalities. Devereux, Young, and Yu (2018) introduce a New-Keynesian framework into Bianchi and Mendoza (2018), similarly to my model. The key difference is the timing when the collateral value of asset is evaluated. They assume that the collateral value of asset is determined based on the expected asset price next period, instead of today’s
realized asset price. They then show that ex ante monetary or macroprudential policies to reduce foreign debt is not optimal, and the government intervenes only when a crisis happens. My model follows the recent literature by assuming that the collateral value of asset is determined by the current asset price. Then ex ante macroprudential policies to reduce foreign debt are welfare improving. Coulibaly (2018) introduces a New-Keynesian framework into the model in Bianchi (2011), which consists of tradable and non-tradable sectors. Price stickiness is introduced in the non-tradable sector, so that monetary policy affects production of non-tradable goods, which in turn affects the non-tradable goods price. The optimal monetary policy is conducted to mitigate externalities associated with the non-tradable goods price. The key contribution of my work relative to these papers is that I develop a novel mechanism in which an amplification loop of real depreciation increases the domestic-currency value of foreign debt and exacerbates crises. Then I provide policy implications for interactions between monetary and macroprudential policies in this context.

There is also a literature that studies implications of foreign-currency debt in policy designs. Aghion, Bacchetta, and Banerjee (2000), Céspedes, Chang, and Velasco (2004), Cook (2004), Choi and Cook (2004), and Devereux, Lane, and Xu (2006) introduce currency mismatches in the financial accelerator models and study monetary and exchange rate policies. This literature typically does not consider macroprudential policies. Aoki, Benigno, and Kiyotaki (2018) introduce currency mismatches in the balance sheet of financial intermediaries in the model of Gertler and Kiyotaki (2011), and study the optimal combination of monetary and macroprudential policy for emerging economies.

3 Model

The model is a small open economy similar to Bianchi and Mendoza (2018), and three innovations are introduced. First, foreign debt is denominated in foreign currency. Second, exports of tradable goods produced in this economy (home tradable goods) face a downward-sloping demand from foreign countries. Third, New-Keynesian style sticky price is introduced. Accordingly, there are intermediate firms which produce differentiated intermediate goods, and a final goods producer who assembles them.
3.1 Households

The economy is inhabited by a unit measure of identical households. The utility by a representative household is given as follows:

\[ E_0 \left[ \sum_{t=0}^{\infty} \beta^t \log \left( c_t - \chi \frac{\ell_{t}^{1+\omega}}{1+\omega} \right) \right], \]  

where \( c_t \) is the amount of goods consumed, and \( \ell_t \) is the labor supply. Households consume only home tradable goods. Households own productive asset and use it for production. They produce wholesale goods and sell them to intermediate firms, which in turn produce differentiated intermediate goods. Wholesale goods are produced using the following production function:

\[ y_w^t = A_t (k_{t-1})^{\alpha_k} (\ell_t)^{\alpha_\ell} (m_t)^{\alpha_m}, \]

where \( A_t \) is a stochastic TFP shock, \( k_{t-1} \) is asset holdings at the beginning of period \( t \), and \( m_t \) is imported inputs, with \( \alpha_k + \alpha_\ell + \alpha_m = 1 \).

Households borrow from abroad in terms of foreign currency, reflecting the fact that most of the outstanding foreign bonds issued by emerging economies are denominated in foreign currency. Let \( S_t \) denote the nominal exchange rate, the value of foreign currency measured in terms of domestic currency. Let \( P_t \) denote the nominal price of home tradable goods in domestic currency, and \( P_t^* \) denote the nominal price of foreign tradable goods in foreign currency. Then the real exchange rate \( e_t \), defined as the relative price of foreign tradable goods to home tradable goods, is given by \( e_t = S_t P_t^*/P_t \). Foreign goods price \( P_t^* \) is assumed to be constant and normalized to one, which implies \( e_t = S_t/P_t \). Higher \( e_t \) corresponds to real depreciation.

The household’s budget constraint in nominal terms is given as follows:

\[ P_t c_t + S_t \left( \frac{B_t^*}{R_t^*} - B_{t-1}^* \right) + \left( \frac{B_t}{R_t} - B_{t-1} \right) + Q_t (k_t - k_{t-1}) \]

\[ = P_t^* y_t^w - (1 + \tau_m) S_t m_t + T_t + \Pi_t, \]

where \( B_t^* \) is foreign nominal bond holdings, and \( R_t^* \) is the nominal interest rate on foreign
bond. Since there is no inflation in foreign countries, $B^*_t$ and $R^*_t$ are also real foreign bond and real foreign interest rate. $R^*_t$ is assumed to be stochastic and satisfy $\beta R^*_t < 1$. This implies that households always borrow from abroad, and $B^*_t < 0$. $B_t$ is domestic nominal bond holdings, and $R_t$ is the nominal interest rate set by the government of this economy. Because households are homogeneous, $B_t = 0$ holds at the equilibrium. $Q_t$ is the nominal price of asset, and $k_t$ is asset holdings. Supply of asset is fixed at 1, thus $k_t = k_{t-1} = 1$ at the equilibrium. $P^w_t$ is the nominal price of wholesale goods. The price of imported inputs is assumed to be the same as the price of foreign tradable goods. This means that the real exchange rate $e_t$ indicates the terms of trade as well. The tax $\tau_m$ on imported inputs is introduced to correct the terms-of-trade externality, and set to $\tau_m = 1/(\rho - 1)$ where $\rho$ is the price elasticity of exports. $T_t$ is the lump-sum transfer that rebates the tax on imported inputs and finances a subsidy on intermediate goods sales explained below. $\Pi_t$ is intermediate firms’ profits paid to households.

Household’s foreign borrowing is subject to the following borrowing constraint:

$$-S_t \frac{B^*_t}{R^*_t} \leq \kappa_t Q_t k_{t-1}. \tag{4}$$

$\kappa_t$ is a stochastic variable that can be interpreted as a change in the global financial conditions. As is discussed in Bianchi and Mendoza (2018), this borrowing constraint can be derived by assuming foreign lenders with limited enforcement.

The household’s problem is to choose $\{c_t, \ell_t, k_t, m_t, B^*_t, B_t\}$ given $\{P_t, P^w_t, Q_t, S_t, R_t, R^*_t\}$ to maximize expected utility (1) subject to the production function (2), the budget constraint (3) and the borrowing constraint (4). Let $\lambda_t$ be the Lagrange multiplier on the budget constraint, and $\mu_t$ be the multiplier on the borrowing constraint. To transform the equations into real terms, I use lower-case letters $p^w_t$ and $q_t$ for the real price, i.e. the relative price compared to $P_t$. I also use $b_t = B_t/P_t$ and $b^*_t = B^*_t$. Then the first order conditions by households are summarized as follows:

$$c_t : \lambda_t = \frac{1}{c_t - \chi^{1+\omega}}, \tag{5}$$

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4 Appendix proves that this constant tax rate corrects the externality associated with the terms of trade.
\[ \ell_t : p_t^w \alpha \ell_t \frac{y_t^w}{\ell_t} = \chi \ell_t^\omega, \] (6)

\[ m_t : p_t^w \alpha m_t \frac{y_t^m}{m_t} = e_t \frac{\rho}{\rho - 1}, \] (7)

\[ b_t : \lambda_t = \beta R_t E_t \left[ \frac{1}{1 + \pi_{t+1}} \right], \] (8)

\[ b_t^* : \lambda_t - \mu_t = \beta R_t^* E_t \left[ \frac{e_{t+1}}{e_t} \right], \] (9)

\[ k_t : q_t \lambda_t = \beta E_t \left[ \frac{1}{k_t} \right], \] (10)

\[ \mu_t \left[ -e_t \frac{b_t^*}{R_t} - \kappa_t q_t k_{t-1} \right] = 0. \] (11)

Equation (6) are (7) are the standard first order conditions for production. Equations (8), (9), and (10) are the Euler equations with respect to domestic bond, foreign bond, and asset respectively. Note that in (9), expected real depreciation rate \( e_{t+1}/e_t \) affects the real interest rate on foreign bond, which plays a key role in the policy analysis. \( \pi_{t+1} = P_{t+1}/P_t - 1 \) is the inflation rate from period \( t \) to \( t + 1 \). Equation (11) is the complementary slackness condition for the borrowing constraint.

### 3.2 Intermediate Firms

Intermediate firms are modeled following standard New-Keynesian models. There are intermediate firms with unit measure, and they produce differentiated intermediate goods using wholesale goods purchased from households in the competitive market. Their production technology is to convert one unit of wholesale good to one unit of intermediate good. They sell their products to the final goods producer given the following demand equation:

\[ y_t(i) = \left( \frac{p_t(i)}{P_t} \right)^{-\theta} Y_t, \] (12)

where \( y_t(i) \) is the amount of intermediate goods sold to the final goods producer, \( p_t(i) \) is its price, with \( i \in [0, 1] \) indicating the type of intermediate goods. \( Y_t \) is output of final goods.
producer. This demand equation is the first order condition with respect to intermediate inputs by the final goods producer explained below.

Intermediate firms’ price setting is subject to a price adjustment cost. I assume a quadratic adjustment cost proposed in Rotemberg (1982), with the target inflation rate zero. In addition, I introduce a subsidy \( \tau_y = 1/(\theta - 1) \) on intermediate goods sales to eliminate distortions by the market power of intermediate firms. The profit maximization problem by an intermediate firm \( i \) is then defined as follows:

\[
\max_{\{p_t(i)\}_{t=0}^{\infty}} E_0 \left[ \sum_{t=0}^{\infty} \Lambda_{0,t} \left\{ \left( (1 + \tau_y) \frac{p_t(i)}{P_t} - \frac{P_t^w}{P_t} \right) y_t(i) - \frac{\psi}{2} \left( \frac{p_t(i)}{p_{t-1}(i)} - 1 \right)^2 Y_t \right\} \right],
\]

subject to the demand equation (12). \( \Lambda_{0,t} \) is households’ stochastic discount factor given as \( \beta^t \lambda_t/\lambda_0 \). The parameter \( \psi \) determines the size of the price adjustment cost, which governs the extent of price stickiness. Rearranging the first order condition with respect to \( p_t(i) \) gives the following New Keynesian Phillips curve:

\[
[-\theta + \theta p_t^w - \psi \pi_t(1 + \pi_t)] Y_t + \beta E_t [\lambda_t(1 + \pi_t) Y_{t+1}] = 0.
\]

As is standard in this class of models, current inflation \( \pi_t \) is increasing in the current marginal cost \( p_t^w \) and the expected future inflation.

### 3.3 Final Goods Producer

A representative final goods producer assembles a unit measure of differentiated intermediate goods into home tradable goods. The production function is a standard CES aggregation:

\[
Y_t = \left( \int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}},
\]

where \( \theta \) is the elasticity of substitution across different intermediate goods. The first order condition for each type of intermediate goods \( y_t(i) \) gives the demand equation (12).

Output of home tradable goods is either consumed by domestic households, exported to foreign countries, or used to pay a price adjustment cost of intermediate firms. The market
clearing condition is therefore given as follows:

$$Y_t = c_t + x_t + \frac{\psi}{2}(\pi_t)^2 Y_t,$$

(14)

where \(x_t\) is exports to foreign countries. Foreign demand for exports of home tradable goods is assumed as follows:

$$x_t = \left( \frac{P_t}{S_tP_t} \right)^{-\rho} Y^*,$$

where \(\rho > 1\) is the parameter for the price elasticity of demand for exports, and \(Y^*\) is a parameter that governs the size of foreign demand.\(^5\) Using the definition of the real exchange rate \(e_t\), exports can be written as a function of \(e_t\) as follows:

$$x_t = e_t^\rho Y^*.$$

(15)

### 3.4 Decentralized Equilibrium

This subsection defines a decentralized equilibrium. Outputs of final, intermediate, and wholesale goods satisfy \(Y_t = y_t(i) = y_w^i = A_t(k_{t-1})^{\alpha_k} (\ell_t)^{\alpha_l} (m_t)^{\alpha_m}\). Domestic bond market clearing condition is \(b_t = 0\). Asset market clearing implies \(k_t = 1\). Transfers in real terms \(T_t/P_t\) consist of financing a subsidy on intermediate goods sales \(-\tau_g Y_t = -1/(\theta - 1)Y_t\) and rebates of a tax on imported inputs \(\tau_m e_t m_t = 1/(\rho - 1)e_t m_t\). Intermediate profits paid to households are given by \(\Pi_t/P_t = (1 + 1/(\theta - 1) - p_t^w) Y_t - (\psi/2)(\pi_t)^2 Y_t\). Substituting these equations, households’ budget constraint (3) can be written in real terms as follows:

$$c_t + e_t \left( \frac{b_t^*}{R_t} - b_{t-1}^* \right) = Y_t - \frac{\psi}{2}(\pi_t)^2 Y_t - e_t m_t.$$

(16)

Combining this equation with the market clearing condition for final goods (14), the following balance-of-payments identity is obtained:

$$e_t^\rho Y^* - e_t m_t = e_t \left( \frac{b_t^*}{R_t^*} - b_{t-1}^* \right).$$

(17)

\(^5\)Simonovska and Waugh (2014) estimate the price elasticity of exports to be 3 - 4.
This equation says that net exports in the left-hand side are equal to net capital outflows in the right-hand side.

The decentralized equilibrium of the model is defined by allocations \( \{Y_t, c_t, \ell_t, m_t, b^*_t, b_t\}_{t=0}^{\infty} \), prices \( \{p^w_t, e_t, q_t, \pi_t\}_{t=0}^{\infty} \), and Lagrange multipliers \( \{\lambda_t, \mu_t\}_{t=0}^{\infty} \) that satisfy (2), (5), (6), (7), (8), (9), (10), (11), (13), (16), (17), \( b_t = 0 \), given the initial state \( b^*_{-1} \) and \( b_{-1} = 0 \), policy \( \{R_t\}_{t=0}^{\infty} \) and exogenous shocks \( \{A_t, R^*_t, \kappa_t\}_{t=0}^{\infty} \). This completes the exposition of the model economy.

4 Flexible Price Model

This section studies the flexible price version of the model. The purpose of this section is to characterize the externalities in the model by putting aside the nominal rigidities. In the flexible price model, households directly produce home tradable goods, and there are no wholesale goods, intermediate goods, intermediate firms, or final goods producer. Because monetary policy is irrelevant, there are no domestic bonds in the model. The decentralized equilibrium of this economy is defined by allocations \( \{Y_t, c_t, \ell_t, m_t, b^*_t\}_{t=0}^{\infty} \), prices \( \{e_t, q_t\}_{t=0}^{\infty} \), and Lagrange multipliers \( \{\lambda_t, \mu_t\}_{t=0}^{\infty} \) that satisfy (2), (5), (6) with \( p^w_t = 1 \), (7) with \( p^w_t = 1 \), (9), (10) with \( p^w_{t+1} = 1 \), (11), (16), (17), given the initial state \( b^*_{-1} \) and exogenous shocks \( \{A_t, R^*_t, \kappa_t\}_{t=0}^{\infty} \).

To characterize the externalities in the model, I set up the social planner’s problem and solve for the first order conditions. Following Bianchi and Mendoza (2018), I focus on the time-consistent planner’s problem who cannot commit to future policies. Specifically, I consider a Markov perfect equilibrium characterized by the following two features: (1) the planner at each period chooses his/her policy rules optimally, taking as given future planners’ policy rules but internalizing how his/her policies affect future planners’ policies; (2) the optimal policy rules coincide with future planner’s policy rules that are taken as given when the current policy rules are chosen. The second feature implies that the planner’s decision rules are time-invariant, and the planner at any point in time has no incentive to deviate from the decision rules expected by the past planners, assuring that the planners’ decision rules are time-consistent.
I consider the Ramsey planner’s problem who chooses foreign bond $b^*_t$ and imported inputs $m_t$ on behalf of households, and everything else is chosen by households. This implies that all the decentralized equilibrium conditions except the first order conditions with respect to foreign bond and imported inputs are included in the implementability constraints of the planner’s problem. The planner’s problem is then defined in a recursive form as follows:

$$V(b^*_{t-1}, s_t) = \max_{c_t, b^*_t, \ell_t, m_t, e_t, q_t, \mu_t} \log \left( c_t - \chi \frac{\ell_t^{1+\omega}}{1+\omega} \right) + \beta E_t V(b^*_t, s_{t+1}),$$

where $s_t = \{A_t, R^*_t, \kappa_t\}$ denotes stochastic shocks to the economy, subject to the borrowing constraint in real terms:

$$-e_t \frac{b^*_t}{R^*_t} \leq \kappa_t q_t k,$$  \hspace{1cm} (18)

the production function (2), and the implementability constraints are (6) with $p^w_t = 1$, (10) with $p^w_{t+1} = 1$, (11), (16), and (17).

The full characterization of the first order conditions is left to the appendix. Rearranging the key first order conditions gives the following equations:

$$c_t : \lambda_t^{RP} = u_c(t) - \xi_t q_t u_{cc}(t),$$  \hspace{1cm} (19)

$$q_t : \mu_t^{RP} \kappa_t k = \xi_t u_c(t),$$  \hspace{1cm} (20)

$$e_t : \gamma_t = \frac{1}{\rho - 1} \lambda_t^{RP} - \frac{b^*_t/R^*_t}{(\rho - 1)e_t^{\rho - 1}y^*} \mu_t^{RP},$$  \hspace{1cm} (21)

$$b^*_t : \lambda_t^{RP} + \gamma_t - \mu_t^{RP} = \beta R^*_t E_t \left[ \frac{e_{t+1}}{e_t} \left( \lambda_{t+1}^{RP} + \gamma_{t+1} \right) \right],$$  \hspace{1cm} (22)

where $u_c(t)$ is the marginal utility of consumption at period $t$, and $u_{cc}(t)$ is the second derivative of the utility function with respect to consumption. $\lambda_t^{RP}$, $\mu_t^{RP}$, $\xi_t$, $\gamma_t$ are the Lagrange multipliers on the budget constraint (16), the borrowing constraint (18), the asset price equation (10) with $p^w_{t+1} = 1$, and the balance-of-payments identity (17) respectively.

$\lambda_t^{RP}$ indicates that the social value of home tradable goods has an additional term on top of the marginal utility of consumption. This additional term is the effect of increasing consumption on the asset price $q_t$. As is clear from (20), $\xi_t$ is positive if and only if $\mu_t^{RP}$ is
positive, i.e. the constraint is binding. The intuition is that increasing consumption has an additional positive social value through increasing the asset price only when the constraint is binding and a higher asset price would relax the constraint.

$\gamma_t$ can be interpreted as the social value of real appreciation through the balance-of-payments adjustment, which is always positive. To see this, suppose the outstanding debt reduces so that the repayment decreases by one unit in domestic currency, which corresponds to an increase in $b_{t-1}^*$ by $1/e_t$. The effect of this reduction in the debt repayment on the real exchange rate $e_t$ can be seen by applying the implicit function theorem to (17):

$$\frac{\partial e_t}{\partial b_{t-1}^*} \frac{1}{e_t} = -\frac{1}{(\rho - 1)e_t^{\rho - 1}Y^*}. \tag{23}$$

Because this value is negative, a reduction in the debt repayment causes real appreciation of this size.\footnote{Reducing imported inputs $m_t$ by $1/e_t$ units would cause real appreciation of the same size, as is clear from (17).} This real appreciation has two consequences, which correspond to the two terms in (21). First, this real appreciation reduces the payment to imported inputs and net capital outflows in the budget constraint by the following amount:

$$-\frac{1}{(\rho - 1)e_t^{\rho - 1}Y^*} \times \left( \frac{b_t^*}{R_t^*} - b_{t-1}^* + m_t \right) = -\frac{1}{\rho - 1}. $$

This means that households have this amount of additional available resources. Its social value is $1/(\rho - 1)\lambda_t^{RP}$, which is the first term in (21). Second, when the borrowing constraint is binding, real appreciation relaxes the constraint by reducing the domestic-currency value of foreign-currency debt. As is clear from the borrowing constraint (18), marginal real appreciation relaxes the borrowing constraint by $b_t^*/R_t^*$ units. Then the social value of the real appreciation of the size (23) by relaxing the binding borrowing constraint is:

$$-\frac{1}{(\rho - 1)e_t^{\rho - 1}Y^*} \times \frac{b_t^*}{R_t^*} \mu_t^{RP} = -\frac{b_t^*/R_t^*}{(\rho - 1)e_t^{\rho - 1}Y^*} \mu_t^{RP},$$

which is the second term in (21). Since $\mu_t^{RP}$ is the Lagrange multiplier on the borrowing constraint, this value is positive only when the constraint is binding.
Substituting (19) and (21) into (22) gives the explicit expression for the Euler equation with respect to foreign bond. In the case that the constraint is not binding today but may bind next period, the Euler equation is given as follows:

\[ u_c(t) = \beta R_t^e E_t \left[ \frac{e_{t+1}}{e_t} \left( u_c(t + 1) - \xi_{t+1} q_{t+1} u_{cc}(t + 1) \right) \right. \]

pecuniary externality

\[ \left. - \frac{b_{t+1}/R_t^e}{\rho e_{t+1}^u Y^*} \mu_{t+1} \right] \right] \right) . \tag{24} \]

Compared to the Euler equation in the decentralized equilibrium (9) with \( \mu_t = 0 \), there are two additional terms in the right-hand side. The first additional term is a pecuniary externality as in Bianchi and Mendoza (2018). This term indicates that the planner internalizes that reducing foreign debt at period \( t \) will increase the asset price and relax the borrowing constraint when it binds at \( t + 1 \). This term being positive implies that households underestimate the value of resources when the constraint binds, thus they over-borrow when there is a risk of binding constraint next period.

The second additional term in (24) comes from the second term of \( \gamma_t \) in (21), thus it captures the social value of real appreciation when the constraint is binding as explained above. This term indicates that the planner internalizes that reducing foreign debt at period \( t \) will reduce the debt repayment at \( t + 1 \), which causes real appreciation at \( t + 1 \). Real appreciation when the borrowing constraint is binding reduces the domestic-currency value of foreign-currency debt and relaxes the binding borrowing constraint. This term being positive implies that this externality also induces households to over-borrow when there is a possibility of the borrowing constraint binding next period. Because this externality works through the real exchange rate and distorts households’ decision ex ante in normal times, I call this an ‘ex ante’ real exchange rate (RER henceforth) externality.

The key mechanism of the model is that when the borrowing constraint actually binds, over-borrowing caused by the pecuniary and ex ante RER externalities results in inefficiently large real depreciation through an amplification loop. To understand this mechanism, plug-
The effect of an additional unit of foreign borrowing and an associated debt repayment (a marginal increase in \( -b_{t-1}^* \)) on the real exchange rate can be obtained by applying the implicit function theorem to this equation:

\[
\frac{\partial e_t}{\partial (-b_{t-1}^*)} = \frac{e_t}{(\rho - 1) e_t^{\rho-1} Y^* + b_t^*/R_t^*} \left\{ 1 + \left( \frac{-b_t^*/R_t^*}{(\rho - 1) e_t^{\rho-1} Y^*} \right) + \left( \frac{-b_t^*/R_t^*}{(\rho - 1) e_t^{\rho-1} Y^*} \right)^2 + \cdots \right\} \quad (26)
\]

The denominator in the first line \((\rho - 1) e_t^{\rho-1} Y^* + b_t^*/R_t^*\) is always strictly positive under the parameter values used in the quantitative analysis. This means that the inside of the parenthesis in the second line \((-b_t^*/R_t^*)(/(\rho - 1) e_t^{\rho-1} Y^*)\) is strictly between 0 and 1. This equation can be understood as follows. Additional unit of foreign borrowing at the previous period increases the debt repayment and causes real depreciation by \(e_t/[(\rho - 1)e_t^{\rho-1} Y^*]\) through the balance-of-payments adjustment, as shown in (23). This is the direct effect of foreign borrowing on the real exchange rate, which is the first term in (26). When the borrowing constraint is binding, however, new borrowing \(-e_t b_t^*/R_t^*\) is constrained by the borrowing limit \(\kappa_t k_q\), thus real depreciation (an increase in \(e_t\)) forces a cut in the borrowing amount \((-b_t^*)\) to meet the borrowing constraint. Real depreciation of the size \(e_t/[(\rho - 1)e_t^{\rho-1} Y^*]\) forces \((-b_t^*)\) to reduce by \(b_t^*/[/(\rho - 1)e_t^{\rho-1} Y^*]\) to keep the domestic-currency value of new borrowing unchanged. This reduction in \((-b_t^*)\) then causes real depreciation through a second-round adjustment in the balance of payments as follows:

\[
\frac{\partial e_t}{\partial (-b_t^*)} \times \frac{b_t^*}{(\rho - 1)e_t^{\rho-1} Y^*} = \frac{e_t}{(\rho - 1) e_t^{\rho-1} Y^*} \times \left( \frac{-b_t^*/R_t^*}{(\rho - 1) e_t^{\rho-1} Y^*} \right),
\]

where the partial derivative comes from the implicit function theorem applied to the balance-of-payments identity (17). This is the second term in (26). In this way, the interaction between the balance-of-payments adjustment and the binding borrowing constraint causes
an amplification loop of real depreciation. Note also that currency mismatches between foreign borrowing and the collateral asset play the key role here. If the collateral asset was denominated in foreign currency, this amplification loop would not occur.

Intuitively, real depreciation increases the domestic-currency value of the foreign debt repayment, but the domestic-currency value of new borrowing is limited by the collateral constraint. Therefore, real depreciation when the constraint is binding causes large capital outflows and further real depreciation. Through this mechanism, over-borrowing induced by the pecuniary and ex ante RER externalities result in inefficiently large real depreciation during crises. The social cost of inefficiently large real depreciation is twofold. First, imported inputs become inefficiently expensive, which reduces output. Second, a large fraction of output is exported, and domestic consumption becomes inefficiently low.

This same mechanism also distorts households’ decision on imported inputs $m_t$ when the borrowing constraint is binding. The first order condition with respect to $m_t$ in the above Ramsey planner’s problem is given as follows:

$$m_t : \alpha_m \frac{y_t}{m_t} = e_t \left[ \frac{\rho}{\rho - 1} - \frac{b_t^* / R_t^*}{(\rho - 1)e_t^{\gamma_t}} \frac{\mu_t^{RP}}{\lambda_t^{RP}} \right]. \quad (27)$$

The first term in the bracket is the terms-of-trade externality. Because this externality works even without a borrowing constraint and is out of the focus of this paper, this externality is corrected by the fixed rate tax $\tau_m$. The second term indicates that when the borrowing constraint is binding, the social cost of buying imported inputs is higher than that in normal times. Notice that this second term comes from the second term of $\gamma_t$ in (21), i.e. the social value of real appreciation by relaxing the binding constraint. Therefore, this term captures the social cost of buying one more unit of imported inputs through real depreciation and tightening the binding constraint. I call this an ‘ex post’ RER externality, because this externality works only when the borrowing constraint is binding. This externality causes too much imported inputs when the borrowing constraint is binding, which deteriorates the trade balance and triggers an amplification loop of real depreciation in the same way as
The Ramsey planner’s optimal allocation can be decentralized by taxes on foreign bond and imported inputs. The pecuniary and ex ante RER externalities associated with foreign bond can be corrected by the following macroprudential tax on foreign bond:

\[ u_c(t) = \beta R_t^*(1 + \tau^b_t) E_t \left[ \frac{e_{t+1}}{e_t} u_c(t+1) \right], \]

\[ \tau^b_t = \frac{E_t \left[ \frac{e_{t+1}}{e_t} \left( -\xi_{t+1} q_{t+1} u_c(t+1) - \frac{b_{t+1}^*/R_{t+1}^*}{\rho e_t^{-1}Y^*} \lambda_t^{RP} \right) \right]}{E_t \left[ \frac{e_{t+1}}{e_t} u_c(t+1) \right]}, \]

which is strictly positive when there is a positive probability that the borrowing constraint binds next period. The ex post RER externality can be corrected by the following tax on imported inputs:

\[ \alpha_m^y \frac{y_t}{m_t} = (1 + \tau^m_t)e_t, \]

\[ \tau^m_t = \frac{1}{\rho - 1} - \frac{b_t^*/R_t^*}{(\rho - 1)e_t^{-1}Y^*} \lambda_t^{RP} = \tau^m_t - \frac{b_t^*/R_t^*}{(\rho - 1)e_t^{-1}Y^*} \lambda_t^{RP}. \]

This tax is strictly positive when the borrowing constraint is binding.

In summary, this section shows three externalities in the model, the pecuniary externality, the ex ante RER externality, and the ex post RER externality. The pecuniary and ex ante RER externalities induce households to socially over-borrow when the constraint is not binding today but may bind next period. This over-borrowing causes an inefficiently large real depreciation when a crisis actually occurs. The ex post RER externality induces households to buy socially too much imported inputs when the constraint is binding. This externality also causes an inefficiently large real depreciation during crises. The next section studies optimal monetary and macroprudential policies in the full model with price stickiness.

5 Policy Analysis

This section studies the optimal policies in the full model with price stickiness. I assume that the government lacks an ability to commit to future policies, and focus on Markov perfect
equilibrium with time-invariant policy rules. The first subsection considers the case where monetary policy is the only policy tool. The second subsection introduces macroprudential taxes on foreign borrowing and studies a combination of monetary and macroprudential policies.

5.1 Monetary Policy

This subsection studies the case where monetary policy is the only policy tool. It is straightforward to show that without a borrowing constraint, the optimal discretionary monetary policy is strict inflation targeting, i.e. $\pi_t = 0$. Therefore, the main analysis here is whether/how the existence of the borrowing constraint and associated externalities deviate the optimal discretionary monetary policy from strict inflation targeting. The following is the first main proposition:

**Proposition 1** In the model described in Section 3, when monetary policy is the only policy tool, strict inflation targeting is not optimal discretionary monetary policy.

To prove this proposition, I set up a Ramsey planner’s problem in which the planner chooses an inflation rate and all the endogenous variables to maximize households’ expected utility, subject to the decentralized equilibrium conditions derived in Section 3. Formally,

$$V(b^*_t, s_t) = \max_{c_t, b^*_t, \ell_t, \pi_t, \ell_t, q_t, \pi_t} \log \left( c_t - \chi \frac{\ell_t^1+\omega}{1+\omega} \right) + \beta E_t V(b^*_t, s_{t+1}),$$

subject to (2), (6), (7), (9), (10), (11), (13), (16), (17), (18). The full description of the first order conditions and the formal proof of the proposition are left to the appendix. Here I show the key first order conditions and discuss the intuition:

$$\pi_t : \eta^PC_t \psi(2\pi_t + 1)Y_t = \lambda^*_t \psi \pi_t Y_t, \quad (28)$$

7The appendix provides the formal proof for this statement. Intuitively, given that the terms of trade externality and the distortion by the market power are corrected by a tax and a subsidy, monetary policy should focus only on minimizing the price adjustment cost.

8The nominal interest rate $R_t$ and the Euler equation with respect to domestic bond are not included in this setup. The nominal interest rate $R_t$ can be obtained using the Euler equation after all the other endogenous variables are pinned down by the first order conditions.
\[ p_t^w : \eta_{t}^{PC} \theta Y_t = \eta_t^{\ell} + \eta_t^{m}, \]  

(29)

where \( \lambda_t^* \) is the Lagrange multiplier on the budget constraint (16), which is the social value of home tradable goods, and \( \eta_t^{PC}, \eta_t^{\ell}, \eta_t^{m} \) are the Lagrange multipliers on the New Keynesian Phillips curve (13), the market clearing conditions for labor and imported inputs (6) and (7). In the first equation (28), the left-hand side is the effect of a marginal increase in \( \pi_t \) on social welfare by increasing the wholesale goods price \( p_t^w \) through the New Keynesian Phillips curve. The right-hand side is a marginal change in the price adjustment cost evaluated by the social value of final goods. Assuming \( 2\pi_t + 1 \) is always strictly positive, which is the case in the quantitative analysis below, the sign of the optimal inflation rate \( \pi_t \) is the same as the sign of the Lagrange multiplier \( \eta_t^{PC} \), and \( \pi_t = 0 \) if and only if \( \eta_t^{PC} = 0 \). The second equation (29) gives the intuition for \( \eta_t^{PC} \). It says that the effect of a marginal increase in \( p_t^w \) on social welfare consists of two terms, \( \eta_t^{\ell} \) and \( \eta_t^{m} \). These Lagrange multipliers capture the effect of a marginal increase in \( p_t^w \) on social welfare through changes in labor inputs and imported inputs. These multipliers take non-zero values if the government has an incentive to distort production factor inputs. Combined with the first equation (28), the key intuition here is that the optimal \( \pi_t \) deviates from 0 (target in general) if the government has an incentive to distort production factor inputs by manipulating \( p_t^w \) using monetary policy.

The proof of the proposition follows the steps of setting \( E_t(\pi_{t+1}) = 0 \) and then showing \( \pi_t = 0 \) does not satisfy the first order conditions. Setting \( E_t(\pi_{t+1}) = 0 \) and combined with the other first order conditions, equation (29) can be written as follows:

\[
\frac{\eta_{t}^{PC}}{\lambda_t^*} = \frac{1}{\theta y_t \left[ 1 - \left( \frac{\alpha_{\ell}}{\alpha_{m} + 1} + \alpha_{m} \right) \right]} \left( \varepsilon_{t}^{\ell} \frac{\eta_{t}^{\ell}}{\lambda_t^*} + \varepsilon_{t}^{m} \frac{\eta_{t}^{m}}{\lambda_t^*} \right),
\]  

(30)

where

\[
\frac{\eta_{t}^{\ell}}{\lambda_t^*} = \left( 1 - \frac{\psi}{2} \right) \alpha_{\ell} \frac{y_t}{\ell_t} - \chi_{t} \varepsilon_{t}^{\ell},
\]  

(31)

\[
\frac{\eta_{t}^{m}}{\lambda_t^*} = \left( 1 - \frac{\psi}{2} \right) \alpha_{m} \frac{y_t}{m_t} - e_t - \gamma_t^*,
\]  

(32)

\( \varepsilon_{t}^{\ell} \) and \( \varepsilon_{t}^{m} \) are changes in \( \ell_t \) and \( m_t \) responding to a marginal increase in \( p_t^w \). \( \gamma_t^* \) in (32) is the Lagrange multiplier on the balance-of-payments identity (17), and captures the social
value of real appreciation as discussed in the previous section. In the right-hand side of (30), the coefficient fraction term, $\varepsilon_t^{pt}$, and $\varepsilon_t^{pm}$ are always strictly positive. Therefore, whether $\eta_t^{PC}$ is equal to zero or not depends on $\eta_t^p$ and $\eta_t^m$. It is straightforward to show that under $E_t(\pi_{t+1}) = 0$ and $\pi_t = 0$, the right-hand side of (31) coincides with the first order condition in the decentralized equilibrium (6), thus $\eta_t^p = 0$. This means that the government has no incentive to distort labor inputs. Regarding the imported inputs, the right-hand side in (32) would coincide with the first order condition in the decentralized equilibrium (7) if $\gamma_t^*/\lambda_t^* = 1/(\rho - 1)$. The expression for $\gamma_t^*/\lambda_t^*$ depends on whether the constraint is binding or not as follows:

\[
\frac{\gamma_t^*}{\lambda_t^*} - \frac{1}{\rho - 1} = \frac{1}{(\rho - 1)\varepsilon_t^{p-1}Y^* + \varepsilon_t^{pm}} \left( \frac{\eta_t^{EE} \mu_c(t)}{\lambda_t^*} e_t \right) \quad \text{when not binding,} \tag{33}
\]

\[
\frac{\gamma_t^*}{\lambda_t^*} - \frac{1}{\rho - 1} = \frac{1}{(\rho - 1)\varepsilon_t^{p-1}Y^* + \varepsilon_t^{pm}} \left( -\frac{\mu_t^* b_t^*}{\lambda_t^* R_t} \right) \quad \text{when binding.} \tag{34}
\]

In the case of not binding, $\eta_t^{EE}$ in (33) is the Lagrange multiplier on the Euler equation with respect to foreign bond in the decentralized equilibrium (9). It is not possible to determine analytically the sign of $\eta_t^{EE}$, but the appendix shows that the pecuniary and the ex ante RER externalities push up the value of $\eta_t^{EE}$. A higher value of $\eta_t^{EE}$ implies a higher value of $\gamma_t^*$, which in turn implies a lower value of $\eta_t^{PC}$ and thus lower inflation $\pi_t$. Therefore, the pecuniary and the ex ante RER externalities in normal times drive the optimal inflation rate to be low, i.e. contractionary monetary policy. The intuition is as follows. As shown in the flexible price model in Section 4, the pecuniary and ex ante RER externalities induce households to over-borrow in normal times. The government has an incentive to discourage households’ foreign borrowing, and this incentive is captured by positive $\eta_t^{EE}$. Given that monetary policy is the only policy tool, the government tries to discourage foreign borrowing by contractionary monetary policy. Contractionary monetary policy reduces wholesale goods price $p_t^w$ through the New Keynesian Phillips curve, which in turn reduces profits for wholesale goods production and discourages imports for production. This effect is captured by $\varepsilon_t^{pm}$, a change in $m_t$ responding to a marginal change in $p_t^w$. Smaller amount of imports

\[\text{and $\mu_t^* > 0$ hold when it is binding.}\]
improves the trade balance and leads to real appreciation through the balance-of-payments adjustment. Real appreciation today increases the expected depreciation rate $E_t(e_{t+1}/e_t)$ from today to tomorrow by lowering $e_t$, which increases the effective real interest rate and thus discourages foreign borrowing. The social benefit of this policy intervention is captured by the left-hand side in (28). The government sets the inflation rate so that this social benefit becomes equal to the marginal change in the inflation cost in the right-hand side.

In the case of binding, the last term in (34) is unambiguously positive, thus $\pi_t = 0$ is clearly not optimal. Higher $\gamma_t^*$ again implies contractionary monetary policy. The intuition is straightforward. When the borrowing constraint is binding, the government tries to reduce imported inputs and cause real appreciation, thereby relaxing the binding constraint and correcting the ex post RER externality. Contractionary monetary policy decreases the wholesale goods price $p^w_t$ and discourages use of imported inputs for production.

It is worth mentioning that the discussion here does not necessarily imply that the optimal discretionary monetary policy dominates strict inflation targeting policy in terms of social welfare. This is because the optimal discretionary monetary policy is the policy rule that is determined by period-by-period optimization without considering how expectations for future policies affect private agents’ behavior. In fact, the later sections show that expectations for monetary policy intervention during crises induce larger borrowing ex ante and exacerbates crises, thereby reducing the expected welfare compared to the strict inflation targeting policy.

### 5.2 Monetary and Macroprudential Policies

This subsection introduces macroprudential taxes on foreign borrowing and studies interactions between monetary and macroprudential policies. Following the previous subsection, the analysis focuses on time-consistent policies without commitment to future policies. Accordingly, I consider the optimal time-consistent macroprudential taxes in the Markov perfect equilibrium. Regarding monetary policy, I consider strict inflation targeting monetary policy and the optimal discretionary monetary policy. For each policy regime, I set up a Ramsey planner’s problem and characterize the optimal policies. The full description of the planner’s problem and the first order conditions are left to the appendix, and here I discuss the key
results and intuitions.

Under strict inflation targeting monetary policy, the planner’s problem is similar to the one in the flexible price model described in Section 4. The difference is that in the present planner’s problem, the first order conditions with respect to labor $\ell_t$ and imported inputs $m_t$ in the decentralized equilibrium are included in the implementability constraints, because the tax on imported inputs is assumed to be not available. The fact that the planner cannot intervene during crises using either monetary policy or taxes affects the design of the macroprudential tax on foreign borrowing ex ante. The Euler equation with respect to foreign borrowing in the planner’s problem when the borrowing constraint is not binding but may bind next period is given as follows:

$$u_c(t) = \beta R_t^t E_t \left[ \frac{e_{t+1}}{e_t} \left( u_c(t+1) - \xi_{t+1} \eta_{t+1} u_{cc}(t+1) \rho_{t+1} \right) - \frac{b_{t+1}}{\rho \epsilon_{t+1}} \frac{1}{Y_{t+1}} \frac{1}{\rho \epsilon_{t+1}} \rho_{t+1} \eta_{t+1}^m \right], \quad (35)$$

where $\eta_{t+1}^m$ is the Lagrange multiplier on the first order condition with respect to imported inputs in the decentralized equilibrium. The pecuniary and ex ante RER externalities indicate that the macroprudential tax should be imposed on foreign borrowing to mitigate over-borrowing. The third term comes from the ex post RER externality, and is negative when the borrowing constraint binds at $t+1$. The intuition is as follows. As shown in the flexible price model in Section 4, the planner wants to reduce imported inputs when the borrowing constraint binds, thereby causing real appreciation and relaxing the binding constraint. However, the planner here does not have any policy tool for interventions during crises. Therefore, the planner lowers the macroprudential tax rate slightly ex ante, so that when the borrowing constraint actually binds, the real exchange rate depreciates slightly more, which discourages use of imported inputs and corrects the ex post RER externality at least partially.

Under the optimal discretionary monetary policy, the planner’s problem is similar to the one with the optimal discretionary monetary policy in the previous subsection. The difference is that the present planner is not subject to the Euler equation with respect to

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$^{10}$ Although there is no externality that directly distorts the labor decision, distortions on imported inputs caused by the ex post RER externality indirectly distort the labor decision through the marginal product. See the appendix for the detail.
foreign debt in the decentralized equilibrium as an implementability constraint, because the macroprudential tax on foreign borrowing is available. The question is then whether/how the optimal discretionary monetary policy is affected by the macroprudential tax. The following proposition and discussion answer the question.

**Proposition 2** In the model described in Section 3, when a macroprudential tax on foreign borrowing is available, strict inflation targeting is not optimal discretionary monetary policy.

The appendix provides the formal proof for this proposition, but the discussion on Proposition 1 in the previous subsection helps to understand Proposition 2. When the borrowing constraint is not binding today but may bind next period, the planner wants to discourage foreign borrowing and correct over-borrowing. In the previous case where monetary policy is the only policy tool, the planner tries to achieve this purpose by contractionary monetary policy, which is captured by positive $\eta_t^{EE}$ in (33). In the present case where a macroprudential tax on foreign borrowing is available, over-borrowing is handled by the tax, and $\eta_t^{EE} = 0$ in the expression for $\gamma_t^*$ in (33). Therefore, the planner has no incentive to use monetary policy to correct over-borrowing, and monetary policy focuses only on minimizing the price adjustment cost.

When the borrowing constraint is binding, the planner still has an incentive to intervene using monetary policy. This result can be understood from the expression for $\gamma_t^*$ in (34) when the constraint is binding. The last term $-\mu_t^* b_t^*/R_t^*$ remains positive and pushes up the value of $\gamma_t^*$, implying contractionary monetary policy to mitigate real depreciation and correct the ex post RER externality. The intuition is that the macroprudential tax on foreign borrowing cannot do anything to mitigate too much imported inputs when the constraint is binding, thus monetary policy should deal with it.$^{11}$ The next section solves the model numerically and conducts quantitative analyses.

$^{11}$As is the case under strict inflation targeting, the tax rate on foreign borrowing in normal times is determined to partially correct the ex post RER externality. But once the constraint actually binds, there is nothing a macroprudential tax can do to correct the ex post RER externality.
6 Quantitative Analysis

This section conducts a quantitative analysis of the model. The model is numerically solved using a global method to deal with an occasionally binding constraint. The detailed algorithm is explained in the appendix. The section starts with setting the parameter values in the model.

6.1 Calibration

One period in the model is meant to be annual. Table 1 presents the parameter values in the model. The discount factor $\beta$ is set to target the foreign debt-to-GDP ratio at 20%, which roughly matches the average of the emerging economies. The baseline interest rate on foreign borrowing 1.04 is standard for annual models. The labor disutility coefficient $\chi$ is set so that labor supply at the steady state is 1. $\omega$ is set so that the Frisch elasticity of labor supply is 1, which is standard in the literature. The labor share in production $\alpha_\ell$ is set to the conventional value 0.7, and the imported input share $\alpha_m$ is set to target the import-to-GDP ratio at 20%, which is the average of middle income countries in 2018. Accordingly, the asset share $\alpha_k$ is set to $1 - \alpha_\ell - \alpha_m = 0.1$. The elasticity of substitution across differentiated intermediate goods $\theta$ and the price adjustment cost parameter $\psi$ are set to 8 and 50, which are the standard values in New-Keynesian models. The price elasticity of demand for exports is set to 3, which is in line with the estimated value 3 – 4 in Simonovska and Waugh (2014). Foreign demand $Y^*$ is normalized at 1. The tight borrowing limit $\kappa_t = 0.12$ is set so that the unconditional probability of crises is 6.5%. The transition matrix for $\kappa_t$ is set following Bianchi and Mendoza (2018).

The stochastic process to TFP and the interest rate are taken from Mendoza (2010). Specifically, TFP takes two values, $A_t = \exp(\pm 0.0134)$, and the interest rate takes two values, $R^*_t = R^* \times \exp(\pm 0.0196)$, with the same autocorrelation 0.59 and the negative correlation $-0.67$ between TFP and the interest rate.
### Table 1: Parameter Values

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<th>Variable</th>
<th>Value</th>
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<td>Debt-to-GDP 20%</td>
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<tr>
<td>$R^*$</td>
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<td>Standard</td>
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<tr>
<td>$\chi$</td>
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<td>Unit labor supply</td>
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<td>$\omega$</td>
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<td>$\alpha_k$</td>
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<td>$1 - \alpha_t - \alpha_m$</td>
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<td>$\alpha_m$</td>
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<tr>
<td>$\psi$</td>
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<td>Simonovska and Waugh (2014)</td>
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<td>$P_{HL}, P_{LH}$</td>
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<td>Bianchi and Mendoza (2018)</td>
</tr>
</tbody>
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#### 6.2 Decision Rules

The following quantitative analysis compares four different policy regimes: strict inflation targeting monetary policy, optimal discretionary monetary policy, and each monetary policy regime with the optimal time-consistent macroprudential tax on foreign borrowing. The section starts with plotting the decision rules for the key variables.

Figure 1 plots the decision rules under the four policy regimes when the collateral shock is not hitting the economy. The horizontal axis in each panel is the amount of the outstanding foreign debt at the beginning of a period, which is the state variable of the model. Business cycle shocks are set to be high TFP and the low interest rate. Panel (a) plots the decision rules for foreign borrowing $b_t^*$. It shows that the amount of foreign borrowing is smaller under the macroprudential tax, and the gap becomes larger as the outstanding foreign debt becomes larger. Panel (c) shows that the macroprudential tax rate becomes higher as the outstanding debt becomes larger, corresponding to larger pecuniary and ex ante RER externalities. This high tax rate creates a large gap in the amount of foreign borrowing in Panel (a). The intersection of the decision rules and the 45-degree line indicates that at the stochastic steady state, the amount of foreign debt is smaller by 8.5% under the macroprudential tax. Another observation in Panel (a) is that foreign borrowing is slightly larger under
Figure 1: Decision rules when collateral shock is not hitting

Note: This figure plots the decision rules under different policy regimes when a collateral shock is not hitting. The horizontal axis is the amount of foreign debt at the beginning of a period.

discretionary monetary policy than under inflation targeting policy, both with and without a tax. This is because, as shown in Figure 2 below, monetary policy intervention under discretionary policy would mitigate real depreciation during crises, and anticipation for this intervention reduces the expected effective interest rate on foreign borrowing, thereby causing larger borrowing ex ante. Panel (b) plots the decision rules for inflation. The inflation rate in normal times is negative under discretionary policy without a tax. As explained in the previous section, the government sets lower inflation in normal times in the aim for correcting over-borrowing. However, anticipation for policy intervention during crises more than offsets the effect of this ex ante policy, and induces larger foreign borrowing. With the macroprudential tax, inflation under discretion is almost zero because the tax deals with over-borrowing.

Figure 2 plots the decision rules when the collateral shock is hitting the economy. Panel (a) shows that when the borrowing constraint binds, which is the left to the kink in the decision rules, foreign borrowing shrinks sharply as the outstanding debt becomes larger. There are two reasons for this sharp shrink. First, as explained in the previous section, a large debt repayment with limited new borrowing triggers an amplification loop of real depreciation, which forces a large cut in new borrowing to meet the borrowing constraint. Second, large real depreciation implies large exports and reduces domestic consumption.
Figure 2: Decision rules when collateral shock hits

(a) Foreign bond  
(b) Inflation rate  
(c) Real exchange rate

Note: These figures plot the decision rules under different policy regimes when a collateral shock hits the economy. The horizontal axis is the amount of foreign bond at the beginning of a period.

Lower consumption in turn reduces the asset price and tightens the borrowing constraint. Panel (b) shows that discretionary monetary policy responds to the binding constraint by lowering inflation, i.e. contractionary monetary policy. Panel (c) shows that contractionary monetary policy actually mitigates real depreciation when the constraint binds, both with and without a tax.

6.3 Crisis Dynamics

This subsection compares the crisis dynamics under different policy regimes. The model is simulated for 100,000 periods with stochastic shocks, dropping the first 1,000 periods and the remaining 99,000 periods are used for the analysis. Following the literature, a crisis is defined as an event in which the current account is more than two standard deviations above its long-run mean. I pick up all the crisis events from the simulation under inflation targeting policy without taxes, and take the average dynamics of the variables around crises under four policy regimes. Figure 3 plots the crisis dynamics of the key variables in the seven-period window around sudden stops at period 0. The real exchange rate, asset price, output, consumption, labor, and imported inputs are expressed in terms of percentage gaps from the values at period \(-1\) under inflation targeting without taxes.
Figure 3: Crisis Dynamics

Note: These figures plot the crisis dynamics under different policy regimes. The horizontal axis is the time, and a sudden stop occurs at period 0. Consumption, output, real exchange rate, asset price, labor, and imported inputs are expressed in percentage deviations from the level at period -1 under inflation targeting policy without tax.
Panel (a) shows that without taxes, a crisis triggers a sharp reversal in capital flows. This capital reversal is accompanied by real depreciation by about 10% in Panel (b) and a fall in asset price by 17 – 18% in Panel (c). Real depreciation increases the cost of imported inputs, thereby reducing imported inputs in Panel (i) and causing a fall in output by 5 – 8% in Panel (d). Low output and large exports associated with real depreciation result in a fall in consumption by 11 – 15% as in Panel (e). These dynamics are consistent with the empirical regularities about sudden stops in emerging economies.

These panels show that the macroprudential tax is effective in stabilizing the economy. By reducing foreign borrowing in normal times, the size of a capital flow reversal becomes muted, and the real exchange rate barely depreciates. Falls in the asset price, output, and consumption are substantially smaller, and they fall only by 1%, 0.4 – 2%, and 1 – 2% respectively. Panel (f) shows that the macroprudential tax rate is about 3% before a crisis occurs.

Comparing different monetary policy regimes, Panel (g) shows that discretionary monetary policy reduces inflation during crises. This contractionary monetary policy mitigates real depreciation both with and without taxes as shown in Panel (b), but this intervention is less effective without a macroprudential tax. This is because, as discussed above, anticipation of monetary policy intervention reduces the effective real interest rate on foreign borrowing, and induces larger borrowing ex ante, as is clear in Panel (a).

Panels (d) and (e) show that contractionary monetary policy during crises has negative impacts on both output and consumption, but the effect on consumption is relatively limited. With the macroprudential tax, a drop in output during crises from the previous period is 0.4% under inflation targeting and 2% under discretionary monetary policy, implying output drops more by 1.6% under discretionary policy. In contrast, a drop in consumption is 1% and 2% respectively, and the gap is only 1%. This difference comes from the fact that contractionary monetary policy mitigates real depreciation and reduces exports, which enables a larger fraction of output to be consumed domestically. Monetary policy regimes also affect the dynamics of labor supply. Panel (h) shows that a limited drop in output under inflation targeting is supported by large labor supply, which implies high labor disutility. These dynamics in consumption and labor are relevant for the welfare analysis. Section 6.5
conducted the formal welfare analysis and shows that with the optimal macroprudential tax, discretionary monetary policy gives higher expected utility than inflation targeting policy.

### 6.4 Business Cycle Moments

Table 2 presents standard deviations of the key variables under different policy regimes. Numbers in parenthesis are percentages compared to the standard deviations under inflation targeting without taxes. It is observed that macroprudential taxes reduce the volatility of all variables. In particular, the standard deviations of foreign bond and the real exchange rate are about 20% of those without taxes. Comparing different monetary policy regimes, discretionary monetary policy leads to higher volatilities of output and consumption both with and without taxes. In contrast, the real exchange rate is more stable under discretionary monetary policy. The effect of monetary policy regimes on volatility of foreign bond depends on whether a macroprudential tax is imposed or not. Discretionary monetary policy increases volatility of foreign bond without taxes, but reduces volatility with taxes. As shown in the previous subsection, discretionary monetary policy without taxes induces larger foreign borrowing in normal times, which leads to higher volatility of foreign bond. If over-borrowing is handled by a macroprudential tax, discretionary monetary policy reduces volatility of foreign bond.

Figure 4 plots the ergodic distribution of foreign bond under the four policy regimes. The left panel shows the distributions without macroprudential taxes, and the right panel plots the distributions under the optimal taxes. The vertical axis is the share of time periods in the stochastic simulation when the amount of foreign debt belongs to each bin. It is observed

---

**Table 2: Business Cycle Moments**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Without tax</th>
<th>With tax</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Targeting</td>
<td>Discretion</td>
</tr>
<tr>
<td>Output</td>
<td>0.024</td>
<td>0.028 (116.3%)</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.030</td>
<td>0.035 (114.3%)</td>
</tr>
<tr>
<td>Foreign bond</td>
<td>0.025</td>
<td>0.026 (105.2%)</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>0.016</td>
<td>0.014 (89.8%)</td>
</tr>
<tr>
<td>Asset price</td>
<td>0.063</td>
<td>0.066 (103.3%)</td>
</tr>
</tbody>
</table>
that the distributions without taxes are centered around larger foreign debt compared to the distributions under the taxes. The distributions without taxes also have low but positive long tails in the side of the small amount of debt, indicating sharp reversals of capital flows during crises. The right panel shows that the distributions are highly concentrated under macroprudential taxes, and they do not have long tails, implying that macroprudential taxes stabilize foreign bond and capital flows. Comparing different monetary policy regimes, the distributions show that foreign debt is on average larger under discretionary monetary policy.

Regarding the crisis probability, the unconditional probability of crises is 6.5% under inflation targeting and 6.2% under discretionary monetary policy without macroprudential taxes. With macroprudential taxes, the current account is substantially more stable and a crisis never happens if it is defined based on the standard deviation of the current account under inflation targeting policy without taxes.

### 6.5 Welfare Analysis

This subsection compares the welfare implications by different policy regimes. I use the expected utility under inflation targeting monetary policy without macroprudential taxes as the benchmark welfare, and express the welfare gain/loss by other policy regimes in terms
Figure 5: Welfare Impact by Different Policy Regimes

![Figure 5: Welfare Impact by Different Policy Regimes](image)

Note: This figure plots the expected welfare gain/loss under different policy regimes, in terms of permanent consumption compared to inflation targeting without taxes.

of permanent consumption gain/loss compared to the benchmark welfare. Specifically, let $V^{IT}(b^*_1, s_0)$ denote the expected utility under inflation targeting without taxes when the initial state is $(b^*_1, s_0)$. Then the welfare gain/loss by another policy regime is expressed as $\gamma(b^*_1, s_0)$ that satisfies the following:

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \log \left( \left[ 1 + \gamma(b^*_1, s_0) \right] c_t - \chi \frac{\ell_{t+1}^{1+\omega}}{1+\omega} \right) \right] = V^{IT}(b^*_1, s_0)$$

Figure 5 plots the results. The horizontal axis is the state foreign debt $b^*_1$, and the vertical axis is the welfare gain/loss in percent, which is $\gamma(b^*_1, s_0) \times 100$. The collateral shock is not hitting the economy. The business cycle shocks are set to high TFP and the low interest rate, but these do not affect the results much. The dashed vertical line is the mode of the distribution of foreign debt in the stochastic simulation under inflation targeting without taxes.

The first observation is that discretionary monetary policy without taxes gives lower expected welfare than inflation targeting policy, by 0.03% at the mode. This result can
be understood from the discussions in the previous subsections: discretionary monetary policy without taxes induces larger borrowing ex ante and destabilizes the economy. In contrast, combined with the optimal macroprudential tax on foreign borrowing, discretionary monetary policy gives higher expected welfare than inflation targeting. At the mode of the distribution, inflation targeting with the optimal tax gives a 0.070% welfare gain, and discretionary policy gives a 0.075% welfare gain.

Although this result suggests that monetary policy intervention during crises can be welfare-improving, the welfare gain over inflation targeting is only 0.005% of permanent consumption. As discussed in the previous section, monetary policy intervention aims at correcting the ex post real exchange rate externality that induces too much use of imported inputs during crises. To measure the effectiveness of monetary policy intervention, I compute the welfare gain in the case where the planner can use the optimal taxes on both foreign debt in normal times and imported inputs during crises. Monetary policy is conducted as inflation targeting. The result is plotted in Figure 6. The welfare gain under the two-tax regime is larger than the two policy regimes above, a 0.092% gain over inflation targeting without taxes.
at the mode of the debt distribution. Compared to inflation targeting with the optimal tax, the welfare gain is 0.022%, which is more than four times as large as a 0.005% gain by discretionary monetary policy. This result suggests that monetary policy is not an effective policy tool for intervention during crises. There are two reasons for this ineffectiveness. First, contractionary monetary policy discourages both labor and imported inputs, causing unnecessarily low output. Second, monetary policy intervention is accompanied by the price adjustment cost.

Finally, I consider the fixed-rate tax on foreign borrowing, given that the optimal time-varying macroprudential tax is difficult to implement in reality. The optimal fixed tax rate is different depending on the monetary policy regime. I find that 2.75% is optimal under discretionary monetary policy, and 2.5% is optimal under inflation targeting policy. The optimal tax rate is higher under discretionary monetary policy because the tax needs to correct over-borrowing induced by monetary policy intervention under discretionary policy. The results are plotted in Figure 7. The welfare gain over inflation targeting without taxes at the mode is 0.055% under both monetary policy regimes. This number corresponds to
about 75% of welfare gains under the optimal taxes. The model suggests that the fixed-rate tax gives a decent welfare gain.

### 7 Conclusion

In this paper, I develop a small open economy model which borrows from abroad in foreign currency subject to an occasionally binding borrowing constraint. The novel mechanism of the model is that the interaction between the balance-of-payments adjustment and the borrowing constraint triggers an amplification loop of real depreciation, which increases the domestic-currency value of foreign debt and exacerbates crises. Because private agents take the real exchange rate as given and do not internalize this mechanism, they over-borrow in normal times and import too much during crises, both of which lead to inefficiently large real depreciation during crises.

Given this model economy, I study the optimal time-consistent monetary and macro-prudential policies. When monetary policy is the only available policy tool, the optimal discretionary monetary policy is contractionary both in normal times and during crises to mitigate real depreciation during crises. When combined with the optimal macroprudential tax on foreign borrowing, the optimal discretionary monetary policy focuses only on minimizing the price adjustment cost in normal times, but still intervenes during crises to mitigate real depreciation.

In the quantitative analysis, I show that discretionary monetary policy induces larger borrowing in normal times and destabilizes the economy through an anticipation of policy intervention during crises. Macroprudential taxes substantially stabilizes the economy regardless of the monetary policy regime.

The welfare analysis shows that discretionary monetary policy gives lower welfare than inflation targeting without macroprudential taxes, but gives higher welfare if combined with the optimal taxes. However, the welfare gain by monetary policy intervention during crises is limited because it has negative side effects on labor inputs and output. Macroprudential taxes improve welfare regardless of the monetary policy regimes, even with the fixed tax rate.
References


Appendix

A Flexible Price Model

This section gives the full description of the Ramsey planner’s problem and the first order conditions in the flexible price model in Section 4. The Ramsey planner’s problem is set up as follows:

\[
V(b_{t-1}^*, s_t) = \max_{c_t, b_t^*, e_t, m_t, q_t} \log \left( c_t - \chi \frac{\ell_t^{1+\omega}}{1 + \omega} \right) + \beta E_t V(b_{t+1}^*, s_{t+1}) \\
- \lambda_t^{RP} \left[ c_t + e_t \left( \frac{b_t^*}{R_t^s} - b_{t-1}^* \right) - y_t + e_t m_t \right] \\
- \mu_t^{RP} \left[ -e_t \frac{b_t^*}{R_t^s} - \kappa_t q_t \right] \\
- \xi_t \left\{ q_t u_c(t) - \beta E_t \left[ u_c \left( \tilde{c} \left( b_t^*, s_{t+1} \right) - \chi \frac{\ell \left( b_t^*, s_{t+1} \right)^{\omega+1}}{\omega + 1} \right) \right] \left( \tilde{q} \left( b_t^*, s_{t+1} \right) + \alpha_k \frac{\tilde{y} \left( b_t^*, s_{t+1} \right)}{k_{ss}} \right) + \tilde{\mu} \left( b_t^*, s_{t+1} \right) \kappa_{t+1} \tilde{q} \left( b_t^*, s_{t+1} \right) \right\} \\
+ \gamma_t \left[ e_t^p Y^* - e_t m_t - e_t \left( \frac{b_t^*}{R_t^s} - b_{t-1}^* \right) \right],
\]

where variables with tilde indicate the decision rules by the next period planner. As discussed in the main text, imported inputs and foreign bond are chosen directly by the planner, thus the first order conditions with respect to these variables (7) and (9) are not included in the implementability constraint. The complementary slackness condition in the decentralized equilibrium is not included either, because private \( \mu_t \) appears only in that condition, thus any \( \mu_t \) is consistent with the first order conditions here. The first order condition with respect to labor (6) with \( p_t^w = 1 \) is not included either, but later I show that this constraint is not binding. The first order conditions are as follows:

\[
c_t : u_c(t) - \lambda_t^{RP} - \xi_t q_t u_{cc}(t) = 0, \quad (36) \\
b_t^* : - \lambda_t^{RP} e_t + \mu_t^{RP} e_t \frac{1}{R_t^s} - \xi_t \frac{\partial R H S^q_{t+1}}{\partial b_t^s} - \gamma_t e_t \frac{1}{R_t^s} + \beta E_t V_b(b_t^*, s_{t+1}) = 0, \quad (37)
\]
\[
\ell_t : u_c(t)(-\chi^{\ell_t}_t) + \lambda_t^{RP} \alpha_t \frac{y_t}{\ell_t} - \xi_t q_t u_{cc}(t)(-\chi^{\ell_t}_t) = 0 \quad (38)
\]

\[
m_t : - \lambda_t^{RP} \left( e_t - \alpha_m \frac{y_t}{m_t} \right) - \gamma_t e_t = 0, \quad (39)
\]

\[
e_t : - \lambda_t^{RP} \left( \frac{b^*_t}{R^*_t} - b^*_{t-1} + m_t \right) + \mu_t^{RP} \frac{b^*_t}{R^*_t} + \gamma_t \left( \rho e_t^{p-1}Y^* - m_t - \frac{b^*_t}{R^*_t} + b^*_{t-1} \right) = 0, \quad (40)
\]

\[
q_t : \mu_t^{RP} \kappa_t k = \xi_t u_c(t). \quad (41)
\]

Rearranging (38),
\[
(u_c(t) - \xi_t q_t u_{cc}(t)) \left( \chi^{\ell_t}_t \right) = \lambda_t^{RP} \alpha_t \frac{y_t}{\ell_t}.
\]

Plugging (36) into this equation,
\[
\chi^{\ell_t}_t = \alpha_t \frac{y_t}{\ell_t},
\]

which coincides with (6) with \( p^w_t = 1 \), the decentralized equilibrium condition with respect to \( \ell_t \).

Rearranging (40), the expression for \( \gamma_t \) is:
\[
\gamma_t = \frac{1}{\rho - 1} \lambda_t^{RP} - \frac{b^*_t/R^*_t}{(\rho - 1)e_t^{p-1}Y^*} \mu_t^{RP}. \quad (42)
\]

Plugging this equation into (39),
\[
\alpha_m \frac{y_t}{m_t} - e_t = e_t \gamma_t \lambda_t^{RP} = e_t \left[ \frac{1}{\rho - 1} - \frac{\mu_t^{RP}}{\lambda_t^{RP}} \frac{b^*_t/R^*_t}{(\rho - 1)e_t^{p-1}Y^*} \right],
\]

which is (27) in the main text. This equation also shows that the terms-of-trade externality can be corrected by the fixed rate tax \( \tau_m = 1/(\rho - 1) \).

Plugging \( \lambda_t^{RP} \) from (36) and \( \gamma_t \) from (40) into the first order condition with respect to \( b^*_t \) (37),
\[
\frac{\rho}{\rho - 1} \left[ u_c(t) - \xi_t q_t u_{cc}(t) \right] - \mu_t^{RP} \left( \frac{b^*_t/R^*_t}{(\rho - 1)e_t^{p-1}Y^*} \right) + \mu_t^{RP} \left( \frac{b^*_t/R^*_t}{(\rho - 1)e_t^{p-1}Y^*} \right) = \beta R_t^{*} E_t \left[ \frac{e^{t+1}_t}{e_t} \left\{ \frac{\rho}{\rho - 1} \left[ u_c(t+1) - \xi_{t+1} q_{t+1} u_{cc}(t+1) \right] - \mu_{t+1}^{RP} \left( \frac{b^*_t/R^*_t}{(\rho - 1)e_t^{p-1}Y^*} \right) \right\} \right] + \beta R_t^{*} \xi_t \frac{\partial R_t^{*}}{\partial b^*_t},
\]

where \( \partial R_t^{*} / \partial b^*_t \) collects all the partial derivatives of the next period planner’s decision rules in the asset price equation with respect to \( b^*_t \). When the borrowing constraint is not
binding today but may bind next period, this equation reduces to the following equation:

\[ u_c(t) = \beta R^*_t E_t \left[ \frac{\epsilon_{t+1}}{e_t} \left\{ [u_c(t + 1) - \xi_{t+1} q_{t+1} u_{cc}(t + 1)] - \mu_{t+1}^* \frac{b_{t+1}^*}{R_{t+1}^*} \right\} \right], \]

which is (24) in the main text.

B  Optimal Discretionary Monetary Policy

This section considers the case where monetary policy is the only policy tool and characterizes the optimal discretionary monetary policy.

B.1 Planner’s Problem

The planner chooses inflation \( \pi_t \) and all the endogenous variables to maximize the households’ expected utility subject to all the decentralized equilibrium conditions as the implementability constraints. With the Lagrange multipliers on the constraints, the Ramsey planner’s problem is defined as follows:

\[
V(b_{t-1}^*, s_t) = \max_{\pi_t, c_t, b_t^*, \epsilon_t, m_t, q_t, p_t^w, \mu_t} \log \left( c_t - \chi \frac{\mu_t^{1+\omega}}{1+\omega} \right) + \beta E_t V(b_t^*, s_{t+1}) \\
- \lambda_t^* \left[ c_t + e_t \left( \frac{b_t^*}{R_t^*} - b_{t-1}^* \right) \right] - \left\{ Y_t - \frac{\psi}{2} Y_t^2 \right\} + e_t m_t \\
- \mu_t^* \left[ -e_t \frac{b_t^*}{R_t^*} - \pi_t q_t k \right] \\
- \xi_t^* \left[ q_t u_c(t) - \beta E_t \left\{ u_c(t + 1) \left( q_{t+1} + p_{t+1}^{w} \frac{\psi_{t+1}}{k} \right) + \mu_{t+1}^* \kappa_{t+1} q_{t+1} \right\} \right] \\
+ \gamma_t^* \left[ e_t^\omega Y^\omega - e_t m_t - e_t \left( \frac{b_t^*}{R_t^*} - b_{t-1}^* \right) \right] \\
- \eta_t^{PC} \left[ (-\theta + \theta p_t^{w} - \psi \pi_t (1 + \pi_t)) Y_t + \beta E_t \left\{ u_c(t + 1) \frac{u_c(t)}{u_c(t)} \psi_{t+1} (1 + \pi_{t+1}) Y_{t+1} \right\} \right] \\
- \eta_t^{EE} \left[ u_c(t) - \mu_t - \beta R_t^* E_t \left\{ \frac{e_t+1}{e_t} u_c(t + 1) \right\} \right] \\
+ \eta_t^l \left[ p_t^{w} - \chi e_t \frac{k_t}{\alpha_t} Y_t \right] \\
+ \eta_t^m \left[ p_t^{w} - e_t \frac{\rho}{\rho - 1} \frac{m_t}{\alpha_m} \frac{1}{Y_t} \right] \\
- \eta_t^a \left[ \mu_t \left( -e_t \frac{b_t^*}{R_t^*} - \kappa_t q_t \right) \right].
\]

41
where $Y_t$ should be taken as the Cobb-Douglas production defined in (2). To simplify the notations, I omit tildes to denote the next period planner's decision rules, but the variables with $t+1$ should be taken as functions of $b^*_t$ and $s_{t+1}$. The first order conditions are given as follows:

$$
\pi_t : \eta^P_t \psi(2\pi_t + 1)Y_t = \lambda^*_t \psi \pi_t Y_t, \quad (53)
$$

$$
c_t : u_c(t) - \lambda^*_t + \eta^P_t \left[ \frac{u_c(t)}{u_c(1 + \pi_{t+1})} \right] - \xi^*_t u_{cc}(t)q_t - \eta^E_t \xi^*_t u_{cc}(t) = 0, \quad (54)
$$

$$
q_t : \xi^*_t u_c(t) = (\mu^*_t + \eta^*_t \mu_t) \kappa_t k, \quad (55)
$$

$$
e_t : \gamma^*_t = \frac{1}{\rho - 1} \lambda^*_t + \frac{1}{\rho - 1} e_t^{\rho - 1} \left[ \frac{\eta^m_t \rho - m_t}{m_t} + \eta^E_t \frac{1}{e_t} (u_c(t) - \mu^*_t) - (\mu^*_t + \eta^*_t \mu_t) \frac{b^*_t}{R^*_t} \right], \quad (56)
$$

$$
\ell_t : \lambda^*_t \left[ \frac{1 - \psi^n_t}{2} \right] Y_t - \frac{\ell_t}{\ell_t} - \eta^P_t \left[ (\theta + \theta p^w_t) \psi \pi_t (1 + \pi_t) \right] \alpha_m \frac{Y_t}{\ell_t} \quad (57)
$$

$$
-\eta^n_t \ell_t \left[ (\theta + \theta p^w_t) \psi \pi_t (1 + \pi_t) \right] \alpha_m \frac{Y_t}{\ell_t} + p^w_t \frac{\alpha_m}{\ell_t} (\eta^* + \eta^{m*}) = 0,
$$

$$
m_t : \lambda^*_t \left[ \frac{1 - \psi^n_t}{2} \right] \alpha_m \frac{Y_t}{m_t} - \ell_t - \gamma^*_t e_t - \eta^P_t \left[ (\theta + \theta p^w_t) \psi \pi_t (1 + \pi_t) \right] \alpha_m \frac{Y_t}{m_t} \quad (58)
$$

$$
-\frac{\eta^m_t \ell_t}{\rho} + \frac{1}{\rho - 1} \frac{1}{\alpha_m Y_t} + p^w_t \frac{\alpha_m}{m_t} (\eta^* + \eta^{m*}) = 0,
$$

$$
b^*_t : \lambda^*_t \ell_t \frac{1}{R^*_t} e_t + \gamma^*_t e_t \frac{1}{R^*_t} - (\mu^*_t + \eta^*_t \mu_t) \ell_t \frac{1}{R^*_t} \quad (59)
$$

$$
+ \eta^P_t \frac{\partial R_H S^P_{t+1}}{\partial b^*_t} - \xi^*_t \frac{\partial R_H S^E_{t+1}}{\partial b^*_t} - \eta^E_t \frac{\partial R_H S^E_{t+1}}{\partial b^*_t} = \beta E_t V_b(b^*_t, s_{t+1}),
$$

$$
p^w_t : \eta^P_t \psi Y_t = \eta^* + \eta^{m*}, \quad (60)
$$

$$
\mu_t : \eta^E_t - \eta^m_t \left[ -\ell_t \frac{b^*_t}{R^*_t} - \kappa_t q_t k \right] = 0, \quad (61)
$$

where the three partial derivatives in (59) are the terms that collect all the partial derivatives of the next planner's decision rules in (46), (48), and (49) with respect to $b^*_t$. The envelope condition is given by:

$$
V_b(b^*_t, s_{t+1}) = \lambda^*_t e_t + \gamma^*_t e_t. \quad (61)
$$
Combining the envelope condition at $t+1$ and (59) gives the following Euler equation with respect to foreign bond:

\[ \lambda_t^* + \gamma_t^* - (\mu_t^* + \eta_t^\mu \mu_t) + \frac{R^t}{e_t} \left[ \eta_t^{PC} \frac{\partial RHS^{PC}_t}{\partial b_t^{t+1}} - \xi_t \frac{\partial RHS^{EE}_t}{\partial b_t^{t+1}} - \eta_t^{EE} \frac{\partial RHS^{EE}_{t+1}}{\partial b_t^{t+1}} \right] = \beta R^t e_t \left[ \frac{e_{t+1}}{e_t} \left( \lambda_{t+1}^* + \gamma_{t+1}^* \right) \right]. \]  

(62)

**B.2 Without Borrowing Constraint**

This subsection proves that without borrowing constraint, strict inflation targeting is the optimal discretionary monetary policy. Removing the borrowing constraint from the model implies $\mu_t^* = \mu_t = \eta_t^\mu = 0$, and $\xi_t = 0$ by (55). The proof proceeds in the following steps: I assume that the next period expected inflation is zero, i.e. $E_t(\pi_{t+1}) = 0$. Then I set $\pi_t = 0$ and show that all the first order conditions in the above planner’s problem are satisfied.

First, $\pi_t = E_t(\pi_{t+1}) = 0$ implies $p_t^\pi = 1$ by the New-Keynesian Phillips curve in (48). $\pi_t = 0$ also implies $\eta_t^{PC} = 0$ by (53), the first order condition with respect to $\pi_t$. Now, I make a guess that $\gamma_t^{EE} = 0$, which implies that the Euler equation with respect to foreign bond in the decentralized equilibrium holds in the planner’s problem as well. Later I will verify that this guess is correct. Given these things, (54) implies:

\[ u_c(t) = \lambda_t^*. \]

Plugging this into (57),

\[ u_c(t) \left[ \frac{\alpha}{\ell_t} \frac{Y_t}{\ell_t} - \chi_{\ell_t}^\omega \right] - \eta_t^\ell \chi(\omega + 1) \ell_t^\omega \frac{1}{\alpha \ell_t Y_t} + \frac{\alpha}{\ell_t} (\eta_t^\ell + \eta_t^m) = 0. \]

The first term is canceled out by (50), thus:

\[ \eta_t^\ell \chi(\omega + 1) \ell_t^\omega \frac{1}{\alpha \ell_t Y_t} = \frac{\alpha}{\ell_t} (\eta_t^\ell + \eta_t^m). \]

(60) with $\eta_t^{PC} = 0$ implies $\eta_t^\ell + \eta_t^m = 0$. Plugging this result into the above equation implies $\eta_t^\ell = 0$, and therefore $\eta_t^m = 0.$

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Now, the first order condition with respect to \( m_t \) (58) is now:

\[
  u_c(t) \left[ \alpha_m \frac{Y_t}{m_t} - e_t \right] = \gamma_t^* e_t.
\]

(51) in the decentralized equilibrium condition with \( p_l^w = 1 \) implies:

\[
  \alpha_m \frac{Y_t}{m_t} = e_t \frac{\rho}{\rho - 1}.
\]

Combining these two equations implies the following:

\[
  \gamma_t^* = \frac{1}{\rho - 1} u_c(t).
\]

Plugging this equation into the first order condition with respect to \( e_t \) (56) proves that (56) is satisfied. Plugging the expression for \( \gamma_t^* \) into (62), the Euler equation with respect to foreign bond in the planner’s problem is:

\[
  \frac{\rho}{\rho - 1} u_c(t) = \beta R_t^E E_t \left[ \frac{e_t + 1}{e_t} \frac{\rho}{\rho - 1} u_c(t + 1) \right],
\]

where \( \rho/(\rho - 1) \) is canceled out. Therefore, the Euler equation coincides in the decentralized equilibrium and in the planner’s allocation, which verifies that the guess \( \eta_t^{EE} = 0 \) is correct. This completes the proof.

**B.3 Interpretations of Lagrange Multipliers**

This subsection provides interpretations and intuitions for Lagrange multipliers \( \eta_t^\ell, \eta_t^m, \) and \( \eta_t^{PC} \). By structure of the planner’s problem, \( \eta_t^\ell \) is the social value of a change in \( \ell_t \) responding to a marginal increase in \( p_l^w \), and \( \eta_t^m \) is that of \( m_t \).

Plugging the first order condition with respect to \( \pi_t \) (53) into the one with respect to \( \ell_t \)
(57) and rearranging,

\[
\eta^\ell_t = \frac{\alpha_t Y_t}{\chi(\omega + 1)\ell_t^\omega} \left[ \lambda^*_t \left\{ \left( 1 - \frac{\psi}{2} \pi_t^2 \right) \alpha_t \frac{Y_t}{\ell_t^\omega} - \chi \ell_t^\omega \right\} - \eta^\text{PC}_t \left\{ LHS_t^\text{PC} \alpha_t \frac{Y_t}{\ell_t} - \theta p_t^w \alpha_t \frac{Y_t}{\ell_t} \right\} \right]
\]

\[
= \varepsilon^\text{pl}_t \left[ \lambda^*_t \left\{ \left( 1 - \frac{\psi}{2} \pi_t^2 \right) \alpha_t \frac{Y_t}{\ell_t^\omega} - \chi \ell_t^\omega \right\} - \eta^\text{PC}_t \frac{1}{\omega + 1} \alpha_t \frac{Y_t}{\ell_t} \left[ LHS_t^\text{PC} - \theta p_t^w \right] \right],
\]

where \( LHS_t^\text{PC} = \{-\theta + \theta p_t^w - \psi \pi_t(1 + \pi_t)\} \). In the first term, \( \varepsilon^\text{pl}_t = \alpha_t Y_t/\chi(\omega + 1)\ell_t^\omega \) indicates a change in \( \ell_t \) responding to a marginal increase in \( p_t^w \), because it is obtained by applying the implicit function theorem to \( \chi \ell_t^\omega \ell_t/(\alpha_t Y_t) - p_t^w = 0 \) to derive \( \partial \ell_t/\partial p_t^w \), which is strictly positive. The first term is then the social value of an increase in \( \ell_t \) responding to a marginal increase in \( p_t^w \) by affecting production and labor disutility. In the second term, the coefficient comes from the following equation, using (50):

\[
\frac{\alpha_t Y_t}{\chi(\omega + 1)\ell_t^\omega} \times \frac{Y_t}{\ell_t^\omega} = \frac{1}{\omega + 1} \frac{\ell_t}{p_t^w} \times \frac{Y_t}{\ell_t} = \frac{1}{\omega + 1} \frac{\alpha_t Y_t}{p_t^w}.
\]

The second term is the social value of an increase in \( \ell_t \) responding to a marginal increase in \( p_t^w \) by affecting \( p_t^w \) through the New-Keynesian Phillips curve. Since \( \eta^\text{PC}_t \) contains \( \eta^\ell_t \) in its expression as shown in (60), there is a feedback loop in which a marginal change in \( p_t^w \) increases \( \ell_t \), which in turn affects \( p_t^w \) through the New-Keynesian Phillips curve, and the loop continues.

Doing the similar algebra with respect to \( m_t \) by plugging (53) into (58),

\[
\eta^m_t = \frac{\alpha_m Y_t}{\varepsilon_t \rho/(\rho - 1)} \left[ \lambda^*_t \left\{ \left( 1 - \frac{\psi}{2} \pi_t^2 \right) \alpha_m \frac{Y_t}{m_t} - \varepsilon_t \right\} - \gamma^*_t \varepsilon_t - \eta^\text{PC}_t \left\{ LHS_t^\text{PC} \alpha_m \frac{Y_t}{m_t} - \theta p_t^w \alpha_m \frac{Y_t}{m_t} \right\} \right]
\]

\[
= \varepsilon^\text{pm}_t \left[ \lambda^*_t \left\{ \left( 1 - \frac{\psi}{2} \pi_t^2 \right) \alpha_m \frac{Y_t}{m_t} - \varepsilon_t \right\} - \gamma^*_t \varepsilon_t \right] - \eta^\text{PC}_t \frac{\alpha_m Y_t}{p_t^w} \left[ LHS_t^\text{PC} - \theta p_t^w \right],
\]

where \( \varepsilon^\text{pm}_t = \alpha_m Y_t/[\varepsilon_t \rho/(\rho - 1)] \) is a change in \( m_t \) responding to a marginal increase in \( p_t^w \), which is strictly positive given \( \rho > 1 \). The first term is the social value of an increase in \( m_t \) responding to a marginal increase in \( p_t^w \) by affecting production, the payment to imported inputs, and the effect on real exchange rate captured by \( \gamma^*_t \). The coefficient in the second
term comes from the following, using (51):

\[
\frac{\alpha_m Y_t}{\epsilon_t \rho/(\rho - 1)} \times \alpha_m \frac{Y_t}{m_t} = \frac{m}{p_t^w} \alpha_m \frac{Y_t}{m_t} = \frac{\alpha_m Y_t}{p_t^w}.
\]

The second term is the social value of an increase in \( m_t \) responding to a marginal increase in \( p_t^w \) by affecting \( p_t^w \) through the New-Keynesian Phillips curve. This term contains the same feedback loop as in the case of \( \ell_t \).

Plugging \( \eta_t^\ell \) and \( \eta_t^m \) into the first order condition with respect to \( p_t^w \) (60) gives the following equation:

\[
\theta Y_t \eta_t^{PC} = \eta_t^\ell + \eta_t^m
\]

\[
= \left( \varepsilon_t^{\ell} \lambda_t^\ell + \varepsilon_t^m \lambda_t^m \right) - \eta_t^{PC} \left[ \text{LHS}^{PC}_t - \theta p_t^w \right] \left[ \frac{1}{\omega_t + 1} \frac{\alpha_t Y_t}{p_t^w} + \frac{\alpha_m Y_t}{p_t^w} \right].
\]

where \( \lambda_t^\ell \) is the bracket term that starts with \( \lambda_t^\pi \) in (63), \( \lambda_t^m \) is the bracket term that starts with \( \lambda_t^\pi \) in (64). Solving for \( \eta_t^{PC} \),

\[
\eta_t^{PC} = \frac{\varepsilon_t^{\ell} \lambda_t^\ell + \varepsilon_t^m \lambda_t^m}{\theta Y_t \left[ 1 + \frac{1}{\theta Y_t} \left\{ (\text{LHS}^{PC}_t - \theta p_t^w) \left( \frac{1}{\omega_t + 1} \frac{\alpha_t Y_t}{p_t^w} + \frac{\alpha_m Y_t}{p_t^w} \right) \right\} \right]}
\]

\[
= \frac{\varepsilon_t^{\ell} \lambda_t^\ell + \varepsilon_t^m \lambda_t^m}{\theta Y_t \left[ 1 + \frac{1}{\theta} \left\{ (-\theta - \psi_t \pi_t (1 + \pi_t)) \left( \frac{1}{\omega_t + 1} \frac{\alpha_t Y_t}{p_t^w} + \frac{\alpha_m Y_t}{p_t^w} \right) \right\} \right]}.
\]

The sign of the denominator is ambiguous. However, in the special case of \( E_t (\pi_{t+1}) = 0 \), the New-Keynesian Phillips curve implies \(-\theta - \psi_t \pi_t (1 + \pi_t) = -\theta p_t^w\), and this equation reduces to the following:

\[
\eta_t^{PC} = \frac{\varepsilon_t^{\ell} \lambda_t^\ell + \varepsilon_t^m \lambda_t^m}{\theta Y_t \left[ 1 - \left( \frac{1}{\omega_t + 1} \alpha_t + \alpha_m \right) \right]},
\]

which is (30) in the main text. The denominator is now strictly positive, and the bracket term is between 0 and 1. The interpretation is as follows. By structure of the implementability constraint (48), \( \eta_t^{PC} \) can be interpreted as the effect of an increase in \( p_t^w \) by \( 1/\theta Y_t \) units on the social value. A direct effect of an increase in \( p_t^w \) are the effect on \( \ell_t \) and \( m_t \), captured by the numerator of (66), which are the first terms in (63) and (64). There is also an indirect effect. These changes in \( \ell_t \) and \( m_t \) affect \( p_t^w \) through the New-Keynesian Phillips curve, captured
by the last terms in (63) and (64). This second-round change in \( p_t^w \) causes the second-round direct effects on \( \ell_t \) and \( m_t \), and also the third-round change in \( p_t^w \). The bracket term in the denominator of (66) captures the amplification effect through this feedback loop. Therefore, \( \eta_t^{PC} \) captures the total effect of an increase in \( p_t^w \) on the social welfare including the effect of the amplifying feedback loops. Although the clear expression in (66) is obtained only under the special case of \( E_t(\pi_{t+1}) = 0 \), the interpretation of \( \eta_t^{PC} \) basically holds in the full model, where \( \eta_t^{PC} \) is given by (65).

Given the interpretation of \( \eta_t^{PC} \), the first order condition with respect to \( \pi_t \) (53) can be understood intuitively. Rewriting the equation just for convenience,

\[
\eta_t^{PC} \psi(2\pi_t + 1)Y_t = \lambda_t^* \psi\pi_t Y_t.
\]

The right-hand side is understood as follows: When inflation \( \pi_t \) is marginally increased, it causes a loss (a gain if \( \pi_t < 0 \)) of resources by \( \psi\pi_t Y_t \) units through the price adjustment cost. The social value of this loss (or gain) is given by the right-hand side. The left-hand side is understood as follows: By increasing \( \pi_t \) marginally, the wholesale goods price \( p_t^w \) increases by \( (1/\theta) \psi(2\pi_t + 1) \) through the New-Keynesian Phillips curve. A marginal increase in \( p_t^w \) gives the social value of \( \eta_t^{PC} \theta Y_t \). Therefore, a marginal increase in \( \pi_t \) gives the social value of \( (1/\theta) \psi(2\pi_t + 1) \times \eta_t^{PC} \theta Y_t = \eta_t^{PC} \psi(2\pi_t + 1)Y_t \) by increasing \( p_t^w \) through the New-Keynesian Phillips curve, which is the left-hand side. The first order condition with respect to \( \pi_t \) (53) states that the government chooses \( \pi_t \) so that the marginal social benefit (cost if \( \pi_t < 0 \)) from increasing \( \pi_t \) and \( p_t^w \) in the left-hand side becomes equal to the marginal cost (benefit if \( \pi_t < 0 \)) from a change in the price adjustment cost in the right-hand side.

**B.4 Proof of Proposition 1**

**Proposition 1** In the model described in Section 3, when monetary policy is the only policy tool, strict inflation targeting is not optimal discretionary monetary policy.

I adopt proof by contradiction: I set the next period expected inflation to zero, i.e. \( E_t(\pi_{t+1}) = 0 \). Then, I assume \( \pi_t = 0 \) and show that the first order conditions in the planner’s problem are not satisfied.
First, \( \pi_t = E_t(\pi_{t+1}) = 0 \) implies \( p_t^w = 1 \) and \( \pi_t = 0 \) implies \( \eta_t^{PC} = 0 \). The previous subsection shows that \( \eta_t^{PC} \) given \( E_t(\pi_{t+1}) = 0 \) satisfies (66). Since the denominator is strictly positive, the proof reduces to show that the numerator in (66) is not zero, which would imply \( \eta_t^{PC} \neq 0 \) and contradict with \( \pi_t = 0 \) by (53).

Dividing the numerator in (66) by \( \lambda_t^* \), which is strictly positive because it is the social value of an additional unit of home tradable goods, I obtain the following:

\[
\varepsilon_t^{\ell} \frac{\lambda_t^\ell}{\lambda_t^*} + \varepsilon_t^{pm} \frac{\lambda_t^m}{\lambda_t^*} = \varepsilon_t^{\ell} \left[ \alpha_t \frac{Y_t}{\ell_t} - \chi_t^{\ell^w} \right] + \varepsilon_t^{pm} \left[ \alpha_t \frac{Y_t}{m_t} - e_t - \frac{\gamma_t^*}{\lambda_t^*} e_t \right].
\] (67)

Given \( p_t^w = 1 \) and \( \pi_t = 0 \), the first bracket coincides with the implementability constraint with respect to \( \ell_t \) in (50), and thus disappears. Given the implementability constraint with respect to \( m_t \) in (51) with \( p_t^w = 1 \), the second term is zero if and only if:

\[
\frac{\gamma_t^*}{\lambda_t^*} = \frac{1}{\rho - 1}.
\]

Note that this equation holds in the model without borrowing constraint as shown in Section B.2. Now I show that this equation does not hold in the full model with the borrowing constraint.

The expression for \( \gamma_t^* \) is given in (56). Before examining this equation, it is useful to focus on the Lagrange multipliers included in (56), \( \eta_t^{EE}, \mu_t^*, \mu_t, \) and \( \eta_t^\mu \). When the borrowing constraint is not binding, \( \mu_t^* = \mu_t = 0 \). As shown below, \( \eta_t^{EE} \neq 0 \) because private agents do not internalize the externalities and over-borrow. \( \eta_t^\mu \) has the opposite sign to \( \eta_t^{EE} \) because the inside of the parenthesis in (61) is negative. \( \eta_t^\mu \neq 0 \) implies the complementary slackness condition is binding with \( \mu_t = 0 \). When the borrowing constraint is binding, \( \mu_t^* > 0 \), and \( \eta_t^{EE} = 0 \) because of (61) and the binding constraint. \( \mu_t \) is determined by the private Euler equation. The complementary slackness condition is satisfied with any \( \mu_t \) because the constraint is binding. This implies that the complementary slackness condition is not binding, and \( \eta_t^\mu = 0 \).

Therefore, \( \gamma_t^* \) in (56) can be written separately in the case of not binding and binding as
follows. In the case of not binding,

\[
\gamma^*_t = \frac{1}{\rho - 1} \lambda^*_t + \frac{1}{\rho - 1} \frac{1}{e_t^{\rho - 1} Y^*} \left[ \eta^m_t \frac{\rho}{\rho - 1} \frac{m_t}{\alpha_m Y_t} + \eta^{EE}_{t} \frac{u_c(t)}{e_t} \right].
\]  

(68)

In the case of binding,

\[
\gamma^*_t = \frac{1}{\rho - 1} \lambda^*_t + \frac{1}{\rho - 1} \frac{1}{e_t^{\rho - 1} Y^*} \left[ \eta^m_t \frac{\rho}{\rho - 1} \frac{m_t}{\alpha_m Y_t} - \mu^*_t b^*_t \right].
\]  

(69)

The expression for \( \eta^m_t \) is given in (64). When \( \pi_t = \eta^P_{tC} = 0 \), this expression reduces to:

\[
\frac{\eta^m_t}{\lambda^*_t} = \varepsilon^m_t \left[ \frac{Y_t}{m_t} - e_t - \frac{\gamma^*_t}{\lambda^*_t} e_t \right].
\]  

(70)

This expression is the same as the second term in (67), and would be zero if \( \gamma^*_t / \lambda^*_t = 1 / (\rho - 1) \) holds. This implies that if the last terms in (68) and (69) are zero, then \( \gamma^*_t / \lambda^*_t = 1 / (\rho - 1) \) holds and \( \eta^m_t = \eta^P_{tC} = 0 \).

To see this point more clearly, going back to the full expression for \( \gamma^*_t \) in (56) and dividing it by \( \lambda^*_t \),

\[
\frac{\gamma^*_t}{\lambda^*_t} = \frac{1}{\rho - 1} \frac{1}{e_t^{\rho - 1} Y^*} \left[ \eta^m_t \frac{\rho}{\rho - 1} \frac{m_t}{\alpha_m Y_t} + \eta^{EE}_{t} \frac{1}{\lambda^*_t} \frac{u_c(t)}{e_t} - \mu^*_t + \eta^m_t \mu_t b^*_t \right].
\]

The implementability constraint with respect to \( m_t \) (51) with \( \rho^m_t = 1 \) implies:

\[
\alpha_m \frac{Y_t}{m_t} = e_t \frac{\rho}{\rho - 1}.
\]

Plugging this expression into \( \gamma^*_t / \lambda^*_t \),

\[
\frac{\gamma^*_t}{\lambda^*_t} = \frac{1}{\rho - 1} \frac{1}{e_t^{\rho - 1} Y^*} \left[ \eta^m_t \frac{1}{\lambda^*_t} \frac{1}{e_t} - \eta^{EE}_{t} \frac{1}{\lambda^*_t} \frac{u_c(t)}{e_t} - \mu^*_t + \eta^m_t \mu_t b^*_t \right].
\]
Plugging \( \eta_t^{\alpha} / \lambda_t^* \) from (70) into this equation,

\[
\frac{\gamma_t^* - \frac{1}{\rho - 1}}{\lambda_t^*} = \frac{1}{\rho - 1 \varepsilon_t^{\rho - 1} Y^*} \left[ \varepsilon_t^{\rho m} \left\{ \alpha - \frac{1}{\lambda_t^*} \frac{\eta_t^{EE}}{\varepsilon_t} \frac{1}{\lambda_t^*} \left( u_c(t) - \mu_t^* \right) - \frac{\mu_t^* + \eta_t^\mu \mu_t b_t^*}{\lambda_t^* R_t^*} \right] \right]
\]

Rearranging and solving for \( \frac{\gamma_t^*}{\lambda_t^*} - 1/(\rho - 1) \),

\[
\frac{\gamma_t^*}{\lambda_t^*} = \frac{1}{\rho - 1 \varepsilon_t^{\rho - 1} Y^*} \left[ \frac{\eta_t^{EE}}{\varepsilon_t} \frac{1}{\lambda_t^*} \left( u_c(t) - \mu_t^* \right) - \frac{\mu_t^* + \eta_t^\mu \mu_t b_t^*}{\lambda_t^* R_t^*} \right].
\]

Since the denominator and the coefficient in the numerator are strictly positive given \( \rho > 1 \), whether \( \frac{\gamma_t^*}{\lambda_t^*} = 1/(\rho - 1) \) holds or not depends on whether the bracket term in the numerator is zero or not. Now it is helpful again to separate the case of not binding and binding. In the case of not binding,

\[
\frac{\gamma_t^*}{\lambda_t^*} - \frac{1}{\rho - 1} = \frac{1}{(\rho - 1) \varepsilon_t^{\rho - 1} Y^* + \varepsilon_t^{pm}} \left( \eta_t^{EE} u_c(t) / \varepsilon_t \right).
\]

In the case of binding,

\[
\frac{\gamma_t^*}{\lambda_t^*} - \frac{1}{\rho - 1} = \frac{1}{(\rho - 1) \varepsilon_t^{\rho - 1} Y^* + \varepsilon_t^{pm}} \left( \frac{\mu_t^* b_t^*}{\lambda_t^* R_t^*} \right).
\]

In the case of binding, the bracket term is clearly strictly positive, because \( \mu_t^* > 0 \) and \(- b_t^* / R_t^* > 0 \). This implies \( \frac{\gamma_t^*}{\lambda_t^*} \neq 1/(\rho - 1) \) and thus \( \eta_t^{PC} \neq 0 \), which contradicts the initial conjecture that \( \pi_t = 0 \). This completes the proof of Proposition 1.

B.5 Optimal Policy in Normal Times and during Crises

Equations (71) and (72) suggest in which direction inflation should deviate from 0, expansionary (\( \pi_t > 0 \)) or contractionary (\( \pi_t < 0 \)). I start with the case of binding. In the case of binding, the right-hand side of (72) is strictly positive, which implies \( \frac{\gamma_t^*}{\lambda_t^*} > 1/(\rho - 1) \). Note that the expression inside the parenthesis comes from the first order condition with respect to \( e_t \) (56), and this term captures the additional social value of real appreciation when
the constraint is binding on top of the effect through the terms of trade. Comparing (56) and (42) in the flexible price model makes clear that this additional term is the ex post RER externality that causes too much use of imported inputs during crises. Therefore, the optimal monetary policy deviates from $\pi_t = 0$ to deal with this externality. $\gamma_t^*/\lambda_t^* > 1/(\rho - 1)$ implies $\lambda_t^m$ is negative in (67), which pushes down the value of $\eta_t^{PC}$ in (65). A lower value of $\eta_t^{PC}$ directly translates into lower $\pi_t$ as shown in the first order condition with respect to $\pi_t$ (53). Therefore, the ex post RER externality that causes too much use of imported inputs when the constraint is binding pushes down the optimal inflation rate.

Next I consider the case where the constraint is not binding today but may bind tomorrow. As is clear from (71), whether $\gamma_t^*/\lambda_t^*$ is greater, lower, or equal to $1/(\rho - 1)$ depends on the Lagrange multiplier on the Euler equation $\eta_t^{EE}$. Given $\mu_t^* = \mu_t = \xi_t^* = 0$ when the constraint is not binding, the Euler equation in the planner’s problem is given as follows:

$$\lambda_t^* + \gamma_t^* - \frac{R_t^*}{e_t} \eta_t^{EE} \frac{\partial RHS_t^{EE}}{\partial b_t} = \beta R_t^* E_t \left[ \frac{e_{t+1}}{e_t} \left( \lambda_{t+1}^* + \gamma_{t+1}^* \right) \right].$$

Given that the constraint is not binding today but may bind tomorrow, the Lagrange multipliers in this equation are given as follows:

$$\lambda_t^* = u_c(t) - \eta_t^{EE} u_{cc}(t),$$  \hspace{1cm} (73)

$$\lambda_{t+1}^* = u_c(t+1) - \eta_{t+1}^{EE} u_{cc}(t+1) - \xi_{t+1}^* u_{cc}(t+1) q_{t+1},$$  \hspace{1cm} (74)

$$\gamma_t^* = \frac{1}{\rho - 1} \lambda_t^* + \frac{1}{(\rho - 1) e_t^{\rho-1} Y^*} \left[ \eta_t^m - \frac{\rho}{\rho - 1} m_t + \eta_t^{EE} u_c(t) \right],$$  \hspace{1cm} (75)

$$\gamma_{t+1}^* = \frac{1}{\rho - 1} \lambda_{t+1}^* + \frac{1}{(\rho - 1) e_t^{\rho-1} Y^*} \left[ \eta_{t+1}^m - \frac{\rho}{\rho - 1} m_{t+1} + \eta_{t+1}^{EE} u_{c}(t+1) e_{t+1} - \mu_{t+1}^* \frac{b_{t+1}^*}{R_{t+1}^*} \right].$$  \hspace{1cm} (76)

Plugging these equations and rearranging,

$$\eta_t^{EE} \left[ \frac{\rho}{\rho - 1} u_{cc}(t) - \frac{R_t^*}{e_t} \frac{\partial RHS_t^{EE}}{\partial b_t} + \frac{1}{(\rho - 1) e_t^{\rho-1} Y^*} \frac{u_c(t)}{e_t} \right] = -\frac{1}{(\rho - 1) e_t^{\rho-1} Y^*} \left[ \eta_t^m - \frac{\rho}{\rho - 1} m_t \right]$$

$$+ \beta R_t^* E_t \left[ \frac{e_{t+1}}{e_t} \left( -\eta_{t+1}^{EE} u_{cc}(t+1) - \xi_{t+1}^* u_{cc}(t+1) q_{t+1} \right) + \frac{1}{(\rho - 1) e_t^{\rho-1} Y^*} \left[ \eta_{t+1}^m - \frac{\rho}{\rho - 1} m_{t+1} - \eta_{t+1}^{EE} u_c(t+1) e_{t+1} - \mu_{t+1}^* \frac{b_{t+1}^*}{R_{t+1}^*} \right] \right].$$

Regarding the coefficient on $\eta_t^{EE}$ in the left-hand side, the first term is positive because
$u_{cc}(t) < 0$. The sign of the second term cannot be determined analytically, but it is likely to be positive because reducing debt at period $t$ would increase available resources at $t + 1$, which would reduce the marginal utility of consumption at $t + 1$. The third term is positive. Therefore, the coefficient on $\eta_t^{EE}$ is likely to be positive. In the right-hand side, the pecuniary externality (the term that starts with $\xi_{t+1}^*$) and the ex ante RER externality (the last term) add to the value of the right-hand side. The relative size of the terms with $\eta_t^m$ and $\eta_{t+1}^m$ is ambiguous. Therefore, although it is not possible to determine the sign of $\eta_t^{EE}$, the pecuniary and ex ante RER externalities add to the value and $\eta_t^{EE}$ is likely to be positive.

Positive $\eta_t^{EE}$ implies $\gamma_t^*/\lambda_t^* > 1/(\rho - 1)$ in (71), which lowers the value of $\eta_t^{FC}$ and the optimal $\pi_t$. Therefore, the pecuniary externality and the RER externality when the constraint is not binding push down the optimal inflation rate. The intuition is as follows: when the borrowing constraint is not binding today but may bind next period, private agents socially over-borrow due to the pecuniary and RER externalities, and the planner has an incentive to correct it. When the policy tool is only monetary policy, the planner lowers $\pi_t$ and $p_t^{ew}$, which discourages production and use of imported inputs. Smaller amount of imported inputs causes real appreciation, which is a decline in $e_t$. Lower $e_t$ increases the effective real interest rate on foreign bond, and discourages foreign borrowing by private agents. This at least partially corrects over-borrowing by private agents.

### C Inflation Targeting and Optimal Tax

This section provides the planner’s problem and the first order conditions under the optimal macroprudential tax and inflation targeting monetary policy. The planner's problem is given as follows:
\[ V(b_{t-1}^*, s_t) = \max_{c_t, b_t^*, \ell_t, m_t, e_t, q_t} \log \left( c_t - \frac{\ell_t^{1+\omega}}{1+\omega} \right) + \beta E_t V(b_t^*, s_{t+1}) \]
\[-\lambda_t^* \left[ c_t + e_t \left( \frac{b_t^*}{R_t^*} - b_{t-1}^* \right) - y_t + e_t m_t \right] \]
\[-\mu_t^* \left[ -e_t \frac{b_t^*}{R_t^*} - \kappa_t q_t k \right] \]
\[-\xi_t \left[ q_t u_c(t) - \beta E_t \left\{ u_c(t+1) \left( q_{t+1} + \alpha_k \frac{y_{t+1}}{k} \right) + \mu_{t+1} \kappa_{t+1} q_{t+1} \right\} \right] \]
\[+ \gamma_t \left[ e_t^\ell Y* - e_t m_t - e_t \left( \frac{b_t^*}{R_t^*} - b_{t-1}^* \right) \right] \]
\[-\eta_t^\ell \left[ \alpha_t \frac{e_t}{\ell_t} - \chi \ell_t^{\omega} \right] \]
\[-\eta_t^m \left[ \alpha_m \frac{y_t}{m_t} - \rho \frac{e_t}{\rho - 1} \right]. \]

This problem is similar to the case in the flexible price version of the model in Section A. The difference is that the first order conditions with respect to \( \ell_t \) and \( m_t \) in the decentralized equilibrium are included in the implementability constraints. This is because the planner here cannot use taxes on imported inputs. The first order conditions are given as follows:

\[ c_t : u_c(t) - \lambda_t^* - \xi_t q_t u_{cc}(t) = 0, \quad (77) \]
\[ b_t^* : -\lambda_t^* e_t \frac{1}{R_t^*} + \mu_t^* e_t \frac{1}{R_t^*} - \xi_t \frac{\partial RHS_{t+1}^q}{\partial b_t^*} - \gamma_t e_t \frac{1}{R_t^*} + \beta E_t V_b(b_t^*, s_{t+1}) = 0, \quad (78) \]
\[ \ell_t : u_c(t)(-\chi \ell_t^{\omega}) + \lambda_t^* \alpha_t \frac{y_t}{\ell_t} - \xi_t q_t u_{cc}(t)(-\chi \ell_t^{\omega}) \]
\[-\eta_t^\ell \left[ \alpha_t \frac{e_t}{\ell_t} (\alpha_t - 1) - \chi \omega \ell_t^{\omega-1} \right] - \eta_t^m \left[ \alpha_t \alpha_m \frac{y_t}{\ell_t m_t} \right] = 0 \quad (79) \]
\[ m_t : \lambda_t^* \left( \alpha_m \frac{y_t}{m_t} - e_t \right) - \gamma_t e_t - \eta_t^m \left[ \alpha_t \alpha_m \frac{y_t}{\ell_t m_t} \right] - \eta_t^m \left[ \frac{\alpha_m y_t}{m_t^2} (\alpha_m - 1) \right] = 0, \quad (80) \]
\[ e_t : -\lambda_t^* \left( \frac{b_t^*}{R_t^*} - b_{t-1}^* + m_t \right) + \mu_t^* \frac{b_t^*}{R_t^*} + \gamma_t (\rho - 1) e_t^{\rho-1} + \eta_t^m \frac{\rho}{\rho - 1} = 0, \quad (81) \]
\[ q_t : \mu_t^* \kappa_t k = \xi_t u_c(t). \quad (82) \]
Plugging the implementability constraint with respect to $\ell_t$ into (79),

$$-\eta_t^\ell \left[ \frac{\alpha_t y_t}{\ell_t^2} (\alpha_t - 1) - \chi \omega_t^{\ell_t - 1} \right] = \eta_t^m \left[ \alpha_t \alpha_m \frac{y_t}{\ell_t m_t} \right].$$

(83)

The inside of the bracket in the left-hand side is negative, which implies $\eta_t^\ell$ and $\eta_t^m$ have the same sign, and $\eta_t^\ell = 0$ if and only if $\eta_t^m = 0$. The intuition is that the social value of distorting $\ell_t$ emerges as a side effect of distorting $m_t$. If the planner has an incentive to distort $m_t$, then the planner also adjusts $\ell_t$ accordingly, because $\ell_t$ and $m_t$ are connected through the marginal product of each other.

Rearranging (83) further using the implementability constraint with respect to $\ell_t$,

$$\eta_t^\ell = \frac{\alpha_m}{1 + \omega - \alpha_t} \frac{\ell_t}{m_t} \eta_t^m.$$

Plugging this equation into (80),

$$\lambda_t^* \left[ \alpha_m \frac{y_t}{m_t} \right] = \lambda_t^* e_t + \gamma_t e_t + \eta_t^m \frac{\alpha_m y_t}{m_t^2} \left[ \frac{\alpha_t \alpha_m}{1 + \omega - \alpha_t} + (\alpha_m - 1) \right].$$

It can be shown that the last bracket term is strictly negative. I denote this value as $\overline{\alpha} < 0$.

Solving the first order condition with respect to $e_t$ (81) for $\gamma_t$ and plugging into this equation,

$$\lambda_t^* \left[ \alpha_m \frac{y_t}{m_t} \right] = \frac{1}{\rho - 1} \lambda_t^* e_t + \frac{-b_t^* / R_t^*}{(\rho - 1) e_t^{\rho-1} Y^* \mu_t^*} - \frac{1}{(\rho - 1) e_t^{\rho-1} Y^* \rho - 1} \eta_t^m \frac{\alpha_m y_t}{m_t^2} e_t^\ell + \eta_t^m \frac{\alpha_m y_t}{m_t^2} \overline{\alpha}.$$

Using the implementability constraint with respect to $m_t$, the first three terms cancel out. Therefore,

$$\frac{-e_t b_t^* / R_t^*}{(\rho - 1) e_t^{\rho-1} Y^* \mu_t^*} - \frac{e_t}{(\rho - 1) e_t^{\rho-1} Y^* \rho - 1} \eta_t^m + \eta_t^m \frac{\alpha_m y_t}{m_t^2} \overline{\alpha} = 0.$$

The first two terms come from $\gamma_t$, thus they are the social value of increasing $m_t$ through real exchange rate. The first term is the effect of a change in $e_t$ on the binding borrowing constraint. The second term is the effect of a change in $e_t$ on the cost of buying $m_t$. The last term comes from the effect of increasing $m_t$ on the production margins of $\ell_t$ and $m_t$. 

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Solving this equation for $\eta_t^m$,

$$\eta_t^m = \frac{-\epsilon_{t+1}^2/R_t^*}{(\rho-1)\epsilon_{t+1}^{Y*}} \frac{\rho}{\rho-1} \frac{\alpha_m y_t}{m_t^*} \mu_t^*.$$

The coefficient on $\mu_t^*$ is strictly positive. Therefore, this equation implies that $\eta_t^m > 0$ if and only if $\mu_t^* > 0$, i.e. the borrowing constraint is binding. Intuitively, the planner wants to distort $m_t$ when the constraint is binding. A positive $\eta_t^m$ implies that the planner wants to increase the cost of $m_t$ and discourages $m_t$.

Finally, I derive the Euler equation when the constraint is not binding but may bind next period. The constraint being not binding implies $\mu_t^* = \xi_t = \eta_t^m = 0$. Plugging the expression for $\gamma_t$ from (81) into (78) and combined with the envelope condition,

$$u_c(t) = \beta R_t^* E_t \left[ \frac{e_{t+1}}{e_t} \left( u_c(t+1) - \xi_{t+1} q_{t+1} u_{cc}(t+1) - \frac{b_{t+1}}{\rho^e_{t+1} Y^*} \mu_{t+1}^* - \frac{1}{\rho^e_{t+1} Y^*} \frac{\rho}{\rho-1} \eta_{t+1}^m \right) \right],$$

which is (35) in the main text. See the main text for interpretations.

When the borrowing constraint is binding, the six constraints in the planner’s problem pin down the six endogenous variables, $c_t, \ell_t, m_t, b_t^*, e_t, q_t$. Thus the Euler equation is just to pin down the Lagrange multiplier $\mu_t^*$. This also implies that imposing a tax on foreign borrowing when the constraint is binding does not affect allocations. It would affect the private Lagrange multiplier on the borrowing constraint $\mu_t$, which would in turn affect the asset price $q_{t-1}$ through the forward-looking asset price equation. Since I focus on time-consistent policies without commitment, I assume that the planner does not internalize this effect and set the tax rate to zero when the constraint is binding.

### D Discretionary Monetary Policy and Optimal Tax

This section provides the planner’s problem and the first order conditions under the optimal macroprudential tax and optimal discretionary monetary policy.
D.1 Planner’s Problem

The planner’s problem is given as follows:

\[
V(b_{t-1}, s_t) = \max_{\pi_t, c_t, b_t, R_t, m_t, e_t, q_t, Y_t} \log \left( e_t - \frac{\ell_t^{1+\omega}}{1+\omega} \right) + \beta E_t V(b^*_t, s_{t+1})
\]

\[
-\lambda_t \left[ e_t + e_t \left( \frac{b^*_t}{R_t} - b^*_{t-1} \right) - \left\{ Y_t - \frac{\psi}{2} \pi_t^2 Y_t \right\} + e_t m_t \right]
\]

\[
-\mu_t \left[ -e_t \left( \frac{b^*_t}{R_t} - \kappa_t q_t \right) \right]
\]

\[
-\xi_t \left[ q_t u_c(t) - \beta E_t \left\{ u_c(t+1) \left( q_t+1 + p^m_{t+1} \right) \right\} + \mu_{t+1} \kappa_{t+1} q_{t+1} \right]
\]

\[
+\gamma_t \left[ e_t^p Y^* - e_t m_t - e_t \left( \frac{b^*_t}{R_t} - b^*_{t-1} \right) \right]
\]

\[
-\eta^P_t \left[ \left\{ -\theta + \theta p^m_{t+1} - \psi \pi_t(1+\pi_t) \right\} Y_t + \beta E_t \left\{ u_c(t+1) \right\} \psi \pi_{t+1} (1+\pi_{t+1}) Y_{t+1} \right]
\]

\[
+\eta^m_t \left[ p^{m^m}_{t+1} \left( \frac{1}{\alpha_m} \right) \right]
\]

\[
+\eta^m_t \left[ p^{m^m}_{t+1} \left( \frac{1}{\alpha_m} \right) \right]
\]

This problem is similar to the one in which monetary policy is the only policy tool in Section B. The difference is that the Euler equation with respect to foreign bond in the decentralized equilibrium is not included in the implementability constraint, because a macroprudential tax on foreign bond is available. I also omit the complementary slackness condition from the implementability constraint because private \( \mu_t \) appears only in this equation, thus any \( \mu_t \) is consistent with the first order conditions here. The first order conditions are the same as those in Section B with setting \( \eta_t^{EE} = 0 \).

D.2 Proof of Proposition 2

**Proposition 2** In the model described in Section 3, when a macroprudential tax on foreign borrowing is available, strict inflation targeting is not optimal discretionary monetary policy.

I follow the same strategy as the proof of Proposition 1 in Section B. Namely, I set \( E_t(\pi_{t+1}) = 0 \). Then, I assume \( \pi_t = 0 \) and show that the first order conditions in the planner’s problem are not satisfied. Most of the steps in Section B carry over to this case. In particular, \( \pi_t = 0 \) if and only if \( \eta_t^{PC} = 0 \), which depends on whether \( \gamma^*_t / \lambda^*_t = 1 / (\rho - 1) \) or not. Given \( \eta_t^{EE} = 0 \), the expressions for \( \gamma^*_t / \lambda^*_t \) when not binding and when binding are
given as follows. In the case of not binding,

$$\frac{\gamma^*_t}{\lambda^*_t} - \frac{1}{\rho - 1} = 0. \quad (92)$$

In the case of binding,

$$\frac{\gamma^*_t}{\lambda^*_t} - \frac{1}{\rho - 1} = \frac{1}{(\rho - 1) e^F_t Y_t^* + \varepsilon^p_t} \left( \mu^*_t \frac{b^*_t}{R_t^*} \right). \quad (93)$$

The first equation proves that in normal times when the borrowing constraint is not binding, the optimal discretionary monetary policy focuses only on minimizing the price adjustment cost. The second equation proves that during crises when the constraint is binding, the optimal discretionary monetary policy is contractionary.

**E  Numerical Solution**

**E.1 Inflation Targeting without Taxes**

The model under inflation targeting without taxes is solved using policy function iterations modified to deal with an occasionally binding constraint. I set 101 grid points for the debt space. There are four states for TFP and interest rate shocks, and two states for a collateral shock $\kappa_t$. Thus, there are 808 grid points in total. The numerical solution is obtained by the following steps:

1. I set the initial guess for the decision rules of labor $\ell(b^*_t, s_t)$, asset price $q(b^*_t, s_t)$, real exchange rate $e(b^*_t, \kappa_t)$, and the right-hand side of the Euler equation with respect to foreign debt $RHSEE(b^*_t, s_t)$.

2. For each grid point $(b^*_t, s_t)$, I solve the simultaneous equations of the equilibrium conditions using a non-linear solver and obtain the decision rules for $\ell(b^*_t, s_t), q(b^*_t, s_t), e(b^*_t, \kappa_t), RHSEE(b^*_t, s_t)$. In doing this, I first assume that the borrowing constraint is not binding and solve the equations. Then I check if the constraint is violated or not. If it is violated, I solve the equations with the binding constraint.
3. After deriving the decision rules at every grid point, I compare the initial guess and the obtained decision rules. If they are close enough, I stop. If not, I update the initial guess by the obtained decision rules and go back to step 2.

E.2 Discretionary Monetary Policy without Taxes

I use a combination of policy function iterations and value function iterations. This method consists of an outer loop of policy function iterations and an inner loop of value function iterations.

1. I set the initial guess for the decision rules of labor $\ell(b_{t-1}^*, s_t)$, imported inputs $m(b_{t-1}^*, \kappa_t)$, asset price $q(b_{t-1}^*, s_t)$, inflation $\pi(b_{t-1}^*, \kappa_t)$, and private Lagrange multiplier on the borrowing constraint $\mu(b_{t-1}^*, \kappa_t)$. I use the same initial guess for the next period decision rules $\ell(b_{t-1}^*, s_t)$, $m(b_{t-1}^*, \kappa_t)$, $q(b_{t-1}^*, s_t)$, $\pi(b_{t-1}^*, \kappa_t)$, $\mu(b_{t-1}^*, \kappa_t)$. I also set the initial guess for the value function $\tilde{V}(b_{t-1}^*, \kappa_t)$, which is households’ expected utility at each state.

2. Inner loop: for each grid point $(b_{t-1}^*, s_t)$, given the next period decision rules $\ell(b_{t-1}^*, s_t)$, $m(b_{t-1}^*, \kappa_t)$, $q(b_{t-1}^*, s_t)$, $\pi(b_{t-1}^*, \kappa_t)$, $\mu(b_{t-1}^*, \kappa_t)$ and the guessed value function $\tilde{V}(b_{t-1}^*, \kappa_t)$, I find the inflation rate $\pi(b_{t-1}^*, \kappa_t)$ that maximizes the value function by using non-linear minimizer such as fminsearch. I also obtain the corresponding value $V(b_{t-1}^*, \kappa_t)$ at each grid point.

3. After obtaining $\pi(b_{t-1}^*, \kappa_t)$ and $V(b_{t-1}^*, \kappa_t)$ for every grid point, I check if $V(b_{t-1}^*, \kappa_t)$ and $\tilde{V}(b_{t-1}^*, \kappa_t)$ are close enough. If they are close enough, I proceed to step 4. If not, I update the guess for the value function $\tilde{V}(b_{t-1}^*, \kappa_t)$ by the obtained value functions $V(b_{t-1}^*, \kappa_t)$, and go back to step 2.

4. Outer loop: I compare the next period decision rules $\ell(b_{t-1}^*, s_t)$, $m(b_{t-1}^*, \kappa_t)$, $q(b_{t-1}^*, s_t)$, $\pi(b_{t-1}^*, \kappa_t)$, $\mu(b_{t-1}^*, \kappa_t)$ and the obtained decision rules $\ell(b_{t-1}^*, s_t)$, $m(b_{t-1}^*, \kappa_t)$, $q(b_{t-1}^*, s_t)$, $\pi(b_{t-1}^*, \kappa_t)$, $\mu(b_{t-1}^*, \kappa_t)$. If they are close enough, I stop. If not, I update the next period decision rules with the obtained decision rules and go back to step 2.
E.3 Inflation Targeting with Optimal Tax

I use a combination of policy function iterations and value function iterations.

1. I set the initial guess for the decision rules of consumption $c(b_{t-1}^*, s_t)$, labor $\ell(b_{t-1}^*, s_t)$, asset price $q(b_{t-1}^*, s_t)$, and private Lagrange multiplier on the borrowing constraint $\mu(b_{t-1}^*, \kappa_t)$. I use the same initial guess for the next period decision rules $\widetilde{c}(b_{t-1}^*, s_t)$, $\widetilde{\ell}(b_{t-1}^*, \kappa_t)$, $\widetilde{q}(b_{t-1}^*, s_t)$, $\widetilde{\mu}(b_{t-1}^*, \kappa_t)$. I also set the initial guess for the value function $\widetilde{V}(b_{t-1}^*, \kappa_t)$, which is households’ expected utility at each state.

2. Inner loop: for each grid point $(b_{t-1}^*, s_t)$, I first assume that the borrowing constraint is not binding. Given the next period decision rules $\widetilde{c}(b_{t-1}^*, s_t)$, $\widetilde{\ell}(b_{t-1}^*, \kappa_t)$, $\widetilde{q}(b_{t-1}^*, s_t)$, $\widetilde{\mu}(b_{t-1}^*, \kappa_t)$ and the guessed value function $\widetilde{V}(b_{t-1}^*, \kappa_t)$, I find foreign debt $b^*(b_{t-1}^*, \kappa_t)$ that maximizes the value function by using a non-linear minimizer such as fminsearch. I also obtain the corresponding value $V(b_{t-1}^*, \kappa_t)$ at each grid point. Then I check the constraint. If the constraint is not violated, I proceed to next grid point. If the constraint is violated, I solve the equilibrium conditions with the binding constraint using a non-linear solver to obtain $b^*(b_{t-1}^*, \kappa_t)$ and $V(b_{t-1}^*, \kappa_t)$.

3. After obtaining $b^*(b_{t-1}^*, \kappa_t)$ and $V(b_{t-1}^*, \kappa_t)$ for every grid point, I check if $V(b_{t-1}^*, \kappa_t)$ and $\widetilde{V}(b_{t-1}^*, \kappa_t)$ are close enough. If they are close enough, I proceed to step 4. If not, I update the guess for the value function $\widetilde{V}(b_{t-1}^*, \kappa_t)$ by the obtained value functions $V(b_{t-1}^*, \kappa_t)$, and go back to step 2.

4. Outer loop: I compare the next period decision rules $\widetilde{c}(b_{t-1}^*, s_t)$, $\widetilde{\ell}(b_{t-1}^*, \kappa_t)$, $\widetilde{q}(b_{t-1}^*, s_t)$, $\widetilde{\mu}(b_{t-1}^*, \kappa_t)$ and the obtained decision rules $c(b_{t-1}^*, s_t)$, $\ell(b_{t-1}^*, \kappa_t)$, $q(b_{t-1}^*, s_t)$, $\mu(b_{t-1}^*, \kappa_t)$. If they are close enough, I stop. If not, I update the next period decision rules with the obtained decision rules and go back to step 2.

E.4 Discretionary Monetary Policy with Optimal Tax

I use a combination of policy function iterations and value function iterations.
1. I set the initial guess for the decision rules of labor \( \ell(b_{t-1}^*, s_t) \), imported inputs \( m(b_{t-1}^*, \kappa_t) \), asset price \( q(b_{t-1}^*, s_t) \), inflation \( \pi(b_{t-1}^*, \kappa_t) \), and private Lagrange multiplier on the borrowing constraint \( \mu(b_{t-1}^*, \kappa_t) \). I use the same initial guess for the next period decision rules \( \ell(b_{t-1}^*, s_t) \), \( m(b_{t-1}^*, \kappa_t) \), \( q(b_{t-1}^*, s_t) \), \( \pi(b_{t-1}^*, \kappa_t) \), \( \mu(b_{t-1}^*, \kappa_t) \). I also set the initial guess for the value function \( V(b_{t-1}^*, \kappa_t) \), which is households’ expected utility at each state.

2. Inner loop: for each grid point \((b_{t-1}^*, s_t)\), I first assume that the borrowing constraint is not binding. Given the next period decision rules \( \ell(b_{t-1}^*, s_t) \), \( m(b_{t-1}^*, \kappa_t) \), \( q(b_{t-1}^*, s_t) \), \( \pi(b_{t-1}^*, \kappa_t) \), \( \mu(b_{t-1}^*, \kappa_t) \) and the guessed value function \( \tilde{V}(b_{t-1}^*, \kappa_t) \), I find a combination of foreign debt \( b^*(b_{t-1}^*, \kappa_t) \) and inflation \( \pi(b_{t-1}^*, \kappa_t) \) that maximizes the value function by using a non-linear minimizer such as fminsearch. I also obtain the corresponding value \( V(b_{t-1}^*, \kappa_t) \) at each grid point. Then I check the constraint. If the constraint is not violated, I proceed to next grid point. If the constraint is violated, I find inflation \( \pi(b_{t-1}^*, \kappa_t) \) that maximizes the value function by using a non-linear minimizer, taking into account this time that the borrowing constraint is binding.

3. After obtaining \( b^*(b_{t-1}^*, \kappa_t) \), \( \pi(b_{t-1}^*, \kappa_t) \), and \( V(b_{t-1}^*, \kappa_t) \) for every grid point, I check if \( V(b_{t-1}^*, \kappa_t) \) and \( \tilde{V}(b_{t-1}^*, \kappa_t) \) are close enough. If they are close enough, I proceed to step 4.

4. If not, I update the guess for the value function \( \tilde{V}(b_{t-1}^*, \kappa_t) \) by the obtained value functions \( V(b_{t-1}^*, \kappa_t) \), and go back to step 2.

4. Outer loop: I compare the next period decision rules \( \ell(b_{t-1}^*, s_t) \), \( m(b_{t-1}^*, \kappa_t) \), \( q(b_{t-1}^*, s_t) \), \( \pi(b_{t-1}^*, \kappa_t) \), \( \mu(b_{t-1}^*, \kappa_t) \) and the obtained decision rules \( \ell(b_{t-1}^*, s_t) \), \( m(b_{t-1}^*, \kappa_t) \), \( q(b_{t-1}^*, s_t) \), \( \pi(b_{t-1}^*, \kappa_t) \), \( \mu(b_{t-1}^*, \kappa_t) \). If they are close enough, I stop. If not, I update the next period decision rules with the obtained decision rules and go back to step 2.