Melitz-type Computable General Equilibrium Model

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1. Model Outline

A Melitz-type computable general equilibrium (CGE) model is developed on the basis of the standard CGE model by Hosoe et al. (2010) with the features of Melitz (2003) model: firm heterogeneity, product differentiation, and love of variety, following Dixon et al. (2016) (Figure 1). The domestic output $Z_{i,r}$ in the $i$-th sector of the $r$-th region is made with primary factors (capital, and skilled and unskilled labor) and intermediate input. (The description of this production process is omitted for simplicity of Figure 1.) The $k$-th variety $Q_{k,j,r,s}$ shipped from the $r$-th region to the $s$-th region is produced with variable input $Z_{k,j,r,s}$, fixed input $F_{i,r,s}^{MLZ}$ for each variety $k$ and fixed firm setup cost $H_{i,r}^{MLZ}$, the latter two of which are measured in the domestic output $Z_{i,r}$ unit, as Itakura and Oyamada (2015) assumed.

Varieties $Q_{k,j,r,s}$ are used to produce a variety aggregate shipped to the $s$-th...
region $Q_{i,r,s}$ with a constant elasticity of substitution (CES) function, à la Dixit and Stiglitz (1977). The inbound shipping from various regions (including the domestic one) $Q_{T,s,r}$ is aggregated into Armington’s (1969) composite good $Q_{i,r}$. This is used for household consumption $X^p_{i,r}$, government consumption $X^g_{i,r}$, investment $X^v_{i,r}$, and intermediate input $X_{i,j,r}$. The representative household in each region is assumed to have a Cobb-Douglas type utility function.

Figure 1: Model Structure

2. Details of the Model

2.1 Melitz Structure

We assume that a variety-producing firm, facing a monopolistically-competitive market with an inverse demand function $p^q_{k,i,r,s}(\cdot)$, maximizes its profit as follows:

$$\max \quad \pi^{MLZ}_{k,i,r,s} = p^q_{k,i,r,s} Q_{k,i,r,s} - \left(1 + r^*_{j,r} \right) p^r_{j,r} \left( ZZ_{k,i,r,s} + F_{k,i,r,s}^{MLZ} \right)$$
subject to

\[ QT_{k,j,r,s} = \Phi_{k,j,r,s}^{MLZ} ZZ_{k,j,r,s} \]

\[ p_{k,j,r,s}^{qt} = p_{k,j,r,s}^{qt} \left( QT_{k,j,r,s} \right) \]

\[ \frac{dQT_{k,j,r,s}}{dT_{k,j,r,s}} \bigg| \frac{dp_{k,j,r,s}^{qt}}{p_{k,j,r,s}^{qt}} = -\sigma_j^{MLZ} \]

where

\[ k, \; k': \text{ variety/firm type} \]

\[ k = \bullet: \text{ firm with average productivity} \]

\[ k = \text{min}: \text{ the least productive firm active in the market} \]

\[ i,j: \; \text{ goods/sector} \]

\[ r,s,s',rr: \; \text{ region} \]

\[ \pi_{k,j,r,s}^{ZZ}: \text{ profit of the } k\text{-th variety producing firm located in the } r\text{-th region shipping} \]

\[ \text{to the } s\text{-th region} \]

\[ Z_{k,j,r,s}: \text{ input for the } k\text{-th variety in the } j\text{-th sector in the } r\text{-th region shipped to the} \]

\[ s\text{-th region} \]

\[ Q_{k,j,r,s}: \text{ output of the } k\text{-th variety in the } j\text{-th sector in the } r\text{-th region shipped to} \]

\[ s\text{-th region} \]

\[ p_{j,r}: \; \text{ price of the domestic output} \; Z_{i,r} \]

\[ p_{k,j,r,s}^{qt}: \; \text{ price of } Q_{k,j,r,s} \]

\[ \tau_{j,r}: \; \text{ production tax rate} \]

\[ F_{j,r,s}^{MLZ}: \; \text{ fixed cost for variety production (or export/domestic supply cost)} \]

\[ p_{k,j,r,s}^{qt}(\cdot): \; \text{ inverse demand function with price elasticity} \; \sigma_j^{MLZ} \]
\( \Phi_{MLZ}^{k,j,r,s} \): productivity parameter, following a Pareto distribution

PDF: \( g(k) = \frac{\alpha_j^{MLZ}}{\Phi(k)^{1+\alpha_j^{MLZ}}} \), \( \Phi_{MLZ}^{k,j,r,s} \geq 1 \)

\[
\Phi_{MLZ}^{k,j,r,s} = \beta_j^{MLZ} \Phi_{MLZ}^{min,j,r,s} = \left( \frac{\alpha_j^{MLZ}}{\alpha_j^{MLZ} - \sigma_j^{MLZ} + 1} \right)^{1\over \sigma_j^{MLZ} - 1} \Phi_{MLZ}^{min,j,r,s}
\]

\( \alpha_j^{MLZ} \): shape parameter of a Pareto distribution (or Pareto’s \( k \))

\( \beta_j^{MLZ} \): productivity parameter \( = \left[ \alpha_j^{MLZ} \left( \alpha_j^{MLZ} - \sigma_j^{MLZ} + 1 \right) \right]^{1\over \sigma_j^{MLZ} - 1} \)

In the following part, we focus on an average firm with \( \Phi_{MLZ}^{k,j,r,s} \) and the least productive firm in the market with \( \Phi_{MLZ}^{min,j,r,s} \). The average firm’s profit maximization problem is:

max \( \pi_{MLZ}^{k,j,r,s} = p_{q,t,r,s}^{MLZ} Q T_{*,j,r,s} - (1 + \tau_{*,j,r,s}) p_{q,t,r,s}^{MLZ} Z Z_{*,j,r,s} + R_{MLZ}^{j,r,s} \)

subject to

\( Q T_{*,j,r,s} = \Phi_{MLZ}^{k,j,r,s} Z Z_{*,j,r,s} \)

\( p_{q,t,r,s}^{MLZ} = p_{q,t,r,s}^{MLZ} (Q T_{*,j,r,s}) \)

\( dQ T_{*,j,r,s} / dp_{q,t,r,s}^{MLZ} = -\sigma_j^{MLZ} \)

FOCs lead to:

\[
p_{q,t,r,s}^{MLZ} = \frac{(1 + \tau_{*,j,r,s}) p_{r,t}^{MLZ}}{\Phi_{MLZ}^{k,j,r,s} \left( \frac{\sigma_j^{MLZ}}{\sigma_j^{MLZ} - 1} \right)} \left( \frac{\sigma_j^{MLZ}}{\sigma_j^{MLZ} - 1} \right)
\]

\( Q T_{*,j,r,s} = \Phi_{MLZ}^{k,j,r,s} Z Z_{*,j,r,s} \)

The least productive and thus marginal firm in the market earns no profit:
\[ \pi^\text{MLZ}_{\min, j, r, s} = 0 : \frac{Q^\text{MLZ}_{\min, j, r, s}}{(\sigma_j^\text{MLZ} - 1) \theta_{\text{MLZ}}^\text{MLZ}_{\min, j, r, s}} - F^\text{MLZ}_{j, r, s} = 0 \]

2.2 Krugman (Dixit-Stiglitz) Structure

A firm that aggregates varieties \( Q^\text{MLZ}_{j, r, s} \) to make a composite good \( Q^\text{MLZ}_{j, r, s} \) faces the following profit maximization problem.

\[
\max \quad \pi^\text{QT}_{j, r, s} = p^\text{qt}_{j, r, s} Q^\text{QT}_{j, r, s} - \sum_k p^\text{qt}_{k, j, r, s} Q^\text{k}_{j, r, s}
\]

subject to

\[
Q^\text{QT}_{j, r, s} = \left( \sum_k Q^\text{QT}_{k, j, r, s} - \frac{\sigma_j^\text{MLZ} - 1}{\theta_j^\text{MLZ}} \right) \frac{\theta_j^\text{MLZ}}{\sigma_j^\text{MLZ}}
\]

where

\[ \pi^\text{QT}_{j, r, s} : \text{profit of variety aggregating firm} \]

\[ p^\text{qt}_{j, r, s} : \text{price of variety composite} \]

\[ Q^\text{QT}_{j, r, s} : \text{variety composite} \]

\[ \sigma_j^\text{MLZ} : \text{elasticity of substitution among varieties} \]

Note: The weight assigned to each variety is unity (=1).

We can rewrite the above problem with average productivity \( \Phi^\text{MLZ}_{\ast, j, r, s} \) and \( Q^\text{HT}_{\ast, j, r, s} \) as follows:

\[
\max \quad \pi^\text{QT}_{\ast, j, r, s} = p^\text{qt}_{\ast, j, r, s} Q^\text{QT}_{\ast, j, r, s} - \sum_k p^\text{qt}_{k, j, r, s} Q^\text{k}_{\ast, j, r, s}
\]

subject to

\[
Q^\text{QT}_{\ast, j, r, s} = \Phi^\text{MLZ}_{\ast, j, r, s} \frac{\theta_j^\text{MLZ}}{\sigma_j^\text{MLZ}} Q^\text{HT}_{\ast, j, r, s}
\]
where

\[ NF_{j,r,s} : \text{the number of firms active in the } j\text{-th sector located in the } r\text{-th region shipping to the } s\text{-th region} \]

The FOCs lead to:

\[
\begin{align*}
p^{\mu}_{j,r,s} &= p^{\mu}_{j,r,s} \cdot NF_{j,r,s} \cdot \frac{1}{\sigma_j} \\
QT_{j,r,s} &= NF_{j,r,s} \cdot \frac{\sigma_j^{\mu}}{\sigma_j^{\mu} - 1} QT_{j,r,s} 
\end{align*}
\]

### 2.3 Armington Structure

As usual, we assume Armington’s composite goods \( Q_{j,s} \), made of shipping from various regions. The profit maximization problem for a firm producing Armington’s composite good is:

\[
\begin{align*}
\max \quad & \pi^Q_{j,r} = p^q_{j,r} Q_{j,r} - \sum_s \left[ (1 + \tau^e_{j,s,r}) p^{\mu}_{j,s,r} \epsilon_{s,r} + \tau^t_{j,s,r} p^{\text{ROW}}_{j,s,r} \right] (1 + \tau^m_{j,s,r}) QT_{j,s,r} \\
\text{subject to} \quad & Q_{j,r} = \gamma_{j,r} \left( \sum_s \delta_{j,s,r} QT_{j,s,r} \right) \frac{\sigma_j^{\mu}}{\sigma_j^{\mu} - 1} 
\end{align*}
\]

where

\[
\begin{align*}
\pi^Q_{j,r} & : \text{Armington’s composite good producer’s profit} \\
Q_{j,r} & : \text{Armington’s composite good} \\
p^q_{j,r} & : \text{Armington’s composite good price} \\
\tau^e_{j,s,r} & : \text{export tax rates imposed on shipping from the } s\text{-th region to the } r\text{-th region} \\
\tau^t_{j,s,r} & : \text{input requirement of international transportation service} \\
\end{align*}
\]
$p^{qqt}$: international shipping service price in the ROW’s currency

$\tau_{j,s,r}^{n}$: import tariff rate imposed on shipping from the $s$-th region to the $r$-th region

$\varepsilon_{s,r}$: exchange rate converting the $s$-th region’s currency into the $r$-th region’s

$\sigma_{j}^{ARM}$: elasticity of substitution among imports and domestic supply

Note that the international shipping service price $p^{qqt}$ is measured in the ROW’s currency terms.

FOCs give:

$$Q_{T,j,s,r} = \left[ \gamma_{j,s} \sigma_{j}^{ARM} \delta_{j,s,r} p_{j,s}^{q} \left( 1 + \tau_{j,s,r}^{e} \right) \right] Q_{j,s}$$

$$Q_{j,s} = \gamma_{j,s} \left( \sum_{r} \delta_{j,s,r} Q_{T,j,s,r} \sigma_{j}^{ARM} \delta_{j,s,r} \right) \frac{\sigma_{j}^{ARM}}{\sigma_{j}^{ARM} - 1}$$

### 2.4 Monopoly Profit and Number of Firms

The average firm’s profit is:

$$\pi^{MLZ}_{j,s} = p^{q}_{j,s} - (1 + \tau_{j,s}^{e}) p_{j,s}^{q} \left( \frac{Q_{T,j,s}^{MLZ}}{\Phi_{j,s}^{MLZ}} + F_{j,s}^{MLZ} \right)$$

The zero-profit condition for the least productive firm determines its output level $Q_{T_{minj,s}}$.

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2 If $\sigma_{j}^{ARM} = \sigma_{j}^{MLZ}$, the two-stage nested CES structure is degenerated into a single-nest CES structure, following the original specification by Melitz (2003).
Industry-wide zero-profit condition determines the total number of firms in this industry

\[ NH_{j,r} \times \]

\[ \sum_s NF_{j,r,s} \pi^{MLZ}_{j,r,s} - NH_{j,r} H^{MLZ}_{j,r} \left( 1 + \tau_{j,r}^* \right) p_{j,r}^* = 0 \]

where

\[ NH_{j,r} : \text{Number of firms in the}\ j\text{-th sector in the}\ r\text{-th region} \]

\[ H^{MLZ}_{j,r} : \text{Fixed firm setup cost of the}\ j\text{-th sector in the}\ r\text{-th region} \]

Market-clearing condition is described with the domestic output and variable and fixed input.

\[ Z_{j,r} = \sum_s \left( \sum_k ZZ_{k,j,r,s} + NF_{j,r,s} F^{MLZ}_{j,r,s} \right) + NH_{j,r} H^{MLZ}_{j,r} + QT_{j,r} \]

\[ = \sum_s NF_{j,r,s} \left( ZZ_{j,r,s} + F^{MLZ}_{j,r,s} \right) + NH_{j,r} H^{MLZ}_{j,r} + QT_{j,r} \]

where \( QT_{j,s} \) denotes international shipping service (\( \bar{S}\)–TRS) exports by the \( s\)-th region.

### 2.5 Other Conventional Structure

- **Household consumption**

\[ X_{i,p}^p = \frac{\alpha_{i,p}}{p_{j,r}^p} \left( \sum_h p_{h,r}^f FF_{h,r} - T^d_r - S_r^p \right) \]

- **Government consumption**

\[ X_{i}^g = \chi_{i}^{g0} \]

\[ ^3 \text{See Hosoe et al. (2010) for details.} \]
- Direct tax (lump-sum)

\[ T_{j,r}^t = \sum_i p_{i,j,r} X_{i,j,r}^e + S_r^p - \left( \sum_j T_{j,r,s}^z + \sum_{j,s} T_{j,j,r,s}^m + \sum_{h,j} T_{h,j,r}^f \right) \]

- Production tax

\[ T_{j,r}^z = \tau_{j,r}^z p_{j,r}^z Z_{j,r} \]

- Factor use tax

\[ T_{h,j,r}^f = \tau_{h,j,r}^f p_{h,j,r}^f F_{h,j,r} \]

- Import tariff

\[ T_{i,r,s}^m = \tau_{i,r,s}^m \left( (1 + \tau_{i,r,s}^e) p_{i,r,s}^q e_{r,s}^e + \tau_{i,r,s}^e p_{i,r,s}^q e_{ROW,s}^e \right) Q T_{i,r,s} \]

- Export tax

\[ T_{i,r,s}^e = \tau_{i,r,s}^e p_{i,r,s}^q Q T_{i,r,s} \]

- Investment

\[ X_{i,r}^v = X_{i,r}^{v0} \]

- Household saving (lump-sum)

\[ S_r^p = \sum_i p_{i,j,r} X_{i,j,r}^v - (S_r^e + e_{ROW,s}^e S_r^f) \]

- Government saving

\[ S_r^e = s_x^e \left( \sum_j T_{j,r,s}^z + \sum_{j,s} T_{j,j,r,s}^m + \sum_{h,j} T_{h,j,r}^f + T_r^d \right) \]

- BOP constraint

\[ \sum_{i,s} (1 + \tau_{i,s,r}^e) p_{i,s,r}^q e_{r,ROW} Q T_{i,r,s} + \sum_i (1 + \tau_{i,r}^e) p_{i,r,s}^q e_{r,ROW} Q T S_{i,r} + S_r^f = \sum_{i,s} (1 + \tau_{i,s,r}^e) p_{i,s,r}^q e_{r,ROW} + \tau_{i,s,r}^e p_{i,s,r}^q Q T_{i,s,r} \]

where the current account \( S_r^f \) is assumed to be constant in the ROW’s currency term.

- Foreign exchange rate arbitrage condition
\[ \varepsilon_{r,s} = \varepsilon_{r,s} \varepsilon_{r,s} \]

- Composite factor production (1st Stage)

\[ Y_{j,r} = b_{j,r} \left( \sum_h \beta_{h,j,r} F_{h,j,r} \frac{\sigma_{j}^{-1}}{\sigma_{j}^{-1}} \right) \]

\[ F_{h,j,r} = \left( \frac{\beta_{h,j,r} p_{j,r}^{y}}{(1 + \tau_{h,j,r}) p_{h,r}^{y}} \right)^{\sigma_{j}^{-1}} Y_{j,r} \]

where \( \sigma_{j}^{-1} \) denotes elasticity of substitution among primary factors, whose values are obtained from the GTAP Database (Hertel (1997)).

- Domestic output production (2nd Stage)

\[ X_{i,j,r} = ax_{i,j,r} Z_{j,r} \]

\[ Y_{j,r} = ay_{j,r} Z_{j,r} \]

\[ p_{j,r}^{z} = ay_{j,r} p_{j,r}^{y} + \sum_{i} ax_{i,j,r} p_{i,r}^{y} \]

- International shipping service

\[ QQT = t \prod_{r} QTS_{TRS,r}^{\nu_{r j}} \]

\[ QTS_{TRS,r} = \frac{\nu_{r j} p_{r j}^{eq} QQT}{(1 + \tau_{r j}^{z}) p_{r j}^{eq} \varepsilon_{r,ROW}} \]

\[ QQT = \sum_{i,r,s} \varepsilon_{i,r,s} QT_{i,r,s} \]

- Utility and social welfare (fictitious objective function for GAMS/CONOPT-NLP)

\[ UU_{r} = \prod_{i} X_{i,r}^{\alpha_{i,r}} \]

\[ SW = \sum_{r} UU_{r} \]
3. Calibration

We need a care in calibration of increasing returns to scale models because some prices deviate from unity in the initial equilibrium due to markups. The following formulas are used to calibrate parameters and initial values of endogenous variables:

\[ NF^{0}_{i,r,s} = \frac{NH^{0}_{i,r}}{\Phi^{MLZ0}_{min,i,r,s}} \]

\[ \beta^{MLZ}_{i} = \left( \frac{\alpha^{MLZ}_{i}}{\alpha^{MLZ}_{i} - \sigma^{MLZ}_{i} + 1} \right) \]

\[ \Phi^{MLZ0}_{i,s,r} = \beta^{MLZ}_{i} \Phi^{MLZ0}_{min,i,s,r} \]

\[ p^{0}_{i,r,s} = 1 + \tau^{i}_{i,r,s} \frac{\sigma^{MLZ}_{i}}{\Phi^{MLZ0}_{i,s,r}} \]

\[ p^{0}_{i,r,s} = p^{0}_{i,r,s} NF^{0}_{i,r,s} \frac{1}{1 - \sigma^{MLZ}_{i}} \]

\[ QT^{0}_{i,r,s} = \frac{SAMDATA_{i,r,s}}{p^{0}_{i,r,s}} \]

\[ QT^{0}_{i,r,s} = NF^{0}_{i,r,s} \frac{\alpha^{MLZ}_{i}}{1 - \sigma^{MLZ}_{i}} QT^{0}_{i,r,s} \]

\[ QT^{0}_{min,i,r,s} = \left( \beta^{MLZ}_{i} \right)^{\sigma^{MLZ}_{i}} \]

\[ \tau^{i}_{i,r,s} = \frac{T^{0}_{i,r,s}}{p^{0}_{i,r,s} QT^{0}_{i,r,s}} \]

\[ 4 \text{ The superscript "0" indicates symbols of initial values of endogenous variables.} \]

\[ 5 \text{ SAMDATA}_{i,r,s} \text{ denotes a trade value of the i-th good shipped from the r-th region to the s-th region recoded in the GTAP Database.} \]
\[ \tau_{i,r,s}^* = \frac{T_{i,r,s}^0}{QT_{i,r,s}^0} \]

\[ \tau_{i,r,s}^w = \frac{T_{i,r,s}^m}{(1 + \tau_{i,r,s}^e)p_{i,r,s}^{0} + \tau_{i,r,s}^e} QT_{i,r,s}^0 \]

\[ F_{i,r,s}^{MLZ} = \frac{QT_{i,r,s}^0}{(\sigma - 1) \Phi_{i,r,s}^{MLZ}} \]

\[ Q_{i,r}^0 = X_{i,r}^{p0} + X_{i,r}^{g0} + X_{i,r}^{v0} + \sum_j X_{i,j,r}^0 \]

\[ \delta_{i,r,s} = \frac{(1 + \tau_{i,r,s}^m)(1 + \tau_{i,r,s}^e)p_{i,r,s}^{0} + \tau_{i,r,s}^e}{\sum_{mr} (1 + \tau_{i,r,s}^m)(1 + \tau_{i,r,s}^e)p_{i,r,s}^{0} + \tau_{i,r,s}^e} QT_{i,r,s}^0 \frac{1}{\sigma_{i,r,s}^{MLZ}} \]

\[ \gamma_{i,s} = \frac{Q_{i,s}^0}{\left( \sum_{r} \delta_{i,r,s} QT_{i,r,s}^0 \frac{\sigma_{i,r,s}^{AM} - 1}{\sigma_{i,r,s}^{AM}} \right)} \]

\[ Z_{i,r,s}^0 = \frac{QT_{i,r,s}^0}{\Phi_{i,r,s}^{MLZ}} \]

\[ \pi_{i,j,r,s}^{MLZ} = P_{i,j,r,s}^{0} - (1 + \tau_{j,r,s}^e) \left( \frac{QT_{i,j,r,s}^0}{\Phi_{i,j,r,s}^{MLZ} + F_{i,j,r,s}^{MLZ}} \right) \]

\[ H_{i,s}^{MLZ} = \frac{\sum_{j} NF_{i,j,s} \pi_{i,j,s}^{MLZ}}{(1 + \tau_{j,s}^e) NH_{i,s}^{0}} \]
References


