Note: Model structure of the CGE model for Jordan

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I owe the nested-CES/CET structure of the CGE model for Jordan employed by Hosoe (1998) to Devarajan et al. (1990)\(^1\). But some modifications are made to equip their original model with the international trade model with three regions. Some parts of the model may be a little bit complicated due to the unique structure of the GTAP version 4 database, particularly in the treatment of international transportation services.

The Armington structure between domestic and imported Goods:

The CES aggregation function:

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Demand functions derived from the CES function:

\[ Q_{j,r} = \lambda_1 \left( \lambda_2 QD_{j,r} \frac{\sigma_{d,j-1}}{\sigma_{d_j}} + (1 - \lambda_2) QD_{j,r} \frac{\sigma_{d,j-1}}{\sigma_{d_j}} \right)^{\frac{\sigma_d}{\sigma_{d,j-1}}}, \quad \forall j, r \]

The Armington structure between import sources:

CES aggregation function:

\[ QM_{j,r} = \omega_1 \left( \sum_i \omega_2 QM_{j,s,r} \frac{\omega_{m,j-1}}{\omega_{m,j}} \right)^{\frac{\omega_{m,j}}{\omega_{m,j-1}}}, \quad \forall j, r \]

\[ QT_{j,s,r} = \left( \frac{\omega_1 \omega_{m,j-1}}{\omega_{m,j}} \omega_2 \frac{pqm_{j,r}}{(1 + \tau_{j,s,r}) pqt_{j,r} \cdot \text{Exch}_{j,r} + \tau_{j,s,r} pqq \cdot \text{Exch}_{ROW,j}} \right)^{\frac{\omega_{m,j}}{\omega_{m,j-1}}} QM_{j,r}, \quad \forall j, s, r \]

\[ Z_{j,r} = \delta_1_{j,r} \left( \delta_2_{j,r} QD_{j,r} \frac{1+\alpha_{j}}{\alpha_{j}} + (1-\delta_2_{j,r}) QE_{j,r} \frac{1+\alpha_{j}}{\alpha_{j}} \right) + QTS_{r}, \quad i = SRV, \forall r \]

\[ = \delta_1_{j,r} \left( \delta_2_{j,r} QD_{j,r} \frac{1+\alpha_{j}}{\alpha_{j}} + (1-\delta_2_{j,r}) QE_{j,r} \frac{1+\alpha_{j}}{\alpha_{j}} \right), \quad i \neq SRV, \forall r \]

Supply functions derived from the CET function:

\[ QD_{j,r} = \left( \frac{\delta_1_{j,r} \frac{1+\alpha_{j}}{\alpha_{j}} \delta_2_{j,r} (1+\tau_{j,r}) p_{z_{j,r}}}{p q d_{j,r}} \right) (Z_{j,r} - QTS_{r}), \quad i = SRV, \forall r \]

\[ = \left( \frac{\delta_1_{j,r} \frac{1+\alpha_{j}}{\alpha_{j}} \delta_2_{j,r} (1+\tau_{j,r}) p_{z_{j,r}}}{p q d_{j,r}} \right) Z_{j,r}, \quad i \neq SRV, \forall r \]

\[ QE_{j,r} = \left( \frac{\delta_1_{j,r} \frac{1+\alpha_{j}}{\alpha_{j}} (1-\delta_2_{j,r})(1+\tau_{j,r}) p_{z_{j,r}}}{p q e_{j,r}} \right) (Z_{j,r} - QTS_{r}), \quad i = SRV, \forall r \]

\[ = \left( \frac{\delta_1_{j,r} \frac{1+\alpha_{j}}{\alpha_{j}} (1-\delta_2_{j,r})(1+\tau_{j,r}) p_{z_{j,r}}}{p q e_{j,r}} \right) Z_{j,r}, \quad i \neq SRV, \forall r \]

The CGE transformation structure between export destinations:

\[ QE_{j,r} = \gamma_{1_{j,r}} \left( \sum_s \gamma_{2_{j,s,r}} QT_{j,s,r} \frac{1+\psi_{j}}{\psi_{j}} \right) \frac{\psi_{j}}{1+\psi_{j}}, \quad \forall j, r \]

\[ QT_{j,s,r} = \left( \frac{\gamma_{1_{j,r}} \frac{1+\psi_{j}}{\psi_{j}} \gamma_{2_{j,s,r}} p q e_{j,r}}{p q t_{j,r}} \right) QE_{j,r}, \quad \forall j, s, r \]

The CES value-added aggregation function and the derived factor demand functions:

\[ Y_{j,r} = \beta_{1_{j,r}} \left( \sum_h \beta_{2_{j,h,r}} F_{h,j,r} \frac{\gamma_{j}^{-1}}{\gamma_{j}} \right) \frac{\gamma_{j}}{1-\gamma_{j}}, \quad \forall j, r \]
\[
F_{h,j,r} = \left( \frac{\beta_1 j, i, \eta_j - 1}{\eta_j} \beta_2 j, i, p_y j, r \right) \left( \frac{p_{f_{h,r}}}{Y_{j,r}} \right), \quad \forall h, j, r
\]

The intermediate goods demand functions and the zero-profit conditions derived from the Leontief function:

\[X_{i,j,r} = ax_{i,j,r} Z_{i,j,r}, \quad \forall i, j, r\]
\[Y_{j,r} = ay_{j,r} Z_{j,r}, \quad \forall j, r\]
\[p_{z_{j,r}} = \sum_{i} ax_{i,j,r} (1 + \tau_{i,j,r}) pq_{i,r} + ay_{j,r} p_y j, r, \quad \forall j, r\]

The final demand functions:

Household:
\[(1 + \tau_{j,HOR,r}) pq_{j,r} QC_{j,HOR,r} = \alpha_{j,r} \left( \sum_{h} p_{f_{h,r}} FF_{h,r} + \sum_{i,s} T_{m_{i,s,r}} + \sum_{i,s} T e_{i,s}, r \right)
+ \sum_{i} T z_{i,r} + \sum_{i,j} T x_{i,j,r} + \sum_{i,l} T c_{i,l,r} + Exch_{ROW,r} CAB_r
- \sum_{i,l} (1 + \tau_{i,l,2,r}) pq_{i,r} QC_{i,l,2,r}, \quad \forall j, r\]

Government and investment:
\[QC_{i,l,2,r} = QC_{i,l,2,r}^0, \quad \forall i, l, 2, r\]

Taxes and freight:

Import tariffs:
\[T m_{i,r,s} = \eta_{m_{i,r,s}} \left( \left( 1 + \tau_{i,r,s} \right) pq t_{i,r,s} Exch_{i,s} + \tau_{i,r,s} pq q t \cdot Exch_{ROM,s} QT_{i,r,s} \right), \quad \forall i, r, s\]

Export taxes:
\[T e_{j,r,s} = \tau_{e_{j,r,s}} pq t_{j,r,s} QT_{j,r,s}, \quad \forall j, r, s\]
Indirect taxes on intermediate uses:

\[ T_{x_{i,j,r}} = \tau_{i,j,r} p_{q_{i,j,r}} X_{i,j,r}, \forall i, j, r \]

Indirect taxes on final demand:

\[ T_{c_{i,l,r}} = \tau_{i,l,r} p_{q_{i,l,r}} Q_{c_{i,l,r}}, \forall i, l, r \]

The BOP constraint:

\[
\sum_{i,r} (1 + \tau_{i,z,r}) Exch_{s,ROW} p_{q_{i,z,r}} QT_{i,z,r} + CAB_s + (1 + \tau_{SRV,s}) Exch_{s,ROW} p_{z_{SRV,s}} QTS_s = \sum_{i,r} (\tau_{i,z,r} p_{q_{i,z,r}} \cdot Exch_{s,ROW} + (1 + \tau_{i,z,r}) Exch_{s,ROW} p_{q_{i,z,r}}) QT_{i,z,s}, \forall s
\]

Market-clearing conditions:

For commodities:

\[ Q_{i,r} = \sum_{j} X_{i,j,r} + \sum_{i} Q_{c_{i,l,r}}, \forall i, r \]

For factors:

\[ FF_{h,r} = \sum_{j} F_{h,j,r}, \forall h, r \]

The International transportation sector:

Generation of the total international trade service with inputs supplied by each region:

\[ QQT = \rho \prod_{r} QTS_{r}^{\phi_{r}} \]

International trade service demand for each region:

\[ QTS_{s} = \frac{\phi_{s} p_{qqt}}{(1 + \tau_{SRV,s}) Exch_{s,ROW} p_{z_{SRV,s}}} QQT, \forall s \]

The Market-clearing condition of the international transportation services:
\[ QT = \sum_{i,r,s} \tau_{i,r,s} QT_{i,r,s}, \]

The arbitrage condition on foreign exchange:

\[ Exch_{i,s} = Exch_{r,r}, Exch_{r,s}. \]

Notations are:

[Indices]

\[ i, j, jj: \text{ goods (AGR, ENG, MIN, TXA, LMN, CHM, NMM, MAN, SRV)}, \]

\[ s, r, rr: \text{ regions (JOR, E_U, ROW)}, \]

\[ h: \text{ factors (LND, LAB, CAP)}, \]

\[ l: \text{ agents (HOH, GOV, INV)}, \]

\[ l2: \text{ the subset of } l \text{ (GOV, INV).} \]

[Endogenous variables]

\[ Tm_{i,r,s}: \text{ the amount of import tariffs on the } i\text{-th good imports from the } r\text{-th region to the } s\text{-th region,} \]

\[ Te_{i,r,s}: \text{ the amount of export taxes on the } i\text{-th good exports from the } r\text{-th region to the } s\text{-th region,} \]

\[ Tz_{j,r}: \text{ the amount of indirect taxes on the } j\text{-th good domestic production in the } r\text{-th} \]
region,

\( T_{x_{i,j,r}} \): the amount of indirect taxes on intermediate uses of the \( i \)-th good by the \( j \)-th sector in the \( r \)-th region,

\( T_{c_{i,l,r}} \): the amount of indirect taxes on final uses of the \( i \)-th good by the \( l \)-th final users in the \( r \)-th region,

\( QT_{i,r,s} \): transportation of the \( i \)-th good from the \( r \)-th region to the \( s \)-th region (including that from the \( r \)-th region to itself),

\( QM_{j,r} \): composite imports of the \( j \)-th good in the \( r \)-th region,

\( QE_{j,r} \): composite exports of the \( j \)-th good in the \( r \)-th region,

\( QD_{j,r} \): the \( j \)-th domestic good in the \( r \)-th region,

\( Q_{i,r} \): the \( i \)-th Armington's composite good in the \( r \)-th region,

\( Z_{j,r} \): domestic output of the \( j \)-th good in the \( r \)-th region,

\( F_{h,j,r} \): uses of the \( h \)-th factor by the \( j \)-th sector in the \( r \)-th region,

\( Y_{j,r} \): uses of composite factor (i.e., value added) by the \( j \)-th sector in the \( r \)-th region,

\( X_{i,j,r} \): uses of the \( i \)-th intermediate inputs by the \( j \)-th sector in the \( r \)-th region,

\( QC_{i,l,r} \): final demand of the \( i \)-th good by the \( l \)-th agent in the \( r \)-th region,

\( QTS_{r} \): exports of international transportation services by the \( r \)-th region,

\( QQT \): aggregate international transportation services,
\[\text{Note 1: which is common among sectors due to mobility of factors across sectors,}\]

\[\text{Note 2: } pf_{LAB,i} \text{ are set at unity as a numéraire,}\]

\[\text{Exch}_{r,s}: \text{exchange rate which convert the } r\text{-th region's currency to that of the } s\text{-th region.}\]

[Exogenous variables and key parameters]

\[\tau_{i,r,s}: \text{the import tariff rate for the } i\text{-th good imports from the } r\text{-th region to the } s\text{-th region,}\]

\[\tau_{e,i,r,s}: \text{the export tax rate for the } i\text{-th good exports from the } r\text{-th region to the } s\text{-th region,}\]

\[\tau_{j,r}: \text{the indirect tax rate for the } j\text{-th good domestic production in the } r\text{-th region,}\]
\( \tau_{i,j,r} \): the indirect tax rate for uses of the \( i \)-th good by the \( j \)-th sector in the \( r \)-th region,

\( \tau_{i,l,r} \): the indirect tax rate for uses of the \( i \)-th good by the \( l \)-th final users in the \( r \)-th region,

\( \tau_{i,r,s} \): coefficients of international transportation service requirement of the \( i \)-th good shipped from the \( r \)-th region to the \( s \)-th region,

\( CAB_r \): the current account balance in the \( r \)-th region,

\( QC_{i,i,r}^0 \): the initial value of \( QC_{i,i,r} \)

\( FF_{h,r} \): the \( h \)-th factor endowment in the \( r \)-th region,

\( \sigma_v \): elasticity of substitution among factors,

\( \sigma_d \): elasticity of substitution/transformation between composite imports/exports and domestic goods,

\( \sigma_m \): elasticity of substitution/transformation among import sources/export destinations,

and the other Greek letters are coefficients.
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<th>sigma-v</th>
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Source: compiled from GTAP version 3.
Figure: The flowchart of commodities
(for the case of Jordan)

Note: "Utility", at the top of the figure, represents utility of the household in each region.