A Consistent Representative Consumer Framework
for Discrete Choice Models with Endogenous Total Demand

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Abstract: Classical discrete choice models are often used to depict a consumer’s micro behavior of which brand to buy; however, they adopt an a priori assumption about a consumer’s individual behavior that he or she demands at most unity. This assumption is sometimes unrealistic, especially when the goods analyzed are daily goods, such as food, because a consumer buys multiple units of multiple brands and different consumers demand different amounts. The purpose of this paper is to construct models that allow more flexible individual behaviors that are consistent with the results of discrete choice models. We formulate utility maximization problems that give demand functions equivalent to those from discrete choice models, explicitly taking into account the aggregation structure of consumers, and examine their characteristics from various aspects, such as the form of the utility function, elasticities, and measurement of welfare.

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1 Introduction

Discrete choice models are suitable for depicting a consumer’s micro behavior. For example, consider a problem in which a consumer chooses a unit of a good from two available brands, A and B. Comparing utilities from brands A and B, a consumer chooses the brand that gives higher utility. This setup is consistent with a consumer’s utility maximization. However, this problem depicts only a part of consumer behavior, because the reason a consumer chooses a unit of the good is left unexplained. If we assume that a consumer chooses multiple units, the same point remains unresolved. A third option of ‘not buy’ involves not purchasing either brand, therefore solving the problem; even in this case, however, we need an a priori assumption that the total demand of a consumer for brand A, brand B, and ‘not buy’ is fixed. Thus, typical discrete choice models explain only a part of consumer behavior in that they need an a priori assumption regarding the controlled total.

What characteristics do discrete choice models have in the case of a noncontrolled total, that is, if they are fully consistent with the utility maximization problem of the consumer? The purpose of this paper is to address this problem. For this purpose, we analyze a deterministic utility maximization problem of a representative consumer, which yields the same result as a probabilistic utility maximization in random utility models. In random utility models, unobservable differences in taste are assumed to be subject to probabilistic distributions, and consequently, the choice probability is derived. The market demand is then obtained from multiplying the choice probability by the controlled total. This approach needs an a priori controlled total. On the contrary, in a deterministic representative consumer model, differences in taste are considered as differences in the form of the utility function of an individual consumer; that is, utility
functions differ by consumer. The demand of each consumer for a good is endogenous, and consequently, the market demand, which is the sum of the demand of each consumer, is also endogenous. Thus, our approach does not need an a priori controlled total. Classical discrete choice models, in which a consumer is supposed to make a single selection among a set of mutually exclusive alternatives, are included as a special case of our analyses. By deriving demand functions consistent with discrete choice models in a representative consumer framework, our analyses provide a theoretical foundation for demand functions derived from discrete choice models, including the case of classical discrete choice models. Our approach can also deal with the relationship between the utility functions of each consumer and a representative consumer, elasticities, and welfare measurement in a consistent manner by adopting a framework of utility maximization of a representative consumer.

Our focus is on the generalized extreme value (GEV) model and its mixed form, because it gives analytically closed-form demand functions. For the sake of simplicity, we analyze the logit and mixed logit models as special cases, before fully analyzing the GEV and mixed GEV models. Our analyses are quite general, given that Dagsvik (1995) and McFadden and Train (2000) show that the GEV and the mixed logit models can approximate any random utility models.

The main results we obtain in this paper are as follows. Focusing on a representative consumer model naturally implies that we follow the framework, and its restriction of a representative consumer, of Gorman (1953, 1961). First, if a representative consumer’s choice is represented by the logit model, his or her demand function for a good has the form of the total market demand, which is the sum of the demand of all consumers for all goods, times the choice probability of a good. This
form of the demand function is derived when the indirect utility function of the representative consumer includes a price index, which is a monotonic transformation of the log-sum term. In our model, the demand of a consumer for a good is endogenous, and thus, his or her total demand for all goods, the market demand for a good, and the total market demand for all goods are also endogenous. Making the total market demand endogenous affects the own and cross-price elasticity of the market demand for a good. The price elasticity of the total market demand is added to the usual own and cross-price elasticity in classical discrete choice models. Regarding welfare analysis, equivalent and compensating variations can be calculated from the demand of a consumer for a good, the market demand for a good, the total demand of a consumer, or the total market demand. When the market demand for a good or the total market demand is adopted to calculate equivalent and compensating variations, the log-sum term plays a role as the price index. Our analysis can easily be extended to cases where the goods are classified into multiple groups, like food and clothes. The analyses in the cases of the GEV model are basically the same.

Second, in the case of the mixed logit model, the analyses must be modified. In the mixed logit model, each consumer has his or her own parameters, which implies that each consumer has his or her own price index. Given this fact, the indirect utility function of the representative consumer must be quasi-linear, because all consumers must have the same income coefficient for their Gorman-form indirect utility functions. Thus, the market demand for a good has no income effect, if these utility functions are consistent with the mixed logit model. The price elasticity of the total market demand of a consumer is added to the usual own and cross-price elasticity in classical discrete choice models. Because the indirect utility function of the representative consumer is
quasi-linear, the three welfare measures, which are equivalent variation, compensating variation, and Marshallian consumer surplus, are equal, and they can be calculated from the demand of a consumer for a good, the market demand for a good, or the total demand of a consumer. When the total demand of a consumer is adopted, we can use the log-sum term as the price index to calculate welfare change. Note that we cannot calculate welfare change using total market demand. The price index depends on a consumer’s parameter and differs by consumer; consequently, there exists no price index with which a change in welfare is calculated for total market demand. As in the case of the logit model, our analysis can easily be extended to cases where the goods are classified to multiple groups and to the mixed GEV model.

Before proceeding, we briefly relate our analysis to the existing literature.

The first line of research related to this paper is analyses on the relationship between discrete choice models and a representative consumer model. Anderson et al. (1988, 1992, Ch.3) derive a representative consumer’s direct utility function consistent with logit demand. They obtain it under the assumption of classical discrete choice models, and then extend their analysis to allow endogenous demand for the group. As we will show, their analyses correspond to a special case of ours, because their direct utility function yields a special form of our indirect utility functions. Verboven (1996) derives a representative consumer’s direct utility function consistent with the nested logit demand within the framework of classical discrete choice models. We focus on the entire GEV family, taking into account endogenous demand for the group. Morisugi and Le (1994) analyzes the logit model with endogenous demand for the group, but their analyses lack theoretical consistency in that they focus on a representative consumer without considering its restriction on utility maximizing
problems; that is, indirect utility functions must have Gorman form.\(^1\)

The second line of research is econometric analyses that do not employ the assumption of classical discrete choice models. Dubin and McFadden (1984) and Hanemann (1984) consider that a consumer first chooses one brand and next decides how many he or she buys. These analyses do not correspond to our analyses, in which a consumer is free to choose multiple brands as well as multiple units. Hendel (1999) focuses on a situation where a firm buys multiple computers of multiple brands depending on its tasks. Although his analysis takes into account that multiple brands are chosen, it is based on a firm’s profit maximization model. It is fundamentally different from a utility maximization problem discussed in this paper that explicitly takes into account an aggregation structure of consumers.

The third line of research is recent empirical applications of discrete choice models, which widely range from durable goods, such as housing (Earnhart (2002)) and automobiles (Berry et al. (1995), Goldberg (1995), and Petrin (2002)), to daily goods, such as ready-to-eat cereal (Nevo (2002)) and tuna (Nevo and Hatzitaskos (2005)). The point is whether or not it is appropriate to assume that a consumer chooses at most one unit. The appropriateness of the assumption would hinge on the characteristics of the goods. If we focus on the choices of housing and automobiles, the assumption would nearly hold. However, in the case of ready-to-eat cereal and tuna, the rationale for the assumption would be weak\(^2\), because different consumers demand different

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\(^1\) Morisugi, Ueda, and Le (1995) try to extend the analyses in Morisugi and Le (1994) to the case of the GEV model, but it also lacks theoretical consistency by disregarding Gorman’s (1953, 1961) restriction.

\(^2\) Nevo (2000) points out, “…even though many of us buy more than one brand at a time, less actually consume more than one at a time. Therefore, the discreteness of choice can be sometimes defended by defining the choice period appropriately.” McFadden (1999) comments on another possibility; an alternative can be interpreted ‘as a ‘portfolio’ of decisions made in sequence, or as
amounts. The analyses in this paper establish a theoretical basis in applying discrete choice models to cases where a consumer chooses multiple units and the demand differs consumer by consumer.

The fourth line of research is analyses on welfare measurement for discrete choice models. First, welfare measurement for discrete choice models is theoretically analyzed in Small and Rosen (1981)\(^3\) in a general form, but the total market demand is assumed exogenous. Our analysis is an extension of their analysis, because we make total market demand endogenous and derive a method of welfare measurement applicable to a case of endogenous demand. Second, Herriges and Kling (1999), McFadden (1999), De Palma and Kilani (2003), and Dagsvik and Karlstrom (2005) analyze welfare measurement for discrete choice models in which an indirect utility function is nonlinear regarding income. These analyses focus on a one-consumer economy, in which a consumer’s indirect utility function coincides with a representative consumer’s indirect utility function, and derive a change in welfare. In a many-consumer economy, their analyses are inapplicable, because an indirect utility function that is nonlinear regarding income yields a difference in the marginal utility of income among consumers. In this case, as Boadway (1974) shows, the aggregated compensating variation is not consistent with the compensation test and its usefulness is lost. Our analysis is complementary to theirs in that we restrict the form of the indirect utility functions to the Gorman form but we can aggregate each consumer’s welfare change consistently.

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\(^3\) Welfare measurement in discrete choice models is developed by Ben Akiva (1973), McFadden (1973), and Domencich and McFadden (1975) for the logit model, by Williams (1977) and McFadden (1978, 1979, 1981) for the nested logit model, and by McFadden (1978, 1979, 1981) for the GEV model.
The rest of the paper is structured as follows. In section 2, we focus on the logit model. In section 3, the analyses are extended to the GEV model. In section 4, we examine the mixed logit model, followed by the mixed GEV model in section 5. Section 6 concludes.

2 Logit

We begin with the logit model. The GEV model, which includes the logit model as a special case, will be discussed in the next section.

Consider an \( N \)-consumer economy with \( M + 1 \) goods. The goods are numbered consecutively from 0 to \( M \). The price of good 0, whose market demand is \( X_0 \), is normalized at unity. The market demand and price of good \( m \) are \( X_m \) and \( p_m \) (\( m = 1, \ldots, M \)). The income of consumer \( n \) is \( y(n) \) (\( n = 1, \ldots, N \)).

Utility maximization by a consumer \( n \) yields an indirect utility function, \( v(p_1, \ldots, p_M, y(n), n) \). In this paper, we assume that each consumer’s preference can be aggregated to a representative consumer’s preference. Without this assumption, no clear relationship exists between the sum of compensating variations of each consumer and the compensation principle, as Boadway (1974) shows.

Gorman (1953, 1961) shows that in order to define the preferences of a representative consumer by aggregating preferences of each consumer, consumer \( n \)’s indirect utility function must have the so-called Gorman form:

\[
(1) \quad v(p_1, \ldots, p_M, y(n), n) = A(p_1, \ldots, p_M, n) + B(p_1, \ldots, p_M)y(n).
\]

Applying Roy’s Identity, consumer \( n \)’s demand for good \( m \), which is denoted by \( x_m(p_1, \ldots, p_M, y(n), n) \), is derived as:
Adding up each consumer’s indirect utility function, a representative consumer’s indirect utility function is derived as:

\[
V = \sum_n v(p_1, \ldots, p_M, y(n), n) = \sum_n A(p_1, \ldots, p_M, n) + B(p_1, \ldots, p_M)Y,
\]

where \(y(n)\) is aggregate income. Applying Roy’s Identity, the market demand for good \(m\), \(X_m(p_1, \ldots, p_M, Y)\), is derived as:

\[
X_m(p_1, \ldots, p_M, Y) = \frac{-\frac{\partial V}{\partial p_m}}{\frac{\partial V}{\partial y(n)}} = \sum_n \left( \frac{-\frac{\partial v}{\partial p_m}}{\frac{\partial v}{\partial y(n)}} \right) = \sum_n x_m(p_1, \ldots, p_M, y(n), n).
\]

### 2-1 Utility maximization problem that yields logit-type demand functions

For the moment, suppose that all the goods belong to the same group. The case of different groups will be focused on in section 2-4. The choice probability of the logit model, \(s_m\), can be written as:

\[
s_m = \frac{\exp(u_m(p_m, Y))}{\sum_{j=1}^M \exp(u_j(p_j, Y))},
\]

where \(u_m(p_m, Y)\) is the utility of a representative consumer from a unit consumption of good \(m\). Suppose that the market demand function of good \(m\) is consistent with the logit model:

\[
X_m(p_1, \ldots, p_M, Y) = C(p_1, \ldots, p_M, Y)s_m,
\]
where $C(p_1, \ldots, p_M, Y)$ is the total market demand, which is the sum of the demands from good 1 to good $M$. Regarding the form of the indirect utility function of the representative consumer, we obtain the following proposition.

**Proposition 1**

The necessary and sufficient condition for the market demand function of good $m$ to have the form of (6) is that the indirect utility function of a representative consumer is:

$$(7) \quad V = \sum_n A(P(LS), p_1, \ldots, p_M, n) + B(P(LS))Y,$$

which satisfies:

i) $P(LS)$ is a monotonic transformation of:

$$(8) \quad LS = -\frac{1}{\beta} \ln \sum_{j=1}^{M} \exp(\alpha_j - \beta p_j),$$

ii) $$(9) \quad \left( \sum_n \frac{\partial A}{\partial P} + \frac{\partial B}{\partial P} Y \right) \frac{dP}{dLS} = \frac{\partial V}{\partial LS} < 0,$$

iii) $$(10) \quad \frac{\partial}{\partial p_m} \left( \sum_n A(P, p_1, \ldots, p_M, n) \right) = 0 \quad \text{for any} \quad p_m.$$

The market demand function of good $m$, (6), satisfies:

$$(11) \quad C(p_1, \ldots, p_M, Y) = C(P(LS), Y) = -\left( \sum_n \frac{\partial A}{\partial P} + \frac{\partial B}{\partial P} Y \right) \frac{dP}{dLS} > 0,$$

$$(12) \quad s_m = \frac{\exp(\alpha_m - \beta p_m)}{\sum_{j=1}^{M} \exp(\alpha_j - \beta p_j)} = \frac{\partial LS}{\partial p_m}.$$
Proof

See Appendix 1.

The content of Proposition 1 can be understood intuitively. From the relationship
in (12), \( A(P(LS),p_1,...,p_M,n) \) and \( B(P(LS)) \) in the indirect utility function of a
representative consumer must include the price index, \( P(LS) \), which is a monotonic
transformation of the log-sum term, \( LS \), so that it produces the logit-type market
demand function. The price index cannot depend on aggregate income, \( Y \); otherwise,
the indirect utility function of a representative consumer does not satisfy the Gorman
form. The Gorman restriction also implies that \( B(P(LS)) \) is common to all consumers.
If \( B(P(LS)) \) includes an argument other than the price index, \( P(LS) \), we cannot
obtain the logit-type market demand function. Thus, \( B(P(LS)) \) includes the price
index, \( P(LS) \), only. On the other hand, \( A(P(LS),p_1,...,p_M,n) \) may vary across
consumers in the Gorman-form indirect utility functions. The requirement here is that
(10) holds; that is, \( \sum_n A(P(LS),p_1,...,p_M,n) \) is a function that depends only on the
price index, \( P(LS) \). Because \( \sum_n A(P(LS),p_1,...,p_M,n) \) and \( B(P(LS)) \) depend only
on the price index, \( P(LS) \) and aggregate income, \( Y \), the total market demand,
\( C(p_1,...,p_M,Y) \), in (11) also depends only on them and can be written as \( C(P(LS),Y) \).
In a two-consumer economy, for example, each person’s indirect utility function can be
written as:

(13) \( v(p_1,...,p_M,y(1),1) = \tilde{A}(P) + p_1 + B(P)y(1) \) and

(14) \( v(p_1,...,p_M,y(2),2) = \tilde{A}(P) - p_1 + B(P)y(2) \).
Adding (13) and (14), we obtain an indirect utility function of a representative consumer, which yields the logit-type market demand function, although each consumer’s demand function is not logit-type.

Classical discrete choice models, in which a consumer is supposed to choose one of several mutually exclusive alternatives, assume:

\[(15) \sum_{n} x_{m}(p_{1}, \ldots, p_{M}, y(n), n) = 1.\]

Accordingly, the total market demand, \(C\), equals the number of consumers, as in (6),

\[(16) C(p_{1}, \ldots, p_{M}, Y) = N.\]

For example, when \(A = -P\), \(B = 1\), and \(P = LS\), (16) is satisfied from (11), and (7) yields classical discrete choice models.

Nevo (2000, 2001) stresses the importance of including an “outside option,” which is typically “not buy” or “buy other goods,” in the choice sets. An outside option is crucial under the assumptions of (15). As an example, suppose that the choice sets consist of “buy brand A,” “buy brand B,” and “not buy,” and that the prices of both brands are reduced. Under the assumptions of (15), each consumer’s demand for brand A or brand B is at most one. The market demand for both brands will be increased because the consumers switch from “not buy” to “buy brand A” or “buy brand B.” Without an outside option of “not buy,” the market demand for brands A and B would remain unchanged. This would sometimes be an unrealistic situation in the choice of daily consumer goods such as food, because a decrease in price never leads to an increase in demand. On the contrary, in the model in this paper, we do not have to consider explicitly an outside option of “not buy”, because a decrease in prices increases the market demand through an increase in each consumer’s demand.
Developing Anderson et al. (1988, 1992 Ch.3), we can show an example of the direct utility function that is consistent with the indirect utility function of the representative consumer, (7).

**Corollary 1**

The following direct utility function of a representative consumer corresponds to a special case of an indirect utility function, (7):

\[
(17) U = X_0 + h \left( \frac{b}{\beta} \sum_{j=1}^{M} X_j \right) + \frac{1}{\beta} \sum_{j=1}^{M} \alpha_j \ln \left( \frac{X_j}{\sum_{j=1}^{M} X_j} \right) X_j ,
\]

where \( b \) is a constant, \( h' > 0 \), and \( h'' < 0 \).

**Proof**

See Appendix 2.

**2-2 Elasticities**

From the market demand function (6), we obtain the following proposition regarding elasticities.

**Proposition 2**

The own price elasticity of the logit-type market demand function of good \( m \) is:

\[
(18) \frac{\partial X_m}{\partial p_m} \frac{p_m}{X_m} = \theta_m - \beta (1 - s_m) p_m ,
\]

13
where \( \theta_m = \frac{\partial C}{\partial p_m} \frac{p_m}{C} \) is the elasticity of the total market demand, \( C \), with respect to the price of good \( m \), \( p_m \). The cross elasticity of the demand for good \( m \) is:

\[
(19) \frac{\partial X_m}{\partial p_m} \frac{p_m'}{X_m} = \theta_m + \beta s m', p m',
\]

where \( m' = 1,...,M \) and \( m' \neq m \).

**Proof**

The results immediately follow from the market demand function, (6).

Both elasticities differ from those in classical discrete choice models, because the elasticities of the total market demand are added on. The cross-price elasticities are the same among the goods and the IIA (independence from irrelevant alternatives) property still holds, even if the total market demand is made endogenous.

Consider typical cases where the elasticities of the total market demand are negative. In classical discrete choice models, \( \theta_m \) and \( \theta_m' \) in (18) and (19) are assumed zero. Thus, classical discrete choice models make the own price elasticity more inelastic when the total market demand is actually endogenous. On the contrary, classical discrete choice models do not take into account the fact that \( \theta_m \) counteracts \( \beta s m', p m' \) in (19); the cross-price elasticities is made positive in classical discrete choice models, but they may not be so if the total market demand is made endogenous.

**2-3 Welfare Analyses**

Here we focus on how to calculate equivalent variation (EV). The same argument
applies for compensating variation if \( V^{WO} \) and \( v^{WO} \) are substituted for \( V^W \) and \( v^W \) in the following analysis, respectively. Henceforth, the superscripts \( WO \) and \( W \) denote without and with a policy, respectively. The results are summarized as Proposition 3.

**Proposition 3**

Equivalent variation can be calculated from the demand of a consumer for a good, the market demand for a good, the total demand of a consumer, or the total market demand, that is:

\[
EV = \sum_n \int_{PM}^P h_m(P, p_1, ..., p_M, v(n)^W)dp_m = \int_{PM}^P H_m(P, p_1, ..., p_M, V^W)dp_m = \int_{LS}^{LSW} \sum_n c(P(LS), p_1, ..., p_M, v(n)^W)dLS = \int_{LS}^{LSW} C(P(LS), V^W)dLS,
\]

where \( h_m(P, p_1, ..., p_M, v(n)) \) is the Hicksian demand function of a consumer \( n \) for good \( m \), \( H_m(P, p_1, ..., p_M, V) \) is the Hicksian market demand function for good \( m \), \( c(P(LS), p_1, ..., p_M, v(n)) \) is the Hicksian total demand function of a consumer \( n \), and \( C(P(LS), V) \) is the Hicksian total market demand function.

**Proof**

See Appendix 3.

The distinctive feature of the logit model, which is also true for GEV models, as we
will see, is that the equivalent variation can be calculated not only from the demand and price of good \( m \) but also from the total demand for a consumer \( n \) or the total market demand using the log-sum term. In particular, if the total market demand is equal to the number of consumers, \( N \), the result of (20) is reduced to the well-known method shown in Small and Rosen (1981); equivalent variation is calculated from a change in the log-sum term times the number of consumers.

2-4 Multiple Groups

The analyses can be readily extended to a case where the goods 1,...,\( M \) are classified into multiple groups and a consumer can choose multiple goods from multiple groups. This extension is an advantage of our model compared with classical discrete choice models, which fail to consider such a situation because of the assumption that a consumer’s choice is restricted to one alternative among one group.

Suppose that the goods are classified into \( G \) groups and good \( m \) is supposed to belong to the group \( g \) \((g = 1,\ldots,G)\) without loss of generality. Suppose that the market demand function of good \( m \) is consistent with the logit model regarding the total market demand for group \( g \).

\[
(21) \quad X_m(p_1,\ldots,p_M,Y) = C_g(p_1,\ldots,p_M,Y)s_{gm},
\]

where
\[
s_{gm} = \frac{\exp(u_m(p_m),Y)}{\sum_{j\in g} \exp(u_j(p_j),Y)}
\]
is the choice probability of good \( m \) within group \( g \) and \( C_g(p_1,\ldots,p_M,Y) \) is the total market demand for group \( g \). Taking into account the classification into multiple groups, proposition 1 is modified as follows.
Proposition 4

The necessary and sufficient condition for the market demand function of good \( m \) to have the form of (21) is that the indirect utility function of a representative consumer is:

\[
V = \sum_{n} A(P_{1}(LS_{1}),...,P_{G}(LS_{G}),p_{1},...,p_{M},n) + B(P_{1}(LS_{1}),...,P_{G}(LS_{G}))Y,
\]

which satisfies:

i) \( P_{g}(LS_{g}) \) is a monotonic transformation of:

\[
LS_{g} = -\frac{1}{\beta} \ln \sum_{j \neq g} \exp(\alpha_{j} - \beta_{g}p_{j}),
\]

ii) \( (24)\)

\[
0 < \frac{\partial}{\partial LS_{g}} \left( \sum_{n} \frac{\partial A}{\partial P_{g}} + \frac{\partial B}{\partial P_{g}} Y \right) dP_{g} + dLS_{g} = \frac{\partial V}{\partial LS_{g}} < 0,
\]

iii) \( (25)\)

\[
\frac{\partial}{\partial p_{m}} \left( \sum_{n} A(P_{1}(LS_{1}),...,P_{G}(LS_{G}),p_{1},...,p_{M},n) \right) = 0 \text{ for any } p_{m}.
\]

The market demand function of good \( m \), (21), satisfies:

\[
C_{g}(p_{1},...,p_{M},Y) = C_{g}(P_{1}(LS_{1}),...,P_{G}(LS_{G}),Y) = \frac{-\left( \sum_{n} \frac{\partial A}{\partial P_{g}} + \frac{\partial B}{\partial P_{g}} Y \right) dP_{g}}{B} > 0,
\]

\[
s_{gm} = \frac{\exp(\alpha_{m} - \beta_{g}p_{m})}{\sum_{j \neq g} \exp(\alpha_{j} - \beta_{g}p_{j})} = \frac{\partial LS_{g}}{\partial p_{m}}.
\]

Proof

The proof is the same as that in proposition 1 in Appendix 1, and is omitted here.
Proposition 4 demonstrates that the indirect utility function of the representative consumer is consistent with the logit-type market demand function regarding the total market demand for group $g$, if it includes the price indices for group $P_g$, which are monotonic transformations of the log-sum term, $LS_g$.

Applying proposition 4, we can construct a demand structure in which the preference between groups is not logit-type but the preference within each group is logit-type. For example, suppose that there exist two groups, $g_1$ and $g_2$, and the preference between them follows the CES utility function. Suppose further that $M_1$ goods exist in group $g_1$ and $M_2$ goods exist in group $g_2$ and that the preference within each group is logit-type. The indirect utility function of a representative consumer is:

$$V = Y \left\{ (P_{g_1})^{1-\sigma} + (P_{g_2})^{1-\sigma} \right\}^{\frac{1}{\sigma-1}},$$

where $\sigma$ is the elasticity of substitution between the two groups. The demands for groups $g_1$ and $g_2$, $C_{g_1}$ and $C_{g_2}$, respectively are:

$$C_{g_1} = \frac{Y(P_{g_1})^{1-\sigma}}{\left\{ (P_{g_1})^{1-\sigma} + (P_{g_2})^{1-\sigma} \right\}} \quad \text{and}$$

$$C_{g_2} = \frac{Y(P_{g_2})^{1-\sigma}}{\left\{ (P_{g_1})^{1-\sigma} + (P_{g_2})^{1-\sigma} \right\}},$$

which have the CES-form. The market demand function of good $m_1$ ($m_1 = 1, \ldots, M_1$) in group $g_1$ and that of good $m_2$ ($m_2 = 1, \ldots, M_2$) in group $g_2$, respectively, are:

$$X_{m_1} = \frac{Y(P_{g_1})^{1-\sigma}}{\left\{ (P_{g_1})^{1-\sigma} + (P_{g_2})^{1-\sigma} \right\}} \sum_{j \neq g_1} \exp(\alpha_j - \beta_{g_1}p_{mj}) \quad \text{and}$$
which have the form of the CES-type total market demand function multiplied by the
logit-type choice probability.

Regarding elasticities, proposition 2 is modified as follows.

**Proposition 5**

The own price elasticity of the market demand for good \( m \), \( X_m \), is:

\[
(33) \quad \frac{\partial X_m}{\partial p_m} = \theta_{gm} - \beta_g (1 - s_{gm}) p_m,
\]

where \( \theta_{gm} \equiv \frac{\partial C_g}{\partial p_m} C_g \) is the elasticity of the total market demand for group \( g \), \( C_g \),
with respect to the price of good \( m \), \( p_m \). The cross-price elasticity is:

\[
(34) \quad \frac{\partial X_m}{\partial p_{m'}} = \theta_{gm'} + \beta_g s_{gm'} P_{m'}
\]

when goods \( m' \) and \( m \) belong to the same group \( g \), and

\[
(35) \quad \frac{\partial X_m}{\partial p_{m'}} = \theta_{gm'}
\]

when they belong to different groups.

**Proof**

The results immediately follow from the market demand function, (21).

The results of (33) and (34) are modified versions of those of (18) and (19), taking
into account the classification of the goods into the groups. The classification of the goods yields the possibility that goods $m$ and $m'$ belong to different groups, in which case, (35) holds.

Regarding welfare analysis, proposition 3 is modified as follows.

**Proposition 6**

Equivalent variation can be calculated from the demand of a consumer for a good, the market demand for a good, the total demand of a consumer for a group, or the total market demand for a group, that is:

\begin{align*}
EV &= \sum_{n,p} \int_{p_m}^{p_{m}'} h_m(p_1, \ldots, P_G, P_1, \ldots, P_M, v(n)^W)dp_m \\
&= \int_{p_m}^{p_{m}'} H_m(p_1, \ldots, P_G, P_1, \ldots, P_M, V^W)dp_m \\
&= \int_{LS_G}^{LS_G} \sum_n c_g(P_1(LS_1), \ldots, P_G(LS_G), p_1, \ldots, p_M, v(n)^W)dLS_g \\
&= \int_{LS_G}^{LS_G} C_g(P_1(LS_1), \ldots, P_G(LS_G), V^W)dLS_g,
\end{align*}

where $c_g(P_1(LS_1), \ldots, P_G(LS_G), p_1, \ldots, p_M, v(n))$ is the Hicksian total demand of a consumer $n$ for group $g$, $C_g(P_1(LS_1), \ldots, P_G(LS_G), V^W)$ is the Hicksian total market demand for group $g$.

**Proof**

The proof is the same as that in proposition 3 in Appendix 3, and is then omitted.

The same argument applies for compensating variation if $V^{WO}$ and $v^{WO}$ are substituted for $V^W$ and $v^W$. 

20
3 GEV

The analyses in section 2 can be extended to GEV models, which are a general form of logit model. From McFadden (1978, Theorem 1), the GEV model can be described using the following function $H(z_1, \cdots, z_M)$, where $z_m = \exp(u_m(p, Y))$:

(GEV-1) $H(z_1, \cdots, z_M)$ is nonnegative,

(GEV-2) $H(z_1, \cdots, z_M)$ is homogenous of degree $n$,

(GEV-3) $\lim_{z_m \to \infty} H(z_1, \cdots, z_M) = \infty$

(GEV-4) The $k$th partial derivative of $H(z_1, \cdots, z_M)$ with respect to any combination of distinct $z_m$ s is nonnegative if $k$ is odd, and nonpositive if $k$ is even.

That is, $\frac{\partial H}{\partial z_i} \geq 0$ for all $i$, $\frac{\partial^3 H}{\partial z_i \partial z_j} \leq 0$ for all $j \neq i$, $\frac{\partial^3 H}{\partial z_i \partial z_j \partial z_k} \geq 0$ for any distinct $i$, $j$, and $k$, and so on for higher order derivatives.

Under the assumptions of (GEV-1) to (GEV-4), from McFadden (1978, Theorem 1), the choice probability of good $m$ is:

$$s_{GEVm} = \frac{\partial H}{\partial z_m} \frac{z_m}{nH}.$$  

(37)

The extension of the analyses in section 2 to the GEV model is straightforward. The aggregation structure is identical. The points to be noted are as follows.

i) The log-sum term is modified from (9) to:

---

4 In the original argument in McFadden (1978, Theorem 1), homogeneity of degree 1 is assumed. Ben-Akiba and Francois (1983) demonstrate that $H$ can be homogeneous of degree $n$. See also Ben-Akiba and Lerman (1985, p.126).
ii) The choice probability of good \( m \) is modified from (13) to:

\[
LS_{\text{GEV}} \equiv -\frac{1}{n \beta} \ln H(\exp(\alpha - \beta p_1), \cdots, \exp(\alpha - \beta p_M)).
\]

iii) In the same way as the logit model, the elasticities of total market demand are added to the ordinary own and cross-price elasticities of the GEV model, when the total market demand is endogenous.

iv) Regarding welfare analysis, the same result as for the logit model holds, although the log-sum term is modified from (8) to (38).

v) The extension to the case of multiple groups is analogous to the analyses in the logit model.

The detailed analyses are deferred to Appendix 4.

4 Mixed Logit

Hereafter, we focus on “mixed” versions of the logit and GEV models. We consider the mixed logit model in this section and then the mixed GEV model in the next section.

Suppose that each consumer has his or her own parameter, \( \gamma \) and income \( y(\gamma) \). A consumer with parameter \( \gamma \) is hereafter called a \( \gamma \)-type consumer. We assume that a stochastic parameter, \( \gamma \), affects the coefficient of price, but the analyses are basically the same if \( \gamma \) is an individually specific constant or affects coefficients of other variables that are not explicitly modeled here.
4-1 Utility maximization problem that yields mixed-logit-type demand functions

From Train (2003), the mixed logit model is defined as the model that has the choice probability of:

$$s_{\text{MLm}} = \int s_{\text{MLm}}(\gamma) f(\gamma) d\gamma,$$

where

$$s_{\text{MLm}}(\gamma) = \frac{\exp(u_{m}(p_{m}, y(\gamma), \gamma))}{\sum_{j=1}^{M} \exp(u_{j}(p_{j}, y(\gamma), \gamma))}$$

and

$$f(\gamma)$$ is a probability density function. For the moment, suppose that all the goods belong to the same group. The case of different groups will be focused on in section 4-4. Denoting the total demand of the $\gamma$-type consumer by $c(p_{1},...,p_{M},y(\gamma),\gamma)$ and taking into account that there exists $N$ consumers, we can write the mixed logit-type market demand function of good $m$ as:

$$X_{m} = N \int c(p_{1},...,p_{M},y(\gamma),\gamma) s_{\text{MLm}}(\gamma) f(\gamma) d\gamma,$$

which coincides with the mixed logit model in classical discrete choice models when $c(p_{1},...,p_{M},y(\gamma),\gamma) = 1$. Because indirect utility functions follow Gorman’s (1953, 1961) restriction, the indirect utility function of the $\gamma$-type consumer is:

$$v(p_{1},...,p_{M},y(\gamma),\gamma) = A(p_{1},...,p_{M},\gamma) + B(p_{1},...,p_{M})y(\gamma),$$

and that of the representative consumer is:

$$V = N \int v(p_{1},...,p_{M},y(\gamma),\gamma) f(\gamma) d\gamma$$

$$= N \int A(p_{1},...,p_{M},\gamma) f(\gamma) d\gamma + B(p_{1},...,p_{M})Y,$$
where \( Y = N \int_{\gamma} y(\gamma)f(\gamma)d\gamma \) is aggregate income in the mixed logit model. Regarding the form of the indirect utility function of the representative consumer, we obtain the following proposition.

**Proposition 7**

The necessary and sufficient condition for the market demand function of good \( m \) to have the form of (42) is that the indirect utility function of the representative consumer is:

\[
(45) \quad V = N \int_{\gamma} A(P(LS_{ML}(\gamma)), p_1, ..., p_M, \gamma)f(\gamma)d\gamma + \bar{B}Y, \quad \text{where} \quad \bar{B} \quad \text{is a fixed constant,}
\]

which satisfies:

i) \( P(LS_{ML}(\gamma)) \) is a monotonic transformation of:

\[
(46) \quad LS_{ML}(\gamma) = -\frac{1}{\beta(\gamma)} \ln \sum_{j=1}^{M} \exp(\alpha_j - \beta(\gamma)p_j),
\]

ii) \( 0 < \frac{\partial A}{\partial P(LS_{ML}(\gamma))} \frac{dP(LS_{ML}(\gamma))}{dLS_{ML}(\gamma)} < 0, \)

\[
\frac{\partial}{\partial p_m} \left( \int_{\gamma} A(P(LS_{ML}(\gamma)), p_1, ..., p_M, \gamma)f(\gamma)d\gamma \right) = 0 \quad \text{for any} \quad p_m.
\]

The market demand function of good \( m \), (42), satisfies:

\[
(49) \quad c(p_1, ..., p_M, \gamma) = -\frac{\partial A}{\partial P(LS_{ML}(\gamma))} \frac{dP(LS_{ML}(\gamma))}{dLS_{ML}(\gamma)} \bar{B} > 0,
\]

\[
(50) \quad s_{MLm}(\gamma) = \frac{\exp(\alpha_m - \beta(\gamma)p_m)}{\sum_{j=1}^{M} \exp(\alpha_j - \beta(\gamma)p_j)} = \frac{\partial LS_{ML}(\gamma)}{\partial p_m}.
\]
Proof

See Appendix 5.

The major difference from the logit model is that the market demand function of good \( m \) must be independent of income. The reason is as follows. In the mixed logit model, each consumer has a price index, \( P(LS_{ML}(\gamma)) \). From Gorman’s (1953, 1961) restriction, the coefficient of income, \( B(p_1, ..., p_M) \), must be common for all consumers. This implies that the coefficient of income, \( B(p_1, ..., p_M) \), does not include each consumer’s price index, \( P(LS_{ML}(\gamma)) \). To be consistent with the market demand function of the mixed logit model, the coefficient of income, \( B(p_1, ..., p_M) \), cannot include \( p_m \). Thus, \( B(p_1, ..., p_M) \) is a fixed constant of \( \bar{B} \), and accordingly, the market demand function of good \( m \) is independent of income.

4-2 Elasticities

From the mixed logit-type market demand function, (41), we obtain the following proposition regarding elasticities.

**Proposition 8**

The own price elasticity of the market demand of good \( m \) is:

\[
(51) \frac{\partial X_m}{\partial p_m} \frac{p_m}{X_m} = N \int_{\gamma} \left\{ \frac{c(\gamma)s_{ML,m}(\gamma)}{X_m} \left( \phi_m(\gamma) - \beta(\gamma)(1 - s_{ML,m}(\gamma))p_m \right) \right\} f(\gamma) d\gamma ,
\]

where \( \phi_m(\gamma) = \frac{\partial c(\gamma)}{\partial p_m} \frac{p_m}{c(\gamma)} \) is the \( \gamma \)-type consumer’s price elasticity of total demand.
The cross-price elasticity is:

\[ \frac{\partial X_m}{\partial p_{m'}} X_m = N \int_\gamma \left\{ \frac{c(\gamma)s_{MLm}(\gamma)}{X_m} \left( \phi_{m'}(\gamma) + \beta(\gamma)s_{MLm}(\gamma) p_{m'} \right) \right\} f(\gamma) d\gamma. \]

**Proof**

The results immediately follow from the market demand function, (42).

The results of (51) and (52) coincide with those of the logit model, (18) and (19), respectively, if parameter $\gamma$ is a fixed constant. Otherwise, the own and cross-price elasticities become more flexible. In particular, the IIA property does not hold, because the cross-price elasticities in (52) depend on $s_{MLm}(\gamma)$ and $X_m$, and accordingly, they differ by good.

**4-3 Welfare Analysis**

The indirect utility function of the representative consumer that yields the mixed logit model, (45), is quasi-linear. Thus, the Hicksian and the Marshallian demand curves coincide, and equivalent variation, compensating variation, and the change in consumer surplus also coincides. Regarding welfare measurement, the following proposition holds.

**Proposition 9**

Equivalent variation can be calculated from the demand of a consumer for a good, the market demand for a good, or the total demand of a consumer; that is:
\[
EV = N \int \left( \int_{p_{m}^{0}}^{p_{m}^{00}} h_{m} dp_{m} \right) f(\gamma) d\gamma \\
= \int_{p_{m}^{0}}^{p_{m}^{00}} H_{m} dp_{m} \\
= N \int \left( \int_{LS_{m}^{0}(\gamma)^{00}}^{LS_{m}^{0}(\gamma)^{0}} c(P(\gamma_{LS_{m}^{0}(\gamma)}), p_{1}, ..., P_{M}, \gamma) d\gamma \right) f(\gamma) d\gamma.
\] (53)

**Proof**

See Appendix 6.

In the mixed logit model, equivalent variation can be calculated by three methods. The first method is to calculate equivalent variation for the \( \gamma \)-type consumer, using his or her demand function for good \( m \), and then sum over all consumers. The second method is to calculate the equivalent variation for all consumers, using the market demand function for good \( m \). The third method is a variation of the first method, where equivalent variation for the \( \gamma \)-type consumer is calculated using his or her total demand function and the log-sum term, \( LS_{mL}(\gamma) \). Note that equivalent variation cannot be calculated using the total market demand function and the log-sum term. This is a different result from proposition 3. The reason is that the log-sum term differs by consumer in the mixed logit model, and consequently, there exists no price index that corresponds to total market demand.

The analyses so far can be easily extended to a case where the goods are classified into multiple groups. The analyses are basically the same as those in the logit model, and then delegated to Appendix 7.
5 Mixed GEV Model

McFadden and Train (2000) demonstrate that the mixed logit model can approximate any random utility model. However, Train (2003, p.171–172) states that the mixed nested-logit model is more appropriate for describing the situation, because of its analytical properties. Therefore, it is useful to derive the properties of the mixed version of the GEV model, which we call the mixed GEV model.

The extension from the mixed logit to the mixed GEV model is analogous to that from the logit to the GEV model. The aggregation structure is identical to that in the mixed logit model. The points to be noted are as follows. Although a stochastic parameter $\gamma$ is assumed to affect the coefficient of price as in the case of the mixed logit, the analyses are basically the same if $\gamma$ is an individually specific constant or affects coefficients of other variables that are not explicitly modeled here.

i) The log-sum term is modified from (46) to:

$$\text{(54)} \quad LS_{MGEV}(\gamma) = -\frac{1}{n}\beta(\gamma)\ln H(\exp(\alpha_1 - \beta(\gamma)p_1), \cdots, \exp(\alpha_m - \beta(\gamma)p_m)).$$

ii) The choice probability of good $m$ is modified from (50) to:

$$\text{(55)} \quad s_{MGEVm}(\gamma) = \frac{\partial H}{\partial z_m} \exp(\alpha_m - \beta(\gamma)p_m) \frac{1}{nH} = \frac{\partial LS_{MGEV}(\gamma)}{\partial p_m}.$$ 

iii) The own and cross-price elasticities of the GEV model are mixed in the same way as the mixed logit model.

iv) Regarding welfare analysis, the same result as the mixed logit model holds, if $LS_{ML}(\gamma)$ is replaced by $LS_{MGEV}(\gamma)$ in (54).

v) The extension to the case of multiple groups is analogous to the analyses in the
mixed logit model.

The detailed analyses are deferred to Appendix 8.

6 Concluding Remarks

In this paper, we formulate a structure of utility maximization problems that are consistent with demand functions derived from the GEV and its mixed models, and clarify the characteristics regarding the form of the utility function, elasticities, and measurement of welfare. Before concluding our analysis, we comment on two issues from the viewpoint of a practical application.

First, in order to be consistent with our analysis, the utility from a good should not depend on income and be linear with respect to its price. Accordingly, the choice probability also does not depend on income. This result is a consequence of our assumption of a Gorman-form indirect utility function. In empirical research, it would be possible that the utility from a good is assumed to be dependent on income or nonlinear with respect to price. If such a setup is adopted, however, the corresponding indirect utility function is not consistent with the Gorman form, in which case applied welfare analysis becomes difficult.5

Second, Berry (1994), Berry et al. (1995), Berry et al. (1999), and Nevo (2000, 2001) apply the mixed logit model to empirical research, using market data only. Their methods have a significant advantage in that there is no need to collect individual data, but they cannot be fully consistent with utility maximization because it needs a priori controlled total. As a result, derived own and cross-price elasticities would be

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5 Pakes et al. (1993), Berry et al. (1999), and Petrin (2002) calculate a change in welfare, assuming that the utility from a good is nonlinear with respect to income. However, they do not clarify how to overcome the aggregation problem with the setup of their utility function, which is inconsistent with the Gorman-form indirect utility function.
biased. Our formulation can avoid these issues, but it is more costly when applied empirically, because we need micro data on each consumer’s behavior.
Appendix 1 Proof of Proposition 1

We prove the necessary condition first. Define \( R(p_1, \ldots, p_M, y) \) by:

\[
R(p_1, \ldots, p_M, y) = R\left(-\ln \sum_{j=1}^{M} \exp(u_j(p_j, y))\right),
\]

which is a monotonic transformation of \(-\ln \sum_{j=1}^{M} \exp(u_j(p_j, y))\). Solving (A1) with respect to \( p_i \) and substituting it for \( p_i \) in (3), we can rewrite (3) as:

\[
V = \sum_{n} A(p_1(R, p_2, \ldots, p_M, y), p_2, \ldots, p_M, n) + B(p_1(R, p_2, \ldots, p_M, y), p_2, \ldots, p_M)Y.
\]

To satisfy the condition that consumer \( n \)'s indirect utility function has the Gorman form, \( A(p_1(R, p_2, \ldots, p_M, y), p_2, \ldots, p_M, n) \) and \( B(p_1(R, p_2, \ldots, p_M, y), p_2, \ldots, p_M) \) must be independent of \( y \). Thus,

\[
V = \sum_{n} A(p_1(R, p_2, \ldots, p_M), p_2, \ldots, p_M, n) + B(p_1(R, p_2, \ldots, p_M), p_2, \ldots, p_M)Y
= \sum_{n} A(R(p_1, \ldots, p_M), p_2, \ldots, p_M, n) + B(R(p_1, \ldots, p_M), p_2, \ldots, p_M)Y,
\]

where:

\[
R(p_1, \ldots, p_M) = R\left(-\ln \sum_{j=1}^{M} \exp(u_j(p_j))\right).
\]

Applying Roy's Identity to (A2), the market demand function for good 1 is derived as:

\[
\left(\sum_{n} \frac{\partial A}{\partial R} + \frac{\partial B}{\partial R} \right) R' u'_1(p_1) \left(-\frac{\exp(u_1(p_1))}{\sum_{j=1}^{M} \exp(u_j(p_j))}\right)
\]

\[
X_1(p_1, \ldots, p_M, Y) = \frac{-\sum_{j=1}^{M} \exp(u_j(p_j))}{B}.
\]

Comparing (6) and (A5) when \( m = 1 \), we obtain:
\begin{align}
C(p_1, \ldots, p_M, Y) &= \frac{\left(\sum_n \frac{\partial A}{\partial R} + \frac{\partial B}{\partial R} Y\right) R'u'_i(p_i)}{B} > 0.
\end{align}

In the same way, the market demand function for good \( l (l = 2, \ldots, m) \) is derived as:

\begin{align}
X_i(p_1, \ldots, p_M, Y) &= -\frac{\left(\sum_n \frac{\partial A}{\partial R} + \frac{\partial B}{\partial R} Y\right) R'u'_i(p_i) \left(-\frac{\exp(u_i(p_i))}{\sum_{j=1}^M \exp(u_j(p_j))}\right) + \sum_n \frac{\partial A}{\partial p_i} + \frac{\partial B}{\partial p_i} Y}{B}.
\end{align}

Comparing (6) and (A7) when \( m = l \), we obtain:

\begin{align}
C(p_1, \ldots, p_M, Y) &= \frac{\left(\sum_n \frac{\partial A}{\partial R} + \frac{\partial B}{\partial R} Y\right) R'u'_i(p_i)}{B} > 0 \quad \text{and}
\end{align}

\begin{align}
\frac{\partial A}{\partial p_i} + \frac{\partial B}{\partial p_i} Y = 0.
\end{align}

Because (A9) holds for any \( Y \), we obtain:

\begin{align}
\sum_n \frac{\partial A}{\partial p_i} = \frac{\partial B}{\partial p_i} = 0.
\end{align}

The above analysis holds when we solve (A1) with respect to any \( p_i \) and substitute it for any \( p_i \) in (3). Thus, including all the cases, we can write the indirect utility function of the representative consumer as:

\begin{align}
V = \sum_n A(R, p_1, \ldots, p_M, n) + B(R) Y,
\end{align}

where, from (A10):

\begin{align}
\frac{\partial \left(\sum_n A(P, p_1, \ldots, p_M, n)\right)}{\partial p_m} = 0 \quad \text{for any} \quad p_m.
\end{align}
Because \( C(p_1, \ldots, p_M, Y) \) is the same for any \( m \), from (A6) and (A8) we obtain:

\[ (A13) \quad u_i'(p_i) = \cdots = u_m'(p_m) = \cdots = u_M'(p_M), \]

which implies that \( u_m(p_m) \) is linear in \( p_m \). Thus, we can rewrite \( R \) as:

\[ (A14) \quad R = R\left(-\ln \sum_{j=1}^{M} \exp(\alpha_j - \beta p_j)\right). \]

\( P(\text{LS}) \) is derived by defining:

\[ (A15) \quad R(\beta \text{LS}) = P(\text{LS}). \]

Because \( C(p_1, \ldots, p_M) > 0 \) and

\[ (A16) \quad u_i'(p_i) = \cdots = u_m'(p_m) = \cdots = u_M'(p_M) = -\beta < 0, \]

we obtain:

\[ (A17) \quad \left( \sum_n \frac{\partial A}{\partial P} + \frac{\partial B}{\partial P} Y \right) \frac{dP}{d\text{LS}} = \frac{\partial V}{\partial \text{LS}} < 0. \]

The proof of sufficiency is straightforward from Roy’s Identity. Applying Roy’s Identity to (7), we obtain:

\[ (A18) \quad X_m = \frac{-\frac{\partial V}{\partial m}}{\frac{\partial V}{\partial Y}} = -\frac{\left( \sum_n \frac{\partial A}{\partial P} + \frac{\partial B}{\partial P} Y \right) \frac{dP}{d\text{LS}} \sum_{j=1}^{M} \exp(\alpha_j - \beta p_j)}{B}, \]

where:

\[ (A19) \quad C(p_1, \ldots, p_M, Y) = C(P(\text{LS}), Y) = \frac{-\left( \sum_n \frac{\partial A}{\partial P} + \frac{\partial B}{\partial P} Y \right) \frac{dP}{d\text{LS}}}{B} > 0. \]

The right-hand side of (A19) is positive from (9).
Appendix 2 Proof of Corollary 1

Maximizing (17) subject to the budget constraint:

\[(A20)\] \[Y = X_0 + \sum_{j=1}^{M} p_j X_j,\]

we obtain the logit-type demand functions:

\[(A21)\] \[X_m = \frac{\beta}{b} h^{-1}(Q) \frac{\exp(\alpha_m - \beta p_m)}{\sum_{j=1}^{M} \exp(\alpha_j - \beta p_j)},\]

where \( Q \) is a special form of \( P \):

\[(A22)\] \[Q = -\frac{1}{b} \ln \sum_{j=1}^{M} \exp(\alpha_j - \beta p_j) = \frac{b}{\beta} L S.\]

Since (17) has a quasi-linear form, (A21) coincides with the Hicksian demand function.

\( h' > 0 \) and \( h'' < 0 \) are sufficient conditions for \( \frac{\partial X_m}{\partial p_m} < 0 \) where:

\[(A23)\] \[\frac{\partial X_m}{\partial p_m} = -b^2 \exp(\alpha_m - \beta p_m) \left\{ bh^{-1}(Q) \sum_{j=m}^{M} \exp(\alpha_j - \beta p_j) - h''^{-1}(Q) \exp(\alpha_m - \beta p_m) \right\} \left( b \sum_{j=1}^{M} \exp(\alpha_j - \beta p_j) \right)^2.\]

Substituting (A21) into (17), a representative consumer’s indirect utility function is derived as a function of \( Q \) and \( Y \):

\[(A24)\] \[V = h(h^{-1}(Q)) - Q h^{-1}(Q) + Y,\]

which satisfies (9) from \( \frac{\partial V}{\partial L S} = \frac{\partial V}{\partial Q} \frac{\partial Q}{L S} = -\frac{b}{\beta} h^{-1}(Q) < 0 \) and (10).

A representative consumer’s indirect utility function (A24) is a special case of (7),
where \( \sum_n A = h(h^{-1}(Q)) - Q h^{-1}(Q) \) and \( B = 1 \).

**Appendix 3 Proof of Proposition 3**

From (7), consumer \( n \)'s and a representative consumer’s expenditure functions are respectively derived as:

\[
(A25) \quad e(n) = \frac{v(n) - A(P, p_1, \ldots, p_M, n)}{B(P)} \quad \text{and} \quad
\]

\[
(A26) \quad E = \frac{V - \sum_n A(P, p_1, \ldots, p_M, n)}{B(P)}.
\]

From (A25) and (A26), we obtain the Hicksian demand function of consumer \( n \) for good \( m \) and the Hicksian market demand function for good \( m \), as:

\[
(A27) \quad h_m(P, p_1, \ldots, p_M, v(n)) = \frac{\partial e(n)}{\partial p_m} = - \left( \frac{\partial A}{\partial p_m} + \frac{\partial A}{\partial p_m} \right) B - \left( v(n) - A \right) B' \frac{\partial p}{\partial p_m} \quad \text{and} \quad
\]

\[
(A28) \quad H_m(P, p_1, \ldots, p_M, V) = \frac{\partial E}{\partial p_m} = - \left( \sum_n \left( \frac{\partial A}{\partial p_m} + \frac{\partial A}{\partial p_m} \right) \right) B - \left( V - \sum_n A \right) B' \frac{\partial p}{\partial p_m}
\]

\[
= \sum_n h_m(P, p_1, \ldots, p_M, v(n)),
\]

where (10) is used in the transformation from the first to the second line.

Denoting consumer \( n \)'s equivalent variation by \( ev(n) \), from (A25)--(A28), we derive:
\[ EV = \sum_{n} ev(n) \]
\[ = \sum_{n} \int_{p_{m}^{n}}^{p_{m}^{W}} h_{m}(P, p_{1}, \ldots, p_{M}, v(n)^{W}) dp_{m} \]
\[ = \int_{p_{m}^{W}}^{p_{m}^{W}} \sum_{n} h_{m}(P, p_{1}, \ldots, p_{M}, v(n)^{W}) dp_{m} \]
\[ = \int_{p_{m}^{W}}^{p_{m}^{W}} H_{m}(P, p_{1}, \ldots, p_{M}, V^{W}) dp_{m}. \]

Substituting (A28) into (A29) and rearranging, we obtain:

\[ EV = \int_{p_{m}^{W}}^{p_{m}^{W}} \sum_{n} h_{m}(P, p_{1}, \ldots, p_{M}, v(n)^{W}) dp_{m} \]
\[ = \int_{p_{m}^{W}}^{p_{m}^{W}} -\left( \sum_{n} \left( \frac{\partial A}{\partial P} \frac{\partial P}{\partial p_{m}} \right) B - \left( V - \sum_{n} A \right) B' \frac{\partial P}{\partial p_{m}} \right) dp_{m} \]
\[ = \int_{p_{m}^{W}}^{p_{m}^{W}} -\left( \frac{\partial A}{\partial P} \right) B - \left( v(n) - A \right) B' \frac{\partial P}{\partial LS} dp_{m} \]
\[ = \int_{LS^{W}}^{LS^{W}} c(P(LS), p_{1}, \ldots, p_{M}, v(n)^{W}) dLS \]
\[ = \int_{LS^{W}}^{LS^{W}} C(P(LS), V^{W}) dLS, \]

where:

\[ c(P(LS), p_{1}, \ldots, p_{M}, v(n)^{W}) = \frac{-\left( \frac{\partial A}{\partial P} \right) B - \left( v(n) - A \right) B'}{(B)^{2}} \frac{dP}{dLS} \]
and

\[ C(P, V) = \frac{-\left( \sum_{n} \frac{\partial A}{\partial P} \right) B - \left( V - \sum_{n} A \right) B'}{(B)^{2}} \frac{dP}{dLS}. \]
Appendix 4 Analyses for the GEV model

Suppose that the market demand function of good \( m \) is consistent with the GEV model:

\[
(A33) \quad X_m(p_1, ..., p_M, Y) = C(p_1, ..., p_M, Y)s_{GEV_m}.
\]

The necessary and sufficient condition for the market demand function of good \( m \) to have the form of (A33) is that the indirect utility function of a representative consumer is:

\[
(A34) \quad V = \sum_n A(P(LS_{GEV}), p_1, ..., p_M, n) + B(P(LS_{GEV}))Y,
\]

which satisfies:

i) \( P(LS_{GEV}) \) is a monotonic transformation of:

\[
(A35) \quad LS_{GEV} = -\frac{1}{n\beta} \ln H(\exp(\alpha_1 - \beta p_1), ..., \exp(\alpha_M - \beta p_M)),
\]

ii) \( (A36) \quad \left( \sum_n \frac{\partial A}{\partial P} + \frac{\partial B}{\partial P} Y \right) \frac{dP}{dLS_{GEV}} = \frac{\partial V}{\partial LS_{GEV}} < 0, \text{ and}
\]

\[
(A37) \quad \frac{\partial}{\partial p_m} \left( \sum_n A(P(LS_{GEV}), p_1, ..., p_M, n) \right) = 0 \text{ for any } p_m.
\]

The market demand function of good \( m \), (A33), satisfies:

\[
(A38) \quad C(p_1, ..., p_M, Y) = C(P(LS_{GEV}), Y) = -\left( \sum_n \frac{\partial A}{\partial P} + \frac{\partial B}{\partial P} Y \right) B \frac{dP}{dLS_{GEV}} > 0 \text{ and}
\]

\[
(A39) \quad s_{GEV_m} = \frac{\partial H}{\partial z_m} \frac{\exp(\alpha_m - \beta p_m)}{nH} = \frac{\partial LS_{GEV}}{\partial p_m}.
\]

The proof of the sufficient and necessary conditions is the same as that in Appendix 1,
and is omitted here.

The own price elasticity of the market demand for good $m$, $X_m$, is:

\[(A40)\]
\[
\frac{\partial X_m}{\partial p_m} p_m = \theta_m - \beta(1 - n s_{GEV m}) p_m + \eta_{m m},
\]

where $\eta_{m m} = \frac{\partial \left( \frac{\partial H}{\partial z_m} \right)}{\partial p_m} p_m$ is the elasticity of $\frac{\partial H}{\partial z_m}$ with respect to the price of good $m$, $p_m$. The cross-price elasticity is:

\[(A41)\]
\[
\frac{\partial X_m}{\partial p_m} p_m = \theta_m + \beta n s_{m'} p_{m'} + \eta_{m m'},
\]

where $\eta_{m m'} = \frac{\partial \left( \frac{\partial H}{\partial z_m} \right)}{\partial p_m} p_m$ is the elasticity of $\frac{\partial H}{\partial z_m}$ with respect to the price of good $m'$, $p_{m'}$.

The results of (A40) and (A41) demonstrates that the elasticities of total market demand are added to the ordinary elasticities of the GEV model. The result of the logit model, (18) and (19), are special cases of (A40) and (A41): (18) and (19) are derived when $n = 1$ and $\frac{\partial H}{\partial z_m} = 1$, which implies $\eta_{m m} = \eta_{m m'} = 0$. From (A41), we also know that the GEV model has no IIA property as long as $\eta_{m m'}$ is nonzero. The proof regarding elasticities is omitted because it immediately follows from the market demand function of good $m$, (A33).

Regarding equivalent variation, (20) holds if (A35) is substituted for (8). The proof is the same as proposition 3, noting that (A39) holds instead of (12).
Our analysis of the GEV model can be readily extended to a case where the goods are classified into multiple groups. The analyses are basically the same as those in section 2-4. Thus, we only show the difference from there.

In the GEV model, the log-sum term for group \( g \) is modified to:

\[
(A42) \quad \frac{1}{n \beta} \ln H_g (\ldots, \exp(\alpha_j - \beta_g p_j), \ldots) \quad \text{for} \quad j \in g,
\]

where \( H_g \) is a function that satisfies (GEV-1) to (GEV-4). The choice probability of good \( m \) within group \( g \) is also changed to:

\[
(A43) \quad \frac{\partial L_{GEVg}}{\partial p_m} \exp(\alpha_m - \beta_g p_m) \quad \text{for} \quad \text{good } m \quad \text{within group } g.
\]

The own price elasticity of the market demand for good, \( m, X_m \), is:

\[
(A44) \quad \frac{\partial X_m}{\partial p_m} \frac{p_m}{X_m} = \theta_m \beta_g (1 - n_{GEVgm}) p_m + \eta_{gam},
\]

where \( \eta_{gam} = \frac{\partial H_g}{\partial p_m} \frac{p_m}{\partial z_m} \) is defined in the same manner as \( \eta_{gam'} \). The cross-price elasticity is:

\[
(A45) \quad \frac{\partial X_m}{\partial p_m} \frac{p_m'}{X_m} = \theta_m + \beta_g n_{GEVgm'} p_m' + \eta_{gam'},
\]

where \( \eta_{gam'} = \frac{\partial H_g}{\partial p_m} \frac{p_m'}{\partial z_m} \), if goods \( m' \) and \( m \) belong to the same group, and

\[
(A46) \quad \frac{\partial X_m}{\partial p_m} \frac{p_m'}{X_m} = \theta_m.
\]
if they belong to different groups.

With respect to welfare analysis, proposition 6 holds if we modify the log-sum term from (23) to (A42).

Appendix 5 Proof of Proposition 7

We prove the necessary condition first. Define $R(p_1, \ldots, p_M, y(\gamma), \gamma)$ by:

$$R(p_1, \ldots, p_M, y(\gamma), \gamma) = R\left(-\ln \sum_{j=1}^{M} \exp(u_j(p_j, y(\gamma), \gamma))\right).$$

which is a monotonic transformation of $-\ln \sum_{j=1}^{M} \exp(u_j(p_j, y(\gamma), \gamma))$. Solving (A47) with respect to $p_1$ and substituting it for $p_1$ in $A(p_1, \ldots, p_M, \gamma)$ in (44), we obtain:

$$V = N \int_{\gamma} A(p_1(R, p_2, \ldots, p_M, y(\gamma), \gamma), p_2, \ldots, p_M, \gamma) f(\gamma)d\gamma + B(p_1, \ldots, p_M)Y.$$

To satisfy the condition that the indirect utility function of the $\gamma$-type consumer has the Gorman form, $A(p_1(R, p_2, \ldots, p_M, y(\gamma), \gamma), p_2, \ldots, p_M, \gamma)$ must be independent of $y$.

Thus,

$$V = N \int_{\gamma} A(p_1(R, p_2, \ldots, p_M, \gamma), p_2, \ldots, p_M, \gamma) f(\gamma)d\gamma + B(p_1, \ldots, p_M)Y$$

$$= N \int_{\gamma} A(R(p_1, \ldots, p_M, \gamma), p_2, \ldots, p_M, \gamma) f(\gamma)d\gamma + B(p_1, \ldots, p_M)Y,$$

where:

$$R(p_1, \ldots, p_M, \gamma) = R\left(-\ln \sum_{j=1}^{M} \exp(u_j(p_j, \gamma))\right).$$

Applying Roy’s Identity to (A46), the market demand function for good 1 is derived as:


\[ X_i(p_1,\ldots,p_M,Y) = -\frac{N\int_{\gamma} \left( \frac{\partial A}{\partial R} R' u'_i(p_i) \left( -\frac{\exp(u_i(p_i))}{\sum_{j=1}^{M} \exp(u_j(p_j))} \right) f(\gamma) d\gamma + \frac{\partial B}{\partial p_i} Y \right)}{B}. \]

Comparing (42) and (A51) when \( m = 1 \), we obtain:

\[ c(p_1,\ldots,p_M,y(\gamma),\gamma) = \frac{\partial A}{\partial R} R' u'_i(p_i) > 0 \text{ and } \frac{\partial B}{\partial p_i} = 0. \]

In the same way, the market demand function for good \( l (l = 2,\ldots,m) \) is derived as:

\[ X_l(p_1,\ldots,p_M,Y) = -\frac{N\int_{\gamma} \left( \frac{\partial A}{\partial R} R' u'_l(p_l) \left( -\frac{\exp(u_l(p_l))}{\sum_{j=1}^{M} \exp(u_j(p_j))} \right) f(\gamma) d\gamma + \frac{\partial B}{\partial p_l} Y \right)}{B}. \]

Comparing (42) and (A54) when \( m = l \), we obtain:

\[ c(p_1,\ldots,p_M,y(\gamma),\gamma) = \frac{\partial A}{\partial R} R' u'_l(p_l) > 0 \text{ and } \frac{\partial B}{\partial p_l} = 0. \]

Because (A56) holds for any \( Y \), we obtain:

\[ \int_{\gamma} \frac{\partial A}{\partial p_l} f(\gamma) d\gamma = 0 \text{ and } \frac{\partial B}{\partial p_l} = 0. \]
From (A53) and (A58), we know that $B$ is a fixed constant, which we denote by $\overline{B}$.

The above analysis holds when we solve (A47) with respect to any $p_t$ and substitute it for any $p_t$ in (44). Thus, including all cases, we can write the indirect utility function of the representative consumer as:

\[(A59)\quad V = N\int_\gamma A(R, p_1, \ldots, p_M, \gamma) f(\gamma) d\gamma + \overline{B}Y,\]

where, from (A57):

\[(A60)\quad \frac{\partial}{\partial p_m} \left( \int A(P(\text{LS}_M(\gamma)), p_1, \ldots, p_M, \gamma) f(\gamma) d\gamma \right) = 0 \quad \text{for any } p_m.

Because $\text{c}(p_1, \ldots, p_M, \gamma(\gamma), \gamma)$ is the same for any $m$, from (A52) and (A55) we obtain:

\[(A61)\quad u_1'(p_1) = \ldots = u_m'(p_m) = \ldots = u_M'(p_M),\]

which implies that $u_m(p_m)$ is linear in $p_m$. Thus, we can rewrite $R$ as:

\[(A62)\quad R = R\left( -\ln \sum_{j=1}^M \exp(\alpha_j - \beta(\gamma)p_j) \right),\]

under the assumption that $\gamma$ affects the price coefficient. $P(\text{LS}_M(\gamma))$ is derived by defining:

\[(A63)\quad R(\beta(\gamma)\text{LS}_M(\gamma)) = P(\text{LS}_M(\gamma)).\]

Because $\text{c}(p_1, \ldots, p_M, \gamma(\gamma), \gamma) > 0$ and

\[(A64)\quad u_1'(p_1) = \ldots = u_m'(p_m) = \ldots = u_M'(p_M) = -\beta(\gamma) < 0,\]

we obtain:
The proof of sufficiency is straightforward from Roy’s Identity. Applying Roy’s Identity to (45) and rearranging, we obtain:

\[
\left( A_{65} \right) \quad \frac{\partial A}{\partial P(LS_{ML}(\gamma))} \frac{dP(LS_{ML}(\gamma))}{dLS_{ML}(\gamma)} < 0.
\]

\[
\text{where:}
\]

\[
\left( A_{66} \right) \quad X_m = \frac{-\frac{\partial V}{\partial P_m}}{\frac{\partial V}{\partial Y}} = \frac{-\int_{\gamma} \left( \frac{\partial A}{\partial R} \exp(u_m(p_m)) \right) \sum_{j=1}^{M} \exp(u_j(p_j)) f(\gamma) d\gamma}{B}.
\]

The right-hand side of (A67) is positive from (47).

**Appendix 6 Proof of Proposition 9**

From (45), the \(\gamma\)-type and a representative consumer’s expenditure functions are respectively derived as:

\[
\left( A_{68} \right) \quad e(\gamma) = \frac{v(\gamma) - A(P(LS_{ML}(\gamma)), p_1, \ldots, p_M, \gamma)}{B} \quad \text{and}
\]

\[
\left( A_{69} \right) \quad E = \frac{V - N \int_{\gamma} A(P(LS_{ML}(\gamma)), p_1, \ldots, p_M, \gamma) f(\gamma) d\gamma}{B}.
\]

From (A68) and (A69), we obtain the Hicksian demand function of the \(\gamma\)-type consumer for good \(m\) and the Hicksian market demand function for good \(m\), as:
\[ \frac{\partial e(\gamma)}{\partial p_m} = -\frac{\partial A}{\partial P(LS_{ML}(\gamma))} \frac{dP(LS_{ML}(\gamma))}{dLS_{ML}(\gamma)} \frac{\partial LS_{ML}(\gamma)}{\partial p_m} \frac{\partial A}{\partial p_m} \] 

and

\[ H_m = \frac{\partial E}{\partial p_m} = -N \int_\gamma \left( \frac{\partial A}{\partial P(LS_{ML}(\gamma))} \frac{dP(LS_{ML}(\gamma))}{dLS_{ML}(\gamma)} \frac{\partial LS_{ML}(\gamma)}{\partial p_m} + \frac{\partial A}{\partial p_m} \right) f(\gamma) d\gamma \]

\[ = N \int_\gamma h_m f(\gamma) d\gamma, \] (A71)

where (48) is used in the transformation from the first to the second line. Denoting the \( \gamma \)-type consumer’s equivalent variation by \( ev(\gamma) \), from (A68)–(A71), we obtain:

\[ EV = N \int_\gamma ev(\gamma) f(\gamma) d\gamma \]

\[ = N \int_\gamma \int_{p_m^{eo}} h_m dp_m f(\gamma) d\gamma \]

\[ = \int_{p_m^{eo}} \left( N \int_\gamma h_m f(\gamma) d\gamma \right) dp_m \]

\[ = \int_{p_m^{eo}} H_m dp_m. \] (A72)

Substituting (A71) into (A72) and rearranging, we can verify a transformation from the second line to the third line on the right-hand side of (53):
Appendix 7 Multiple Groups in the Case of Mixed Logit Model

Suppose that the goods are classified into $G$ groups. Good $m$ is supposed to belong to group $g$ ($g = 1, \ldots, G$), without loss of generality. Suppose that the market demand function of good $m$ is consistent with the mixed logit model:

(A74) \[ X_m = \int \sum_{\gamma} c_g(p_1, \ldots, p_M, y(\gamma), \gamma) s_{ML, gm}(\gamma) f(\gamma) d\gamma, \]

where:

(A75) \[ s_{ML, gm}(\gamma) = \frac{\exp(u_m(p_m, y(\gamma), \gamma))}{\sum_{j \in g} \exp(u_j(p_j, y(\gamma), \gamma))} \]

$c_g(p_1, \ldots, p_M, y(\gamma), \gamma)$ is the $\gamma$-type consumer’s total demand for group $g$.

The necessary and sufficient condition for the market demand function of good $m$ to have the form of (A74) is that the indirect utility function of a representative consumer is:

(A76) \[ V = \int \sum_{\gamma} A(p_1, \ldots, p_G, p_1, \ldots, p_M, \gamma) f(\gamma) d\gamma + \bar{B}Y, \]

where $\bar{B}$ is a fixed constant, which satisfies:
i) $P_g(LS_{MLg}(\gamma))$ is a monotonic transformation of:

$$LS_{MLg}(\gamma) = -\frac{1}{\beta_g(\gamma)} \ln \sum_{j \in g} \exp(\alpha_j - \beta_g(\gamma)p_j),$$

ii) (A78) $\frac{\partial A}{\partial p_g(LS_{MLg}(\gamma))} \frac{dP_g(LS_{MLg}(\gamma))}{dLS_{MLg}(\gamma)} < 0$, and

$$\frac{\partial}{\partial p_m} \left( \int_{\gamma} A(P_1(LS_{ML1}(\gamma)),...,P_g(LS_{MLg}(\gamma)),p_1,...,p_M,\gamma)f(\gamma)d\gamma \right) = 0 \text{ for any } p_m.$$

The market demand function of good $m$, (A70), satisfies:

$$c_g(p_1,...,p_M,\gamma) = c_g(P_1,...,P_G,p_1,...,p_M,\gamma)$$

(A80) $\frac{\partial A}{\partial p_g(LS_{MLg}(\gamma))} \frac{dP_g(LS_{MLg}(\gamma))}{dLS_{MLg}(\gamma)} = \frac{\partial A}{\partial p_g(LS_{MLg}(\gamma))} \frac{dP_g(LS_{MLg}(\gamma))}{dLS_{MLg}(\gamma)} > 0$

(A81) $s_{MLgm}(\gamma) = \frac{\sum_{j \in g} \exp(\alpha_j - \beta_g(\gamma)p_j)}{\sum_{j \in g} \exp(\alpha_j - \beta_g(\gamma)p_j)} = \frac{\partial LS_{MLg}}{\partial p_m}.$

The proof of sufficiency and necessity is the same as that for proposition 7 in Appendix 5, and is omitted here.

The own price elasticity of the market demand for good $m$, $X_m$, is:

$$\frac{\partial X_m}{\partial p_m} \frac{p_m}{X_m} = N \int_{\gamma} \left\{ \frac{c_g(\gamma)s_{MLgm}(\gamma)}{X_m} \left( \phi_{gm}(\gamma) - \beta_g(\gamma)(1-s_{MLgm}(\gamma))p_m \right) \right\} f(\gamma)d\gamma,$$

where $\phi_{gm}(\gamma) = \frac{\partial c_g(\gamma)}{\partial p_m} \frac{p_m}{c_g(\gamma)}$ is the $\gamma$-type consumer’s elasticity of demand for group $g$, with respect to the price of good $m$, $p_m$. The cross-price elasticity is:
when goods $m'$ and $m$ belong to the same group $g$, and

$$ \frac{\partial X_m}{\partial p_m} \frac{P_{m'}}{X_m} = N \int_{\gamma} c_g(\gamma) s_{ML, gm}(\gamma) \left( \frac{\phi_{gm}(\gamma)}{X_m} + \beta_g(\gamma) s_{ML, gm}(\gamma) p_m \right) f(\gamma) d\gamma, $$

when they belong to different groups. The proof regarding elasticities is omitted because it immediately follows from the market demand function of good $m$, (A74). The results of (A83) and (A84) are modified versions of those of (51) and (52), taking into account the classification of the goods into groups.

Equivalent variation can be calculated from the demand of a consumer for a good, the market demand for a good, or the total demand of a consumer for a group, that is:

$$ \text{EV} = N \int_{\gamma} \left( \int_{p_m}^{p_{m'}} p_m h_m dp_m \right) f(\gamma) d\gamma $$

$$ = \int_{p_m}^{p_{m'}} \frac{H_m dp_m}{p_m} $$

$$ = N \int_{\gamma} \left( \int_{LS_{m, g}(\gamma)^{p_m}}^{LS_{m, g}(\gamma)^{p_{m'}}} c_g(P_1, ..., P_G, p_1, ..., p_M, \gamma) dLS_{ML, g}(\gamma) \right) f(\gamma) d\gamma $$

The proof is the same as that for proposition 9 in Appendix 6, and is omitted here.

**Appendix 8 Analyses for the Mixed GEV Model**

Suppose that the market demand function of good $m$ is consistent with the mixed GEV model:

$$ X_m = N \int_{\gamma} c(p_1, ..., p_M, \gamma, \gamma) s_{MGEV, m}(\gamma) f(\gamma) d\gamma, $$

where:
The necessary and sufficient condition for the market demand function of good \( m \) to have the form of (A86) is that the indirect utility function of a representative consumer is:

\[
\text{(A88)} \quad V = N \int_{\gamma} A(P(LS_{MGEV}(\gamma)), p_1, \ldots, p_m, \gamma) f(\gamma)d\gamma + BY,
\]

which satisfies:

i) \( LS_{MGEV}(\gamma) \) is a monotonic transformation of:

\[
\text{(A89)} \quad LS_{MGEV}(\gamma) = -\frac{1}{n\beta(\gamma)} \ln H(\exp(\alpha_1 - \beta(\gamma)p_1), \ldots, \exp(\alpha_m - \beta(\gamma)p_m)),
\]

ii) \( \frac{\partial A}{\partial P(LS_{MGEV}(\gamma))} \frac{dP(LS_{MGEV}(\gamma))}{dLS_{MGEV}(\gamma)} < 0 \), and

\[
\frac{\partial}{\partial p_m} \left( \int_{\gamma} A(P(LS_{MGEV}(\gamma)), p_1, \ldots, p_m, \gamma) f(\gamma)d\gamma \right) = 0 \quad \text{for any} \quad p_m.
\]

The market demand function of good \( m \), (A86), satisfies:

\[
\text{(A92)} \quad c(p, \ldots, p_M, \gamma) = -\frac{\partial A}{\partial P(LS_{MGEV}(\gamma))} \frac{dP(LS_{MGEV}(\gamma))}{dLS_{MGEV}(\gamma)} > 0 \quad \text{and}
\]

\[
\text{(A93)} \quad s_{MGEV_m}(\gamma) = \frac{\partial H}{\partial \gamma} \frac{\exp(\alpha_m - \beta(\gamma)p_m)}{nH} = \frac{\partial LS_{MGEV}(\gamma)}{\partial p_m}.
\]

The proof is basically the same as that developed in Appendix 5, and is omitted here.

The own price elasticity of the market demand for good \( m \), \( X_m \), is:

\[
\text{(A94)}
\]
The cross-price elasticity is:

\[ \begin{align*}
\frac{\partial X_m}{\partial p_m} \frac{p_m}{X_m} &= N \int_{\gamma} \left\{ c(\gamma) s_{MGEV_m}(\gamma) \left( \phi_m(\gamma) - \beta(\gamma)(1 - n s_{MGEV_m}(\gamma)) p_m + \eta_{mm}(\gamma) \right) \right\} f(\gamma) d\gamma.
\end{align*} \]

The results of (A94) and (A95) coincide with those of the GEV model, (A40) and (A41), respectively, if parameter \( \gamma \) is a fixed constant. Otherwise, the own and cross-price elasticities are mixed versions of those of the GEV model over a density of \( \gamma \). The proof regarding elasticities is omitted because it immediately follows from the market demand function of good \( m \), (A86).

Regarding equivalent variation, proposition 9 holds if (A89) is substituted for (46). The proof is the same as Appendix 6, noting that (A93) holds instead of (50).

The analyses for the case of multiple groups are basically the same as those in the case of the mixed logit model, developed in Appendix 7, and accordingly, we note points of difference only.

Suppose again that the goods are classified into \( G \) groups. (54) is modified to:

\[ \begin{align*}
LS_{MGEV_g}(\gamma) &= -\frac{1}{n \beta_g(\gamma)} \ln H_g(\ldots, \exp(\alpha_j - \beta_g(\gamma) p_j), \ldots) \quad \text{for } j \in g.
\end{align*} \]

(A93) is also changed to:

\[ \begin{align*}
\frac{\partial H_g}{\partial \zeta_m} \exp(\alpha_m - \beta_g(\gamma) p_m) &= \frac{\partial LS_{MGEV_g}(\gamma)}{\partial p_m}.
\end{align*} \]

The own price elasticity of the market demand for good, \( m \), \( X_m \), is:
The cross-price elasticity is:

(A99)

\[
\frac{\partial X_m}{\partial p_{m'}} \frac{p_{m'}}{X_m} = N \int_\gamma \left\{ \frac{c_\gamma(\gamma)s_{MGEV_{gm}}(\gamma)}{X_m} \left( \phi_{gm}(\gamma) + \beta_g(\gamma)n_{MGEV_{gm}}(\gamma)p_{m'} + \eta_{gm}(\gamma) \right) \right\} f(\gamma) \, d\gamma,
\]

if goods \( m' \) and \( m \) belong to the same group, and

(A100)

\[
\frac{\partial X_m}{\partial p_{m'}} \frac{p_{m'}}{X_m} = N \int_\gamma \left\{ \frac{c_\gamma(\gamma)s_{MGEV_{gm}}(\gamma)}{X_m} \phi_{gm}(\gamma) \right\} f(\gamma) \, d\gamma
\]

if they belong to different groups.

Regarding equivalent variation, (A85) holds if \( LS_{MGEV}(\gamma) \) is substituted for \( LS_{M_{LS}}(\gamma) \).
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