Final Exam

Date: March 21, 2008
Subject: Game Theory (ECO290E)
Professor: Yosuke YASUDA

1. **True or False (15 points, moderate)**

   Answer whether each of the following statements is true or false. You DON'T need to explain the reason.

   (a) Every subgame perfect Nash equilibrium is a Nash equilibrium.

   (b) In the Stackelberg game, the follower becomes better off than in the Cournot game, since she can move after observing the leader's strategy.

   (c) If a static game is played repeatedly, then some outcome other than static Nash equilibria can possibly be achieved as a subgame perfect Nash equilibrium.

2. **Game tree (20 points, easy)**

   See the game tree in Figure 1.

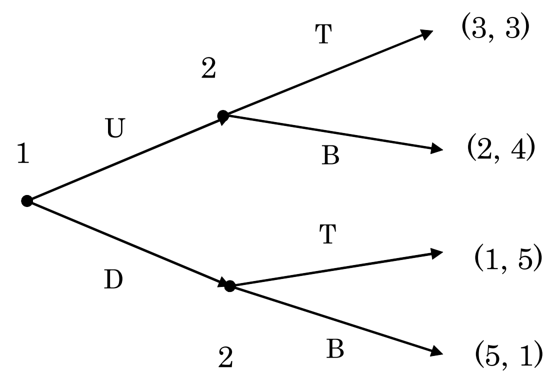

Figure 1

(a) Solve this game by backward induction.

(b) What are the strategies for each player in a normal-form?
Hint: A strategy is a "complete action plan."

(c) Draw the payoff matrix by using your answer in (b).

(d) Find all Nash equilibria.

3. **Subgame perfect Nash equilibrium (20 points, moderate)**

See the game tree in Figure 2. Note that the player 1's payoff for (U,A,X) is assumed to be $a \geq 0$.

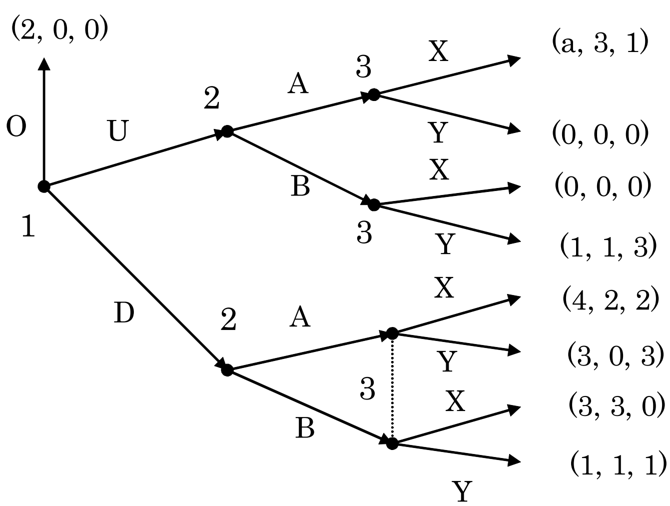

Figure 2

(a) How many subgames are there (including the entire game)?

(b) Does the subgame perfect Nash equilibrium outcome depend on a? If so, explain how the value a changes the outcome of the game. If not, specify the unique equilibrium outcome.

4. **Repeated games (20 points, tricky)**

Consider the following stage game.

1\2	X	Y
A	$5, 6$	$0, 0$
B	$8, 2$	$2, 2$

(a) Find all of the pure-strategy Nash equilibria of this game.

(b) Consider the two-period repeated game in which this stage game is played twice. Suppose the repeated game payoffs are simply the sum of the payoffs in each of the two periods. Then, is there a subgame perfect Nash equilibrium of this repeated game in which (A,X) is played in the first period? If so, fully describe the equilibrium. If not, explain why.

5. **Auction (15 points, moderate)**

Consider the following sealed bid auction, called a second-price auction. There are two bidders 1 and 2, and players simultaneously and independently submit buds b_1 and b_2 (given their valuations v_1 and v_2 that are private information for each bidder). The object is awarded to the highest bidder at a price equal to the *second*-highest bid. For example, if player 1 bids 200 and player 2 bids 150, then player 1 gets the object and is required to pay 150. In this outcome, player 1's payoff would be $v_1 - 150$ and player 2's payoff would be 0 (since player 2 neither gets the object nor has to pay anything). Now, prove it is a weakly dominant strategy for each bidder to truthfully bid her own valuation, i.e., $b_i = v_i, i = 1, 2$.

Hint: A weakly dominant strategy is a strategy that becomes a weakly best response to every strategy of the other player.

6. **Signaling game (30 points, challenging)**

Consider a game between two friends, Amy and Brenda. Amy wants Brenda to give her a ride to the mall. Brenda has no interest in going to the mall unless her favorite shoes are on sale (S) at the large department store there. Amy likes these shoes as well, but she wants to go to the mall even if the shoes are not on sale (N). Only Amy subscribes to the newspaper, which carries a daily advertisement of the department store. The advertisement lists all items that are on sale, so Amy learns whether or not the shoes are on sale. Amy can prove whether or not the shoes are on sale by showing the newspaper to Brenda. But this is costly for Amy, because she will have to take the newspaper away from her sister, who will yell her later for doing so.

In this game, nature first decides whether or not the shoes are on sale, and this information is made known to Amy. That is, Amy observes whether nature chose S or N. Nature chooses S with probability p and N with probability $1-p$. Then Amy decides whether or not to take the newspaper to Brenda (T or D). If she takes the newspaper to Brenda, then it reveals to Brenda whether the shoes are on sale. In any case, Brenda must then decide whether to take Amy to the mall (Y) or to forget it (F). If the shoes are on sale, then going to the mall is worth 1 unit of utility to Brenda and 3 to Amy. If the shoes are not on sale, then traveling to the mall is worth 1 to Amy and -1 to Brenda. Both players obtain 0 utility when they do not go to the mall. Amy's personal cost of taking the newspaper to Brenda is 2 units of utility, which is subtracted from her other utility amounts.

(a) Draw the game tree of this game.

Hint: The tree looks quite similar to what we saw in the practice exam (question 6: signaling game), but there is an important difference.

(b) Does this game have a *separating* perfect Bayesian Nash equilibrium? If so, fully describe it.

(c) Does this game have a *pooling* perfect Bayesian Nash equilibrium? If so, fully describe it.

Hint: A separating equilibrium means that Amy takes different strategies in S and N, while she chooses the same strategy in a pooling equilibrium. Your answer in (c) might depend on the value p.