

# Midterm Exam

Date: February 22, 2008

Subject: Game Theory (ECO290E)

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## 1. Nash equilibrium (10 points, easy)

State the definition of Nash equilibrium (either by words or mathematically) within THREE lines.

## 2. True or False (15 points, moderate)

Answer whether each of the following statements is true or false. You DON'T need to explain the reason.

- a) If a game has finite number of players and strategies, there always exists at least one dominant strategy for some player.
- b) When there are multiple Nash equilibria, one equilibrium can be more efficient than the other.
- c) If a strategy is played in some Nash equilibrium, this strategy will not be eliminated by the iterated elimination of strictly dominated strategies.

## 3. Nash equilibrium and dominated strategies (25 points)

### Part 1

Consider the following static game.

|   | X   | Y   | Z   |
|---|-----|-----|-----|
| A | 3,3 | 0,5 | 0,4 |
| B | 0,0 | 3,1 | 1,2 |

- a) Find the pure-strategy Nash equilibrium in this game. (5 points, easy)
- b) Solve this game by the iterated elimination of strictly dominated strategies. Does the resulting combination of strategies coincide with your answer in (a)? (10 points, moderate)

## Part 2

Players 1 and 2 are bargaining over how to split 100 yen. Both players simultaneously name shares they would like to have,  $x$  and  $y$ . Assume  $x$  and  $y$  can take any integer numbers between 0 and 100, i.e.,  $\{0,1,2,\dots,100\}$ . If  $x+y \leq 100$ , then the players receive the shares they named; if  $x+y > 100$ , then both receive zero.

c) What are the pure-strategy Nash equilibria of this game? How many equilibria are there exist? (10 points, challenging)

## **4. Mixed strategy equilibrium (25 points)**

### Part 1

The following payoff matrix shows the game of “Rock-Paper-Scissors.” (called “JANKEN” in Japanese)

|          | Rock | Paper | Scissors |
|----------|------|-------|----------|
| Rock     | 0,0  | -1,1  | 1,-1     |
| Paper    | 1,-1 | 0,0   | -1,1     |
| Scissors | -1,1 | 1,-1  | 0,0      |

This is a traditional zero-sum game whose rule can be explained as follows: Two players simultaneously chose one action from {Rock, Paper and Scissors}. Rock wins Scissors, Scissors wins Paper, and Paper wins Rock. A winner gets the payoff of 1 while a loser receives -1. If the players choose the same action, both receive 0.

a) Verify that the following mixed-strategy constitutes a Nash equilibrium: both players randomly choose each strategy with equal probability ( $=1/3$  each). (5 points, moderate)

b) Suppose the payoffs are slightly changed in the following way, i.e., when a player wins with/loses against Rock, her payoff becomes double. Compute the mixed-strategy equilibrium in this modified game. (10 points, challenging)

|          |      |       |          |
|----------|------|-------|----------|
|          | Rock | Paper | Scissors |
| Rock     | 0,0  | -1,1  | 2,-2     |
| Paper    | 1,-1 | 0,0   | -1,1     |
| Scissors | -2,2 | 1,-1  | 0,0      |

**Part 2**

- c) Find all the pure-strategy equilibria of the following game. (5 points, easy)  
d) Compute the mixed-strategy equilibrium. (5 points, moderate)

|   |     |     |
|---|-----|-----|
|   | X   | Y   |
| A | 2,4 | 0,0 |
| B | 1,6 | 3,7 |

**5. Focal point (5 points, bonus)**

Choose one course in the winter term, and write down the name. (You must not choose more than one courses.) If the course you choose coincides with the most popular answer, you would get 5 points. Otherwise, you would get 0 point. You DON'T need to explain the reason why you choose the course.

*Hint: It is important for you to guess which course other students are likely to choose.*