

Solution Keys to the Midterm Exam (Game Theory, Winter 2008)

1. Nash Equilibrium (10 points)

A strategy profile s^* is a Nash equilibrium if

$$u_i(s^*) \geq u_i(s_{-i}^*, s_i) \text{ for all } i \text{ and } s_i.$$

2. True or False (15 points)

(a) False (b) True (c) True

3. Nash equilibrium and dominated strategies (25 points)

(a) (B, Z) is the unique Nash equilibrium.

(b) We will get (B, Z) in the following iterated elimination process:

Step 1: We can erase X since X is strictly dominated by Z .

Step 2: Given step 1, we can erase A since A is strictly dominated by B .

Step 3: Given steps 1 and 2, we can erase Y since Y is strictly dominated by Z .

(c) Any combinations of x and y that satisfy $x + y = 100$ are Nash equilibria. Clearly, there are 101 such equilibria, i.e., $(0, 100)(1, 99)\dots(100, 0)$.

4. Mixed strategy equilibrium (25 points)

(a) Given that your opponent chooses actions completely randomly, your expected payoff becomes 0 no matter how you play. Therefore, choosing each action with probability $1/3$ is indeed a (weakly) best response. Thus, when both players choose actions completely randomly, their strategies are mutually best response to each other, and hence constitute a Nash equilibrium.

(b) Let p be a probability that player 2 would choose Rock, and q be a probability that she chooses Paper. Note that her probability of choosing Scissors is written as $1 - p - q$. Under mixed strategy Nash equilibrium, player 1 must be indifferent amongst choosing Rock, Paper and Scissors, which implies that these three actions must give him the same expected payoffs. Let u_R, u_P, u_S be his expected payoffs by selecting Rock, Paper and Scissor respectively. Then, they can be calculated as follows:

$$\begin{aligned} u_R &= -q + 2\{1 - p - q\} \\ u_P &= p - \{1 - p - q\} \\ u_S &= -2p + q \end{aligned}$$

Since $u_R = u_P$, we obtain

$$\begin{aligned} -q + 2\{1 - p - q\} &= p - \{1 - p - q\} \\ \Leftrightarrow & 3\{1 - (p + q)\} = p + q \\ \Leftrightarrow & p + q = 3/4. \end{aligned}$$

Since $u_P = u_S$, we obtain

$$\begin{aligned} p - \{1 - p - q\} &= -2p + q \\ \Leftrightarrow & 4p = 1 \\ \Leftrightarrow & p = 1/4. \end{aligned}$$

Substituting into $p + q = 3/4$, we achieve $q = 1/2$. Since the game is symmetric, we can derive exactly the same result for Player 1's mixed action as well. Therefore, we get the mixed-strategy Nash equilibrium: both players choose Rock, Paper and Scissors with probabilities $1/4, 1/2, 1/4$ respectively.

(c) There are two pure-strategy Nash equilibria: (A, X) and (B, Y) .

(d) Let p be a probability that player 2 chooses X and q be a probability that player 1 chooses A . Since player 1 must be indifferent amongst choosing A and B , we obtain

$$\begin{aligned} 2p &= p + 3(1 - p) \\ \Leftrightarrow & 4p = 3 \Leftrightarrow p = 3/4. \end{aligned}$$

Similarly, player 2 must be indifferent amongst choosing X and Y , which implies

$$\begin{aligned} 4q + 6(1 - q) &= 7(1 - q) \\ \Leftrightarrow & 5q = 1 \Leftrightarrow q = 1/5. \end{aligned}$$

Thus, the mixed-strategy equilibrium is that player 1 takes A with probability $1/5$ (and B with probability $4/5$) and player 2 takes X with probability $3/4$ (and Y with probability $1/4$).

5. Focal point (5 points)

There is no fixed answer of this question. But, you would be likely receive 5 points if you chose "Game Theory."