Practice Exam

1. Game tree (20 points)

See the game tree in Figure 1.

(a) Solve this game by backward induction.
(b) Translate this game into strategic-form (normal-form) and draw the payoff matrix.
(c) Find all Nash equilibria.

2. Subgame perfect Nash equilibrium (20 points)

See the game tree in Figure 2.
(a) How many information sets are there (including the one with the initial node)?

(b) How many subgames are there (including the entire game)?

(c) Find all subgame perfect equilibria.

3. Alternative bargaining (20 points)

Players 1 and 2 are bargaining over how to split the ice-cream of size 1. In the first stage, player 1 proposes a share \((x, 1 - x)\) to player 2 where \(0 \leq x \leq 1\). If player 2 accepts the player 1’s offer, then the game finishes and the players get their shares. If player 2 rejects, the game moves to the second stage, in which the size of the ice-cream becomes half due to melting. In the second stage, player 2 proposes a share \((y, 1/2 - y)\) to player 1 where \(0 \leq y \leq 1/2\). If player 1 accepts the offer, then the game finishes and the players get their shares. If player 1 rejects, the game moves to the third stage, in which the size of the ice-cream becomes 1/4 of the original size. In the third stage, each player randomly gets the ice-cream with probability half by flipping a coin, then game finishes.
Suppose each player maximizes expected size of the ice-cream she can get. Solve this game by backward induction, and derive a subgame perfect Nash equilibrium.

4. **Repeated games (20 points)**

Consider the repeated play of a prisoner’s dilemma game, whose payoffs are given in the following payoff matrix.

\[
\begin{array}{ccc}
1 & 2 & C & D \\
C & 3, 3 & 0, 5 \\
D & 5, 0 & 2, 2 \\
\end{array}
\]

(a) Suppose the game is played only three times. Then, how many subgames (including the entire game) do we have, and what is a subgame perfect equilibrium?

(b) Suppose the game is played indefinitely and players discount future payoffs with a common discount factor delta. Find the range of a discount factor which can sustain cooperation, i.e., repeated play of (C,C), by employing the trigger strategies.

5. **Auction (20 points)**

Consider the following first-price, sealed bid auction. There are three bidders, labeled \( i = 1, 2, 3 \). Bidder \( i \) has a valuation \( v_i \) for the good, and the bidders’ valuations are independently and uniformly distributed on \([0, 1]\). The bidders simultaneously submit their bids \( b_i, i = 1, 2, 3 \).

Compute the Bayesian Nash equilibrium. To do so, you can assume that 1) an equilibrium is symmetric and 2) an equilibrium strategy is a linear function, i.e., \( b_i = a + cv_i \) for some \( a \) and \( c \).

6. **Signaling game (20 points)**

See the game tree in Figure 3. Note that “Nature” first draws a type of player 1 either \( a \) or \( b \) with equal probability, and player 2 cannot observe player 1’s true type.
Find all perfect Bayesian Nash equilibria of this game.