Practice Exam

1. **Game tree (20 points)**

   See the game tree in Figure 1.

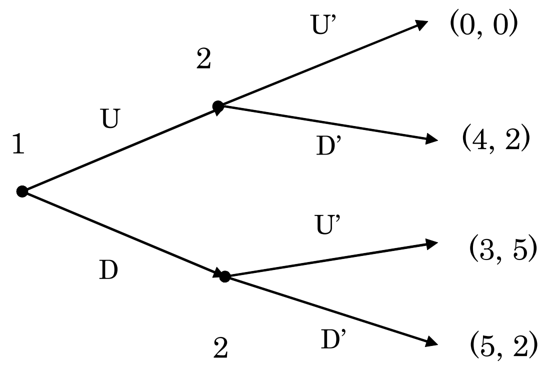

Figure 1

   (a) Solve this game by backward induction.

   (b) Translate this game into strategic-form (normal-form) and draw the payoff matrix.

   (c) Find all Nash equilibria.

2. **Subgame perfect Nash equilibrium (20 points)**

   See the game tree in Figure 2.

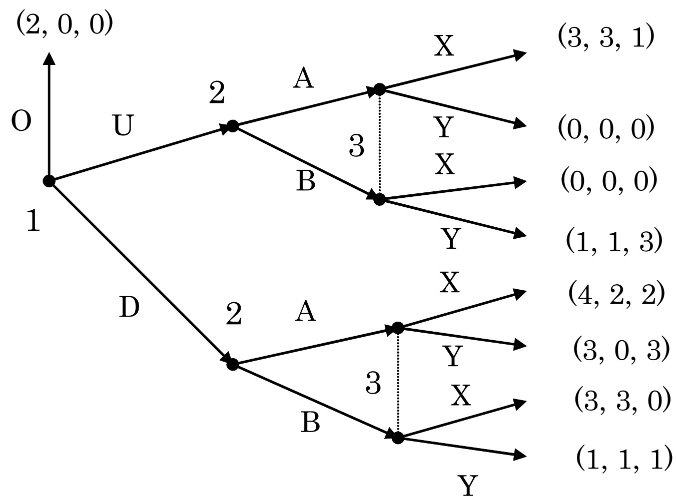

Figure 2

(a) How many information sets are there (including the one with the initial node)?

(b) How many subgames are there (including the entire game)?

(c) Find all subgame perfect equilibria.

3. **Alternative bargaining (20 points)**

Players 1 and 2 are bargaining over how to split the ice-cream of size 1. In the first stage, player 1 proposes a share $(x, 1-x)$ to player 2 where $0 \leq x \leq 1$. If player 2 accepts the player 1's offer, then the game finishes and the players get their shares. If player 2 rejects, the game moves to the second stage, in which the size of the ice-cream becomes half due to melting. In the second stage, player 2 proposes a share $(y, 1/2 - y)$ to player 1 where $0 \leq y \leq 1/2$. If player 1 accepts the offer, then the game finishes and the players get their shares. If player 1 rejects, the game moves to the third stage, in which the size of the ice-cream becomes $1/4$ of the original size. In the third stage, each player randomly gets the ice-cream with probability half by flipping a coin, then game finishes.

Suppose each player maximizes expected size of the ice-cream she can get. Solve this game by backward induction, and derive a subgame perfect Nash equilibrium.

4. **Repeated games (20 points)**

   Consider the repeated play of a prisoner's dilemma game, whose payoffs are given in the following payoff matrix.

$1\backslash 2$	C	D
C	$3,3$	$0,5$
D	$5,0$	$2,2$

   (a) Suppose the game is played only three times. Then, how many subgames (including the entire game) do we have, and what is a subgame perfect equilibrium?

   (b) Suppose the game is played indefinitely and players discount future payoffs with a common discount factor delta. Find the range of a discount factor which can sustain cooperation, i.e., repeated play of (C,C), by employing the trigger strategies.

5. **Auction (20 points)**

   Consider the following first-price, sealed bid auction. There are three bidders, labeled $i = 1, 2, 3$. Bidder i has a valuation v_i for the good, and the bidders' valuations are independently and uniformly distributed on $[0, 1]$. The bidders simultaneously submit their bids $b_i, i = 1, 2, 3$.

   Compute the Bayesian Nash equilibrium. To do so, you can assume that 1) an equilibrium is symmetric and 2) an equilibrium strategy is a linear function, i.e., $b_i = a + cv_i$ for some a and c.

6. **Signaling game (20 points)**

   See the game tree in Figure 3. Note that "Nature" first draws a type of player 1 either a or b with equal probability, and player 2 cannot observe player 1's true type.

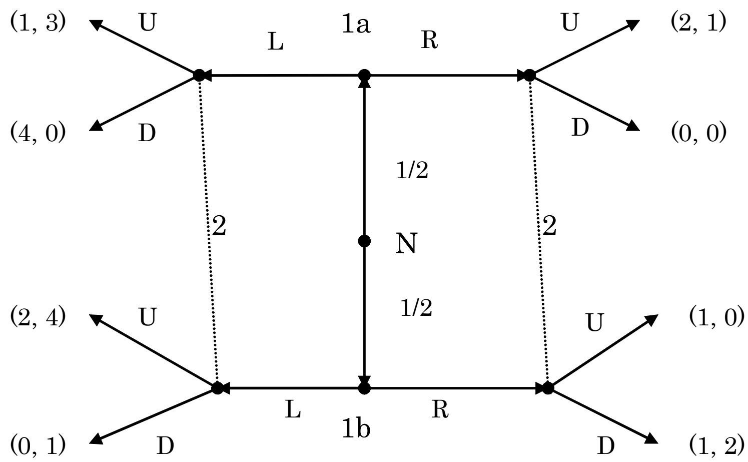

Figure 3

Find all perfect Bayesian Nash equilibria of this game.