

Problem Set 3
Due in class on May 27

1. Question 1 (20 points)

Consider an exchange economy with n consumers, and suppose their preferences are (strictly) monotonic. Then, show that a general equilibrium allocation \mathbf{x} is Pareto efficient.

Note: You should NOT rely on intuitive graphical analysis or MRS argument. Instead, try to give a formal proof of the first welfare theorem.

2. Question 2 (40 points)

A firm can hire at a rental price r and labor at a wage w . To produce anything at all requires one unit of capital, i.e. r is a fixed cost; this is sunk in the short run, but not sunk in the long run. If in a unit of time the firm employs L units of labor, and rents K units of capital (in addition to the one unit needed as a fixed cost), its output Q is given by one of the following production functions:

$$Q = K^{1/4}L^{1/8} \tag{1}$$

$$Q = \min(K, L) \tag{2}$$

For each production function, answer the following questions.

- (a) In the short run, the firm is committed to hire a fixed amount of capital $K(+1)$, and can vary its output Q only by employing an appropriate amount of labor L . Find algebraic expressions for the firm's short-run total, average, and marginal cost functions.
- (b) In the long run, the firm can vary both capital and labor. Find algebraic expressions for the firm's long-run total, average, and marginal cost functions.
- (c) To link the short-run and the long-run cost curves, take the short-run average cost curve, and for given Q , find the K (as a function of Q) that minimizes short-run average cost. Substitute this in the short-run average cost function, reducing it to a function of Q, r and w . Verify that it is the same as the long-run average cost function.

3. Question 3 (40 points)

Ann has an endowment of 200 units of good X and 5 units of good Y . Bob has an endowment of 100 units of good X and 5 units of good Y . Answer the following questions of each of the following two cases where U_A is Ann's utility function and U_B is Bob's utility function:

Case 1: $U_A(X_A, Y_A) = X_A Y_A$ and $U_B(X_B, Y_B) = X_B Y_B$

Case 2: $U_A(X_A, Y_A) = X_A + 30Y_A$ and $U_B(X_B, Y_B) = \min(X_B, 30Y_B)$

Answer the following questions algebraically but illustrate your answers in an Edgeworth box.

- (a) Describe the set of efficient allocations in this economy.
- (b) Describe the set of allocations which "Pareto-improve," i.e., make both individuals better off, on the endowment allocation.
- (c) Describe the "contract curve."
- (d) Find a competitive equilibrium price vector and allocation for this economy.