Problem Set 3
Due in class on May 27

1. Question 1 (20 points)
Consider an exchange economy with \( n \) consumers, and suppose their preferences are (strictly) monotonic. Then, show that a general equilibrium allocation \( \mathbf{x} \) is Pareto efficient.

Note: You should NOT rely on intuitive graphical analysis or MRS argument. Instead, try to give a formal proof of the first welfare theorem.

2. Question 2 (40 points)
A firm can hire at a rental price \( r \) and labor at a wage \( w \). To produce anything at all requires one unit of capital, i.e. \( r \) is a fixed cost; this is sunk in the short run, but not sunk in the long run. If in a unit of time the firm employs \( L \) units of labor, and rents \( K \) units of capital (in addition to the one unit needed as a fixed cost), its output \( Q \) is given by one of the following production functions:

\[
Q = K^{1/4}L^{1/8} \\
Q = \min(K, L)
\]

For each production function, answer the following questions.

(a) In the short run, the firm is committed to hire a fixed amount of capital \( K(+1) \), and can vary its output \( Q \) only by employing an appropriate amount of labor \( L \). Find algebraic expressions for the firm’s short-run total, average, and marginal cost functions.

(b) In the long run, the firm can vary both capital and labor. Find algebraic expressions for the firm’s long-run total, average, and marginal cost functions.

(c) To link the short-run and the long-run cost curves, take the short-run average cost curve, and for given \( Q \), find the \( K \) (as a function of \( Q \)) that minimizes short-run average cost. Substitute this in the short-run average cost function, reducing it to a function of \( Q, r \) and \( w \). Verify that it is the same as the long-run average cost function.
3. Question 3 (40 points)

Ann has an endowment of 200 units of good $X$ and 5 units of good $Y$. Bob has an endowment of 100 units of good $X$ and 5 units of good $Y$. Answer the following questions of each of the following two cases where $U_A$ is Ann’s utility function and $U_B$ is Bob’s utility function:

**Case 1**: $U_A(X_A, Y_A) = X_A Y_A$ and $U_B(X_B, Y_B) = X_B Y_B$

**Case 2**: $U_A(X_A, Y_A) = X_A + 30Y_A$ and $U_B(X_B, Y_B) = \min(X_B, 30Y_B)$

Answer the following questions algebraically but illustrate your answers in an Edgeworth box.

(a) Describe the set of efficient allocations in this economy.

(b) Describe the set of allocations which “Pareto-improve,” i.e., make both individuals better off, on the endowment allocation.

(c) Describe the “contract curve.”

(d) Find a competitive equilibrium price vector and allocation for this economy.