

Note on Consumer Preference

The consumer's preferences are captured by a preference relation \succsim (an "at-least-as-good" relation) defined on the commodity bundles in the consumption set $X \subset R_+^L$.

Definition: The preference relation \succsim on X is *rational* if it possesses the following two properties:

- (i) Completeness: For all $x, y \in X$, we have $x \succsim y$ or $y \succsim x$ (or both).
- (ii) Transitivity: For all $x, y, z \in X$, if $x \succsim y$ and $y \succsim z$, then $x \succsim z$.

In Economics, we often describe preference relations by means of a *utility function*. A utility function $u(x)$ assigns a numerical value to each element in X , ranking the elements of X in accordance with the individual's preferences.

Definition: A function $u : X \rightarrow R$ is a utility function representing preference relation \succsim if, for all $x, y \in X$,

$$x \succsim y \Leftrightarrow u(x) \geq u(y)$$

Proposition: A preference relation \succsim on X can be represented by a utility function only if it is rational.

We cannot use utility functions to describe choice behaviors of an "irrational" person. Can any rational preference relation be described by some utility function?

Proposition: Suppose a preference relation \succsim on X is rational and X is finite, i.e., X has a finite number of elements. Then there is a utility function $u(x)$ that represents \succsim .

Example (Lexicographic Preference Relation): Assume $X = R_+^2$. Define $x \succsim y$ if either " $x_1 > y_1$ " or " $x_1 = y_1$ and $x_2 \geq y_2$."

Definition: The preference relation \succsim on X is *continuous* if it is preserved under limits. That is, for any sequence of pairs $\{(x^n, y^n)\}_{n=1}^\infty$ with $x^n \succsim y^n$ for all n , we have $\lim_{n \rightarrow \infty} x^n \succsim \lim_{n \rightarrow \infty} y^n$.

Intuitively, continuity says that the consumer's preferences cannot exhibit "jumps" For example, if A is preferred to B , then the commodity bundles sufficiently close to A must also be preferred to B .

Proposition: Suppose that the rational preference relation \succsim on X is continuous. Then there is a continuous utility function $u(x)$ that represents \succsim .