Eco 601E Advanced Microeconomics II
Term: Spring (2nd), 2008
Lecturer: Yosuke Yasuda

Problem Set 6: Solutions
Due in class on July 22

1. Question 1 (30 points)

   Consider an alternating offer game (splitting \$1 between two risk neutral players) in which each player i loses c_i during each period of delay, rather than discounting her payoff by δ_i. Then, answer the following questions.

   (a) Show that if $c_1 < c_2$, then this game has a subgame perfect equilibrium in which player 1 always proposes $(1, 0)$.

   Hint: You can show that player 2 doesn't have an incentive to deviate. That is, declining 0 offer from player 1 cannot give player 2 non-negative payoff.

   (b) Show that if $c_1 = c_2 = c$, then for every value of z_1 with $c \leq z_1 \leq 1$ the game has a subgame perfect equilibrium in which player 1 always proposes $(z_1, 1 - z_1)$.

   Skipped.

2. Question 2 (40 points)

   In each period, two firms produce an identical good. The inverse demand curve for the good is $P = 241 - X$, where X is the total quantity produced by the two firms. Each firm has a constant marginal cost 1 of producing the good.

   (a) Find the Cournot Nash equilibrium of this game. What quantity would each firm produce? What would be the market price? What would be the profits of each firm?

   $x_1 = x_2 = 80$, $p = 81$, $\pi_1 = \pi_2 = 6400$

   (b) Suppose the firms formed a cartel: each firm produced the same output and maximized their joint profits. What quantity would each firm produce? What would be the market price? What would be the profits of each firm?

   $x_1 = x_2 = 60$, $p = 121$, $\pi_1 = \pi_2 = 7200$

   (c) Suppose that one firm produced the cartel output (from your answer in part (b)). Suppose that the other firm maximized profits knowing that the other firm would produce cartel output. How much would it produce? What would be the market price? What would be the profits of each firm?

   Suppose firm 1 deviates while firm 2 follows the cartel in (b).

   $x_1 = 90, x_2 = 60$, $p = 91$, $\pi_1 = 8100, \pi_2 = 5400$

(d) Now suppose that the firms interact indefinitely through time. They discount future profits at a discount factor δ. For what value of δ is there an equilibrium where firms follow the "trigger strategies" discussed in class, i.e., they produce cartel output as long as the other firm has always produced cartel output and otherwise they produce Cournot Nash output?

$\delta \geq \frac{9}{17}$

3. Question 3 (30 points)

Consider a "common-value auction" with two players, where the value of the object being auctioned is the same for both players. Call this value V and suppose that $V = v_1 + v_2$, where v_i is independently and uniformly distributed between 0 and 1. The players simultaneously submit bids, b_1 and b_2. If player i bids higher than does player j, then player i wins the auction and gets the payoff $V - b_i$ whereas player j gets 0. What is a symmetric Bayesian Nash equilibrium in this game?

Hint: You can assume the equilibrium strategy is linear, i.e., $b_i = \alpha + \beta v_i$**. It should be noticed that the expected value of** v_j **conditional on** $b_i > b_j$ **is not necessarily** 0.5**.**

The payoff for player i is expressed as

$$u_i = \begin{cases} V - b_i & \text{if } b_i > b_j \\ 0 & \text{if } b_i < b_j \end{cases} .$$

Assume player j takes a linear strategy $b_j = \alpha + \beta v_j$*. Then, player i's expected payoff by submitting* b_i *given her signal* v_i *becomes the following.*

$$\begin{aligned} E[u_i|v_i] &= \Pr(b_i > b_j) \times E[Y - b_i|b_i > b_j, v_i] \\ &= \Pr(\frac{b_i - \alpha}{\beta} > v_j)\{(v_i + E[v_j|b_i > b_j, v_i]) - b_i\} \\ &= \frac{b_i - \alpha}{\beta}(v_i + \frac{b_i - \alpha}{2\beta} - b_i) \end{aligned}$$

So, the optimal strategy for player i is derived by solving the next maximization problem.

$$\max_{b_i} \frac{b_i - \alpha}{\beta}(v_i + \frac{b_i - \alpha}{2\beta} - b_i)$$

The first order condition is:

$$
\begin{aligned}
0 &= \frac{1}{\beta}(v_i + \frac{b_i - \alpha}{2\beta} - b_i) + (\frac{1}{2\beta} - 1)\frac{b_i - \alpha}{\beta} \\
\Rightarrow b_i &= \frac{\beta}{2\beta - 1}v_i + \frac{(\beta - 1)\alpha}{2\beta - 1}
\end{aligned}
$$

It is clear that $\frac{\beta}{2\beta-1} = \beta$ *and* $\frac{(\beta-1)\alpha}{2\beta-1} = \alpha$ *when* $\beta = 1$ *and* $\alpha = 0$. *Since the game is symmetric, we can obtain the same condition for player* j. *Thus, there exists a symmetric Bayesian Nash equilibrium, which is* $b_i = v_i$ *for* $i = 1, 2$.